6.01 Midterm 2 Spring 2017

Name: Solutions **Section Number:**

Kerberos (Athena) name:

Section	Design Lab Time
1:	Wednesday 9:30am
2:	Wednesday 2:00pm
3:	Thursday 2:00pm

Please WAIT until we tell you to begin.

During the exam, you may refer to any written or printed paper material.
You may NOT use any electronic devices (including calculators, phones, etc).

If you have questions, please **come to us at the front** to ask them.

Enter all answers in the boxes provided.

Extra work may be taken into account when assigning partial credit,
but only work shown on pages with QR codes will be considered.

Question 1: 17 Points
Question 2: 16 Points
Question 3: 14 Points
Question 4: 14 Points
Question 5: 16 Points
Question 6: 17 Points
Total: 94 Points

1 *Valar Rhobotus* (17 Points)

In a deleted scene from the first season of *Game of Thrones*, Arya and Jon come across an ancient Wildling robot fashioned from sharp sticks and mammoth skins. They then scheme to use the robot to take over the Seven Kingdoms.

1.1 System Functional

Along with the robot, they unearth a stone tablet which explains that, given an input $a[n]$, the robot will output

$$b[n] = a[n-1] + \frac{1}{2}b[n-1]$$

Determine the system functional relating the input signal A to the output signal B:

$$\frac{B}{A} = \frac{\mathcal{R}}{1 - \frac{1}{2}\mathcal{R}}$$

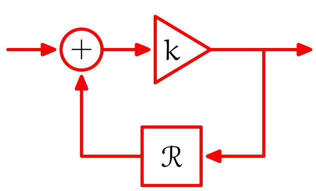
1.2 Control

Arya has a dream in which she sees a vision of a controller they can use to control the robot. However, upon waking, all she remembers is that the controller's system functional was:

$$\frac{k}{1 - \mathcal{R}k}$$

where k is an adjustable gain.

Draw a block diagram for the controller in the box below, using only finitely many gains, delays, and adders:



1.3 Gain

Jon and Arya decide to build an LTI model to see how their robot will behave when controlled using this controller.

Since Jon and Arya both took 6.01 (background in EECS was, of course, required of all noble children at the time), they are able to think about choosing the gain k for optimum behavior.

Solving, they find that the entire system (controller, plant, and sensor) has the following system functional:

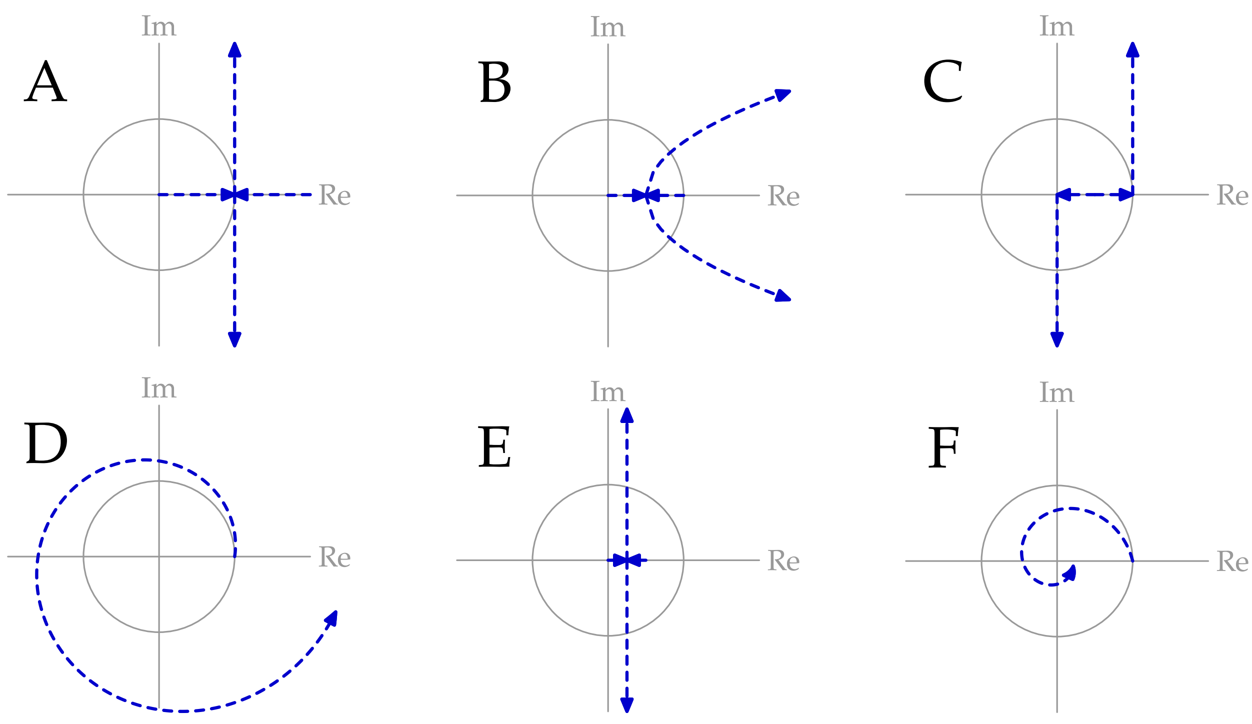
$$\frac{Y}{X} = \frac{k\mathcal{R}}{1 - \frac{1}{2}\mathcal{R} + \frac{k}{2}\mathcal{R}^2}$$

What value of k leads to the fastest convergence of the system? $\frac{1}{8}$

What is the maximum value of k for which the system is stable? 2

1.4 Root Locus

Arya then recalls some cryptic images from her dream, and realizes that (spoiler alert) they were root locus plots. Which of the following root locus plots best describes the system from part 1.3? Note that the arrows below are drawn in the direction of increasing k.



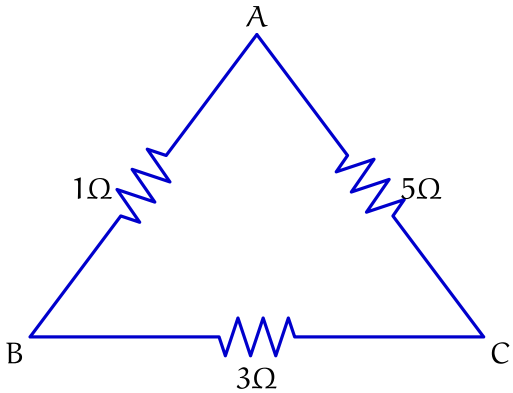
Root Locus that Best Describes the System (circle one): **A** **B** **C** **D** **E** **F**

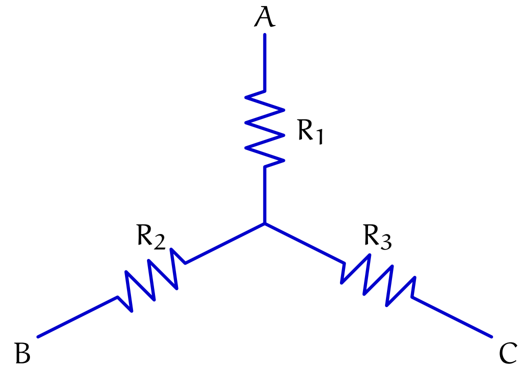
2 Circuit Similarity (16 Points)

2.1 Part I

Find R_1, R_2, and R_3 such that the resistance between all pairs of the nodes A, B, and C is the same in the right-hand circuit as in the left-hand circuit.

If there is not enough information to find such values, enter NEI (for "Not Enough Information") in each box, and explain briefly.



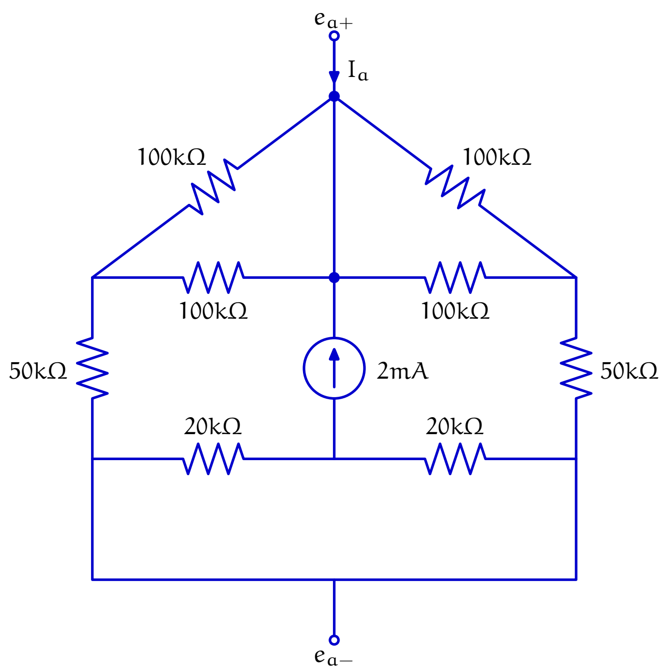


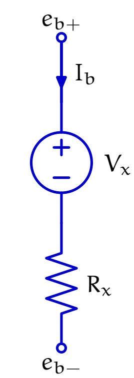
$R_1 =$ $\frac{5}{9}\Omega$ $R_2 =$ $\frac{3}{9}\Omega$ $R_3 =$ $\frac{15}{9}\Omega$

If there is not enough information to solve, explain briefly (1-2 sentences):

2.2 Part II

Consider the following circuits:





Let $V_a = e_{a+} - e_{a-}$, and let $V_b = e_{b+} - e_{b-}$. Determine values of V_x and R_x in circuit B such that the relationship between V_a and I_a is identical to the relationship between V_b and I_b.

If there is not enough information to find such values, enter `NEI` (for "Not Enough Information") in each box, and explain briefly.

V_x = 100V R_x = 50kΩ

If there is not enough information to solve, explain briefly (1-2 sentences):

3 Sweet Ride (14 Points)

You are looking for a new car, and you see the following advertisement for a car for sale on the web:

You're interested in buying it, but the high mileage and the fact that the car's model name is mis-pelled in the title leave you somewhat reluctant to purchase it without investigating further. You decide that the car is either "good" or "bad", and, from what you can tell, the probability that it is "good" is 0.3.

A garage offers to test it for you. Regardless of the condition of the car, their test will say either "pass", "fail", or come back "inconclusive":

- A "good" car will pass with probability 0.8 and will fail with probability 0.1.
- A "bad" car will fail with probability 0.6 and will give an inconclusive result with probability 0.3.

1. What is the probability that a "good" car will produce an inconclusive test?

0.1

2. From what you can tell before doing any tests, what is the overall probability that the car will fail the test?

0.45

3. From what you can tell before doing any tests, what is the overall probability that the car will pass the test?

0.31

4. From what you can tell before doing any tests, what is the overall probability that the test will be inconclusive?

0.24

5. You decide to pay for the test, and you are not told the exact results; only that the test was not "inconclusive". Given this information, what is the probability that the car failed the test?

$\frac{45}{76}$

6. Given this same information, what is the probability that the car is "good"?

$\frac{27}{76}$

7. If, instead, you are told that the car passed the test, what is the probability that the car is "good"?

$\frac{24}{31}$

4 Going Offline (14 Points)

After seeing the limitations of linear approximations of nonlinear systems in lab, Diana Verge decides that she wants to spend her spring break studying *nonlinear* time-invariant systems. Diana starts by considering the following system:

$$y[n] = y[n-1] + x[n] + (x[n-1])^2 - x[n-2]$$

However, she quickly runs into trouble thinking about this system, and so comes to you for help.

4.1 Outputs

Assume that the system is initially at rest.

Below are the first four samples of the system's output when the input is the unit sample signal ($x[n] = \delta[n]$):

$y[0] = 1$ $y[1] = 2$ $y[2] = 1$ $y[3] = 1$

Now suppose the system's input is the unit sample signal delayed by one time step ($x[n] = \delta[n-1]$). What are the first 4 samples of its output $y[n]$?

$y[0] =$ 0 $y[1] =$ 1 $y[2] =$ 2 $y[3] =$ 1

Now suppose the system's input is the unit sample signal scaled by 2 ($x[n] = 2\delta[n]$). What are the first 4 samples of its output $y[n]$?

$y[0] =$ 2 $y[1] =$ 6 $y[2] =$ 4 $y[3] =$ 4

4.2 NLSystem

Diana's sister Connie decides that it would be great to have a Python class designed for simulating these kinds of systems. She implements the following class using 6.01's simulation framework:

```
class NLSystem:
    def __init__(self, f, x_init_vals, y_init_vals):
        self.f = f
        self.initial_state = (x_init_vals, y_init_vals)  # initial state of the system

    def calculate_step(self, state, inp):
        (past_x, past_y) = state
        out = self.f(inp, past_x, past_y)

        past_x = [inp] + past_x[:-1]
        past_y = [out] + past_y[:-1]

        return ((past_x, past_y), out)  # a tuple containing the updated state, and the output
```

In the box below, define a new instance of `NLSystem` to represent the system from the first part of this problem, assuming the system starts at rest. Store the resulting instance in a variable called `system`.

```
system = NLSystem(lambda i,px,py: py[0] + i + px[0]**2 - px[1], [0,0], [0])
```

4.3 The Great Debate

Connie and Diana soon get in an argument. Connie believes that her `NLSystem` class can also be used to simulate all LTI systems. However, Diana argues that because it was built for nonlinear systems, it cannot be used to simulate linear systems. It is up to you to settle the argument. Can the `NLSystem` class be used to simulate all LTI systems, some but not all LTI systems, or no LTI systems?

All, Some, or None: All

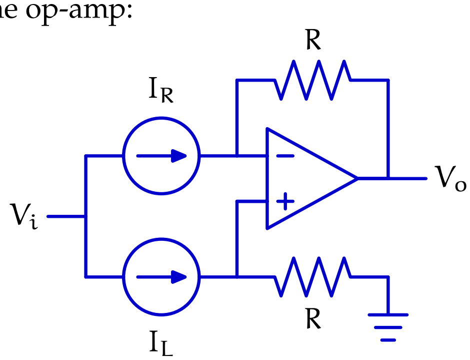
Provide a brief explanation:

Whenever the function `f` passed in to the `NLSystem` constructor returns a linear combination of the `past_x` and `past_y` values, the `NLSystem` will simulate an LTI system. Because `NL-System` can simulate all time-invariant systems, and the set of all LTI systems is a subset of time-invariant systems, `NLSystem` can simulate any LTI system.

5 We Can Work It Out (16 Points)

5.1 Design 1

Ben Bitdiddle decides that many people's solutions to the light tracking problem from Design Lab 7 used too many op-amps, and so he proposes the circuit below for generating a voltage proportional to $I_L - I_R$, which only uses one op-amp:



Make the ideal op-amp assumption and ignore the output limitations of the op-amp, and assume that the "eyes" can *always* be modeled as current sources, regardless of the voltage drop across them.

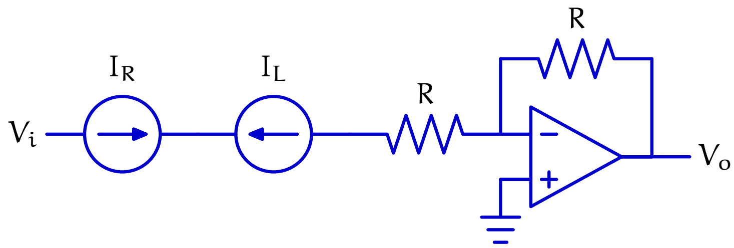
Will Ben's design result in a voltage V_o that is proportional to $I_L - I_R$? (Circle one): **Yes** **No**

If yes, write an expression for V_o in terms of V_i, I_R, I_L, and/or R. If not, briefly explain why:

$V_o = RI_L - RI_R = R(I_L - I_R)$

5.2 Design 2

Ben's friend Lem E. Tweakit decides that there is no need for the op-amp to do subtraction. Instead, he can what he knows about basic circuit laws to subtract the two currents directly, and then use an op-amp to produce a voltage proportional to the resulting current. Lem comes up with the following circuit:



Make the ideal op-amp assumption and ignore the output limitations of the op-amp, and assume that the "eyes" can *always* be modeled as current sources, regardless of the voltage drop across them.

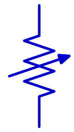
Will Lem's design result in a voltage V_o that is proportional to $I_L - I_R$? (Circle one): **Yes** **No**

If yes, write an expression for V_o in terms of V_i, I_L, I_R, and/or R. If not, briefly explain why:

Two current sources with different currents, connected as shown above, will violate KCL. In the case where $I_L = 2A$ and $I_R = 4A$, for example, the KCL equation at that node would be $2A = -4A$, which cannot be solved.

5.3 Vive La Resistor

You decide to give up on these photosensors, and to use a *photoresistor* as a sensor instead. A photoresistor's resistance decreases as the intensity of the light falling on it increases. In a schematic, a photoresistor is commonly drawn like this:

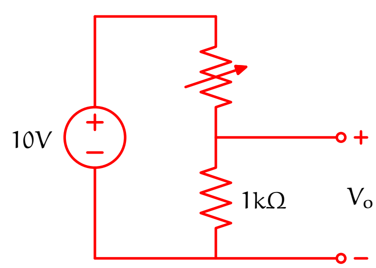


For this problem, consider a photoresistor whose resistance in ambient light (i.e., when there is no light source nearby) is 10kΩ, and whose resistance with a light source 1 foot away is 1kΩ.

Design a circuit using a single photoresistor (and any other resistors, voltage sources, current sources, or op-amps you need) to generate a voltage that varies as the light falling on the sensor changes, **with more light leading to higher voltages**. For full credit, ensure that there is at least a 3V difference between the output voltage in ambient light versus the output voltage with a light source 1 foot away.

Draw your schematic in the box below (we have already drawn the photoresistor for you). **Clearly label your output voltage as V_o, and write expressions for the value of V_o under the different lighting conditions described above.**

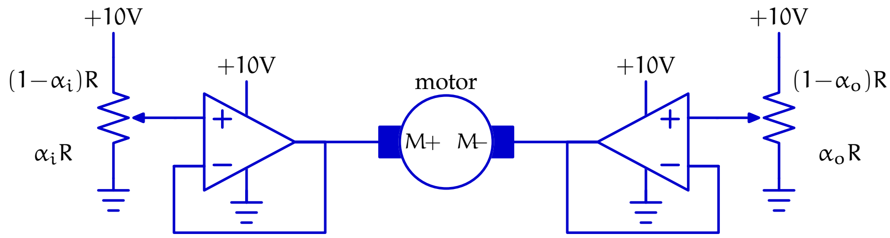
One possible solution:



V_o in ambient light:	$\frac{10}{11}$V
V_o with light 1ft away:	5V

6 Angular Position Controller (17 Points)

The following figure shows an angular position controller of the type that you constructed in Design Lab 6 / Software Lab 7. When a human turns the left potentiometer (the *input pot*), the motor will turn the right potentiometer (the *output pot*) so that the angle of the output potentiometer (which is proportional to α_o) is approximately equal to the angle of the input potentiometer (which is proportional to α_i).



The dependence of the pot resistances on shaft angle is given in terms of α, which varies from 0 (most clockwise position) to 1 (most counterclockwise position). The resistance of the lower part of the pot is αR and that of the upper part is $(1-\alpha)R$.

Notice that if $\alpha_i > \alpha_o$, then the voltage drop across the motor ($V_{M+} - V_{M-}$) is positive, and the motor turns counterclockwise (so as to increase α_o) — i.e., **positive motor voltage → counterclockwise rotation**.

6.1 Motor voltage

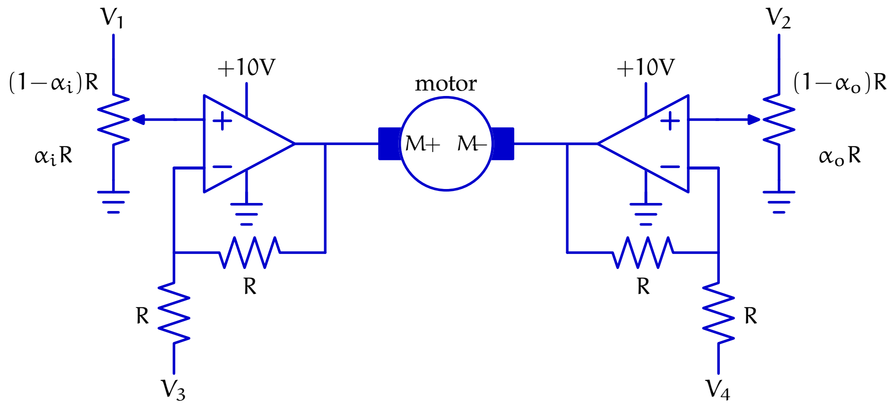
Determine an expression for $V_M = V_{M+} - V_{M-}$ in terms of α_i, α_o, and R. Enter your expression in the box below.

$V_M =$ $(\alpha_i - \alpha_o) \times 10V$

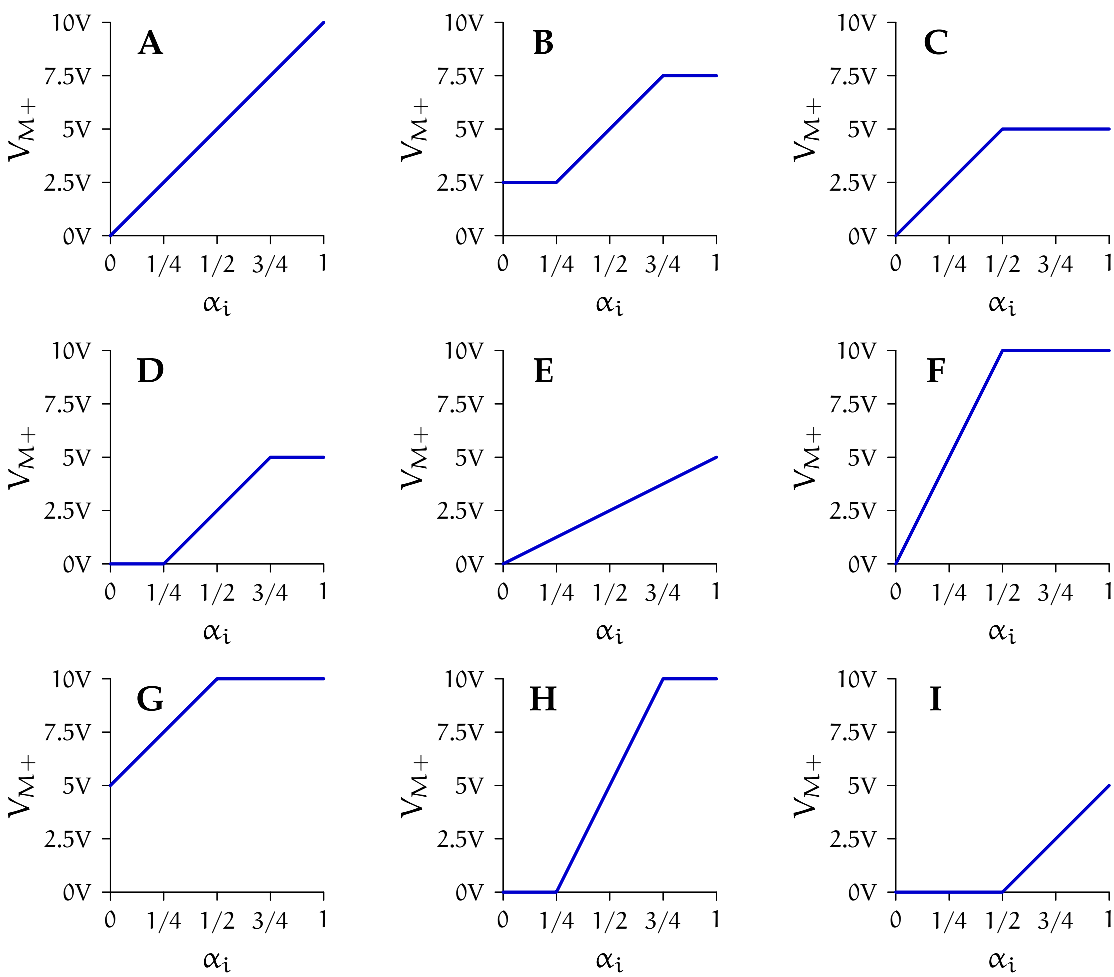
6.2 Power supply limitations

The output voltage of an op amp must lie within the range of voltages provided by the power supply. For this problem, assume that the output voltages of all op amps are between 0V and 10V.

Consider consequences for the following circuit, where V_1, V_2, V_3, and V_4 represent constant voltage sources. Assume that the motor and pots are the same as those in the previous circuit.



The following plots show relations between α_i and V_{M+} that could result for different values of V_1 and V_3. Use these plots to answer the questions on the next page.



Part a. Determine which (if any) of the plots on the previous page show the relation between α_i and V_{M+} when $V_1 = 10$, $V_3 = 0$.

A-I or **none**: F

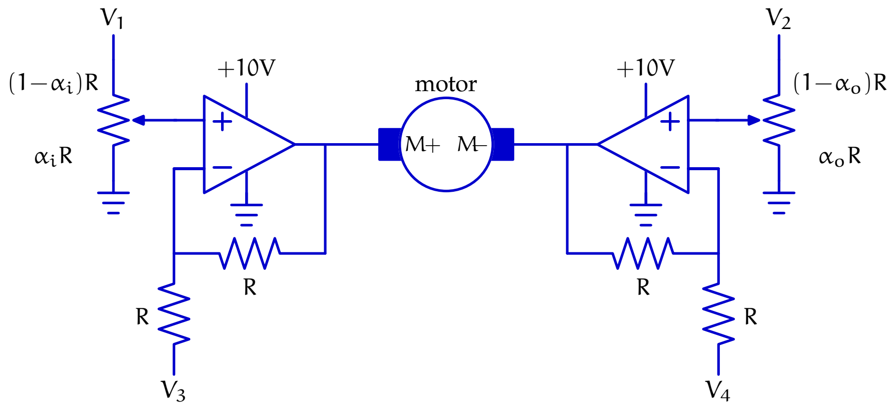
Part b. Determine which (if any) of the plots on the previous page show the relation between α_i and V_{M+} when $V_1 = 10$, $V_3 = 5$:

A-I or **none**: H

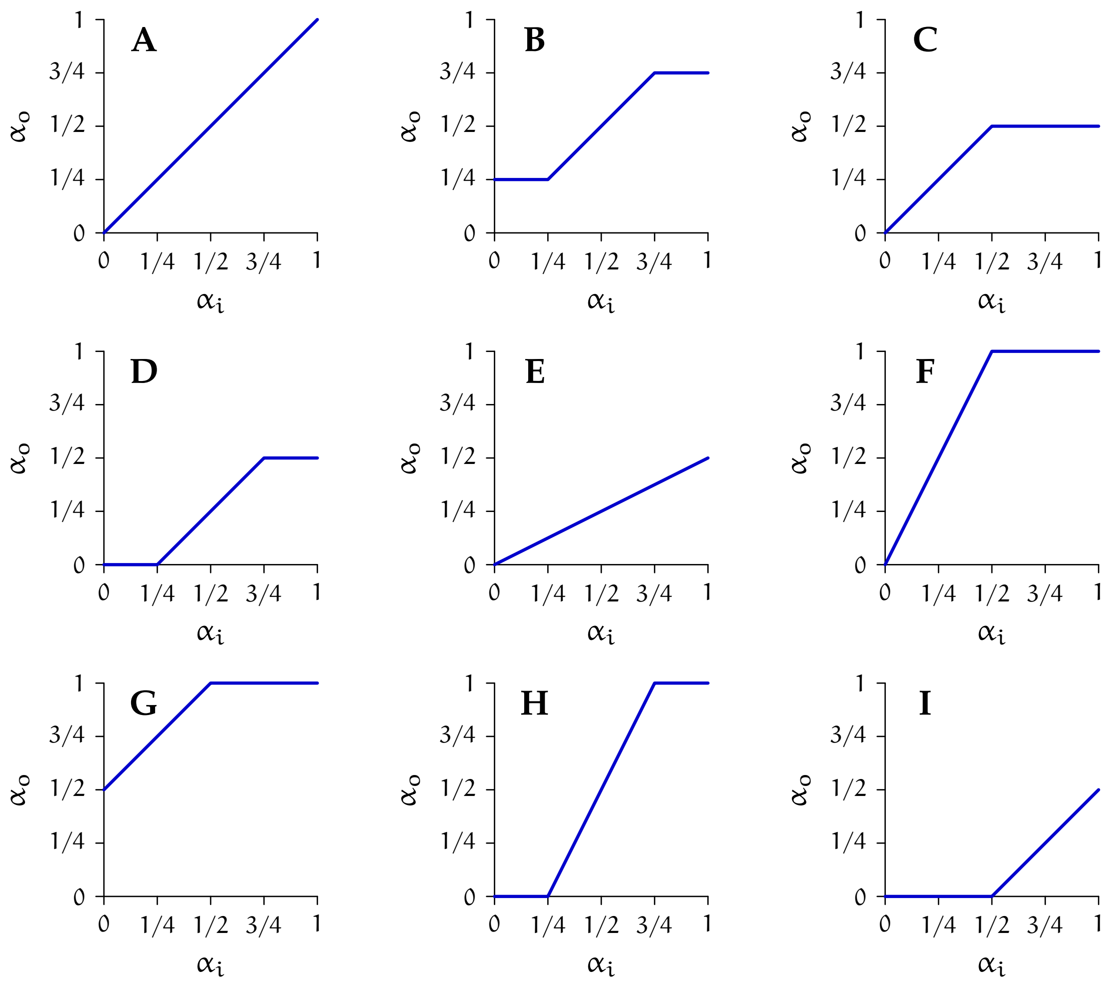
Part c. Determine which (if any) of the plots on the previous page show the relation between α_i and V_{M+} when $V_1 = 5$, $V_3 = 0$:

A-I or **none**: A

When the input pot is turned, the left op-amp will apply a voltage to the motor and the motor will turn until it reaches a new stable position.



The following plots show the relation between α_i and the resulting stable value of α_o, **when** α_i **is slowly, smoothly adjusted from 0 to 1**. Use these plots to answer the questions on the next page.



Part d. Determine which (if any) of the plots on the previous page show a possible relation between α_i and α_o when $V_1 = 10$, $V_2 = 10$, $V_3 = 0$, and $V_4 = 0$.

A-I or **none**: C

Part e. Determine which (if any) of the plots on the previous page show a possible relation between α_i and α_o when $V_1 = 10$, $V_2 = 10$, $V_3 = 5$, and $V_4 = 5$.

A-I or **none**: B

Part f. Determine which (if any) of the plots on the previous page show a possible relation between α_i and α_o when $V_1 = 10$, $V_2 = 10$, $V_3 = 5$, and $V_4 = 0$.

A-I or **none**: D