

# 6.01 Midterm 2

Fall 2016

**Name:**

**Section Number:**

**Kerberos (Athena) name:**

Section	Design Lab Time
1:	Thursday 9:30am
2:	Thursday 2:00pm

**Please WAIT until we tell you to begin.**

During the exam, you may refer to any written or printed paper material.  
**You may NOT use any electronic devices (including calculators, phones, etc).**

If you have questions, please **come to us at the front** to ask them.

**Enter all answers in the boxes provided.**

Extra work may be taken into account when assigning partial credit.

**Question 1:** 11 Points

**Question 2:** 10 Points

**Question 3:** 16 Points

**Question 4:** 12 Points

**Question 5:** 15 Points

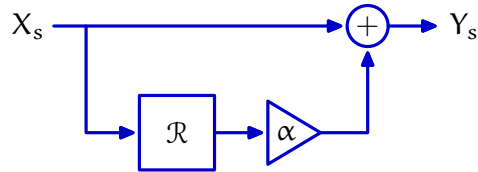
**Question 6:** 10 Points

**Total:** 74 Points

## 1 Decoding (11 Points)

### 1.1 Sender

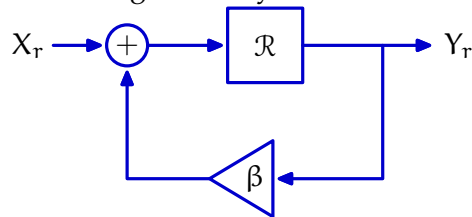
Consider the following system, designed to take in an input signal, and to output a different, coded form of the signal for transmission.



For what values of  $\alpha$  is this system stable? Enter an equation or inequality in the box below:

### 1.2 Receiver

Consider also this system, which is designed to try to decode the signal from the sender.



For what values of  $\beta$  is this system stable? Enter an equation or inequality in the box below:

### 1.3 Combination

Now consider the case when the two systems are connected in cascade (so that  $Y_s = X_r$ ).

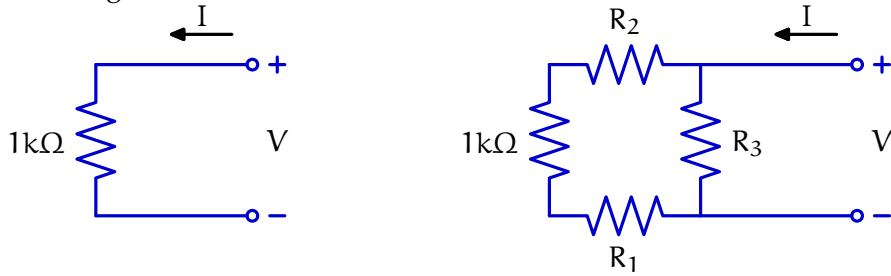
For what values of  $\alpha$  and  $\beta$  is this system stable? Enter equations and/or inequalities in the box below:

Assume a fixed, nonzero value of  $\alpha$ . For what values of  $\beta$  is the unit sample response of the system a delayed unit sample signal? Enter equations and/or inequalities in the box below, or enter None if no value of  $\beta$  would cause this behavior:

Under the conditions from the previous question, what would be the output of the system when the input is the sequence  $[1, 3, 5, 6, 7, 0, 0, 0, \dots]$ ? Calculate the first 6 samples of the systems output in this case, and enter them in the box below:

## 2 The Pièces de Résistance (10 Points)

Consider the following two resistive networks:



Is it possible to choose finite, nonzero positive resistances  $R_1$ ,  $R_2$ , and  $R_3$  such that these two circuits exert the same constraints on the current  $I$  and the voltage  $V$ ? If so, enter a constraint on  $R_1$ ,  $R_2$ , and  $R_3$  that ensures this condition, and provide one specific example of  $R_1$ ,  $R_2$ , and  $R_3$  that satisfy this condition. You can use  $+$  to represent series combinations and  $||$  to represent parallel combinations; you do not need to simplify completely. If this is not possible, briefly explain why.

Constraint, or explanation if not possible:

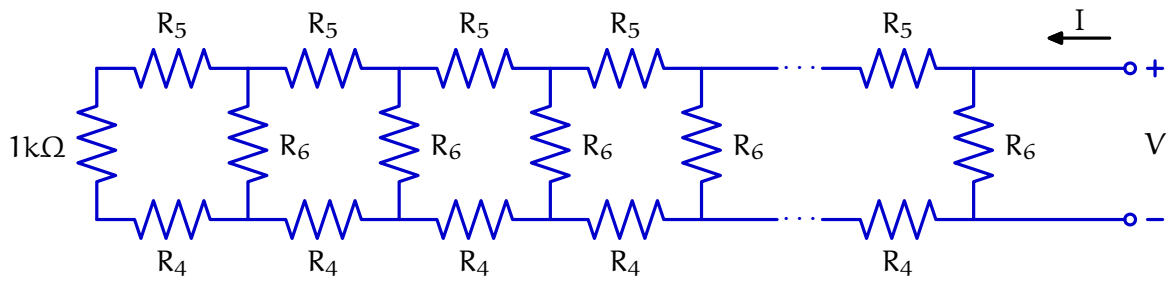
Specific values if possible:

$R_1 = \boxed{\phantom{000}}$

$R_2 = \boxed{\phantom{000}}$

$R_3 = \boxed{\phantom{000}}$

Now consider the following network:



Is it possible to choose finite, nonzero, positive resistances  $R_4$ ,  $R_5$ , and  $R_6$  such that this circuit and the original  $1\text{k}\Omega$  resistor exert the same constraints on the current  $I$  and the voltage  $V$ ? If so, enter a constraint on  $R_4$ ,  $R_5$ , and  $R_6$  that ensures this condition, and provide one specific example of  $R_4$ ,  $R_5$ , and  $R_6$  that satisfy this condition. You can use  $+$  to represent series combinations and  $||$  to represent parallel combinations; you do not need to simplify completely. If this is not possible, briefly explain why.

Constraint, or explanation if not possible:

Specific values if possible:

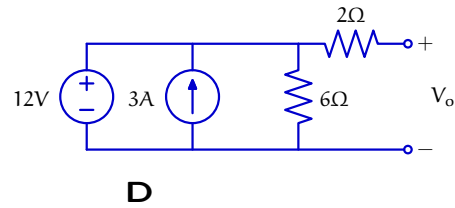
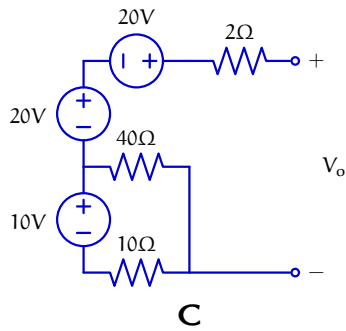
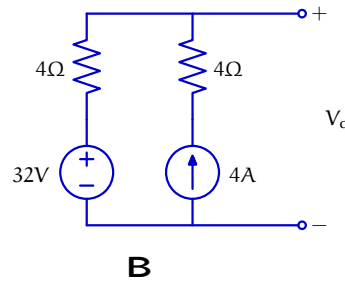
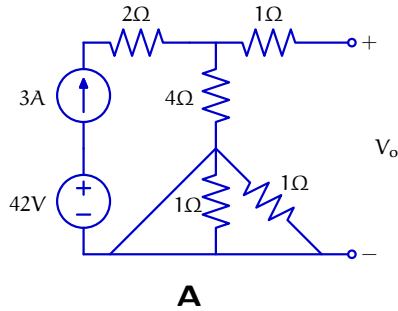
$R_4 =$

$R_5 =$

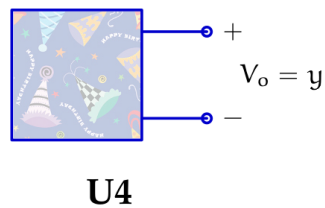
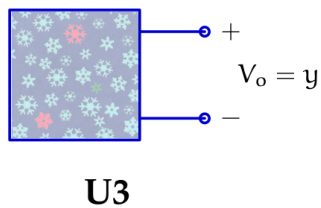
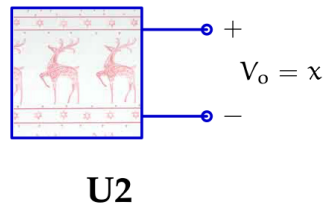
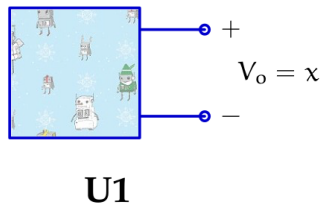
$R_6 =$

### 3 There's No Time Like the Presents (16 Points)

Deciding to get an early start on holiday gifts for friends and family, and wanting these gifts to be personal and meaningful, you decide to wrap up some 6.01 circuits to give as gifts. You build and wrap the following circuits, labeled A through D:



Unfortunately, you fall asleep before you can label each package. Luckily, though, the  $V_o$  port is still visible on each package. You grab your voltmeter and measure  $V_o$  on each package, labeled U1 through U4 below. You find that U1 and U2 measure the same value  $x$  (measured in Volts), and that U3 and U4 measure  $y$  (also in Volts), where  $y < x$ :



### 3.1 Single Measurements

What are the values of  $x$  and  $y$  (Volts)? Recall that  $y < x$ .

$$x = \boxed{\phantom{000}} \qquad y = \boxed{\phantom{000}}$$

Based on these measurements for  $V_o$ , which of the circuits (A, B, C, and/or D) could be in the packages labeled U1 and U2?

Enter one or more letters:

Which of the circuits (A, B, C, and/or D) could be in the packages labeled U3 and U4?

Enter one or more letters:

### 3.2 Parallel

You are discouraged, thinking to yourself that, since two circuits measured  $x$  and two measured  $y$ , you will never be able to tell them apart with just a voltmeter.

However, just before you tear apart the packages, your roommate saves you from wasting that valuable wrapping paper by suggesting that you can figure out which circuit is which by connecting the different boxes together and measuring the output voltage of the combined circuit. You connect two pairs of circuits in parallel by wiring their positive terminals together, and wiring their negative terminals together. You then measure the voltage drop between these terminals.

You take the following measurements:

- When U2 and U3 are connected in parallel, the output voltage is 18V.
- When U1 and U4 are connected in parallel, the output voltage is 32V.

Which circuit is in each package? Enter one letter (A, B, C, or D) in each box:

U1:	<input type="text"/>	U2:	<input type="text"/>
U3:	<input type="text"/>	U4:	<input type="text"/>

Briefly (1-2 sentences) explain your strategy for partial credit:

## 4 (Pet) Food For Thought (12 Points)

Generic Pet Food Inc. (GPF) has a web site that sells pet food for cats, dogs, and snakes. GPF is entirely dependent on online internet ads it places to drive customers to its web site. The ads are very effective: if clicked (bringing the customer to GPF's web site), ALL of those click-throughs result in GPF pet food purchases and customers. However, GPF would like to know what ad clicks are most effective in forming its customer base.

GPF has six kinds of ads: cat, dog, and snake photos (still pictures), and cat, dog, or snake movies (videos). Note that GPF found it was not a good idea to have any movie or photo advertisements with more than one kind of animal in them, e.g., cats and snakes do not mix well, so they don't have any such mixed ads.

GPF hires Animal Advertising Analytics (AAA), a startup that originally specialized in big data analytics on internet cat videos. AAA has expanded to dogs and snakes, and starts to send ad click facts as it discovers them to GPF. In the questions below, facts are revealed by AAA one at a time in the sequence below. Based on the knowledge and data you have up to that part, answer the question. If you do not (yet) have enough information, write "NEI" (not enough information). Do NOT use later facts from AAA to go back and answer earlier questions.

### 4.1 Part 1

AAA tells us that fully 40% of ad clicks to GPF come from cat videos (movies). Also, snake clicks (movies and photos) are 30% of ad clicks to GPF.

1. What is the probability of a snake photo being the source of a customer?

### 4.2 Part 2

AAA finds that movie ads are more effective: 80% of clicks for GPF customers are due to movies.

1. Given that a customer clicked a snake ad, what is the probability that it was a snake movie (as opposed to a snake photo)?



### 4.3 Part 3

AAA sends a new piece of data: given that a customer clicked a movie, the probability that it was a snake movie is 0.3.

1. Given that a customer clicked a movie, what is the probability it was a cat video?

2. Given that a customer clicked a movie, what is the probability it was a dog video?

3. what is the (overall) probability that a customer clicked a snake photo?

4. What is the (overall) probability that a customer clicked a cat photo?

### 4.4 Part 4

Perhaps surprisingly, AAA reports that if a photo of a dog or cat is clicked by a GPF customer, it is 13 times more likely to be a dog photo than a cat photo.

1. What is the (overall) probability that a customer clicked a dog photo?

2. What is the probability that a GPF customer is a cat lover, i.e., that a GPF customer clicked a picture or a movie of a cat?

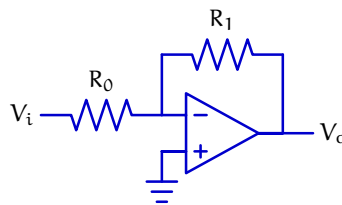
## 5 Ohm Sweet Ohm (15 Points)

### 5.1 Circuit Analysis

For each of the following circuits, determine the gain  $G = \frac{V_o}{V_i}$ . Make the ideal op-amp assumption, and ignore the output limitations of the op-amps. For now, assume that nothing is connected to the circuits other than what is drawn. All labeled voltages are measured relative to the same ground node.

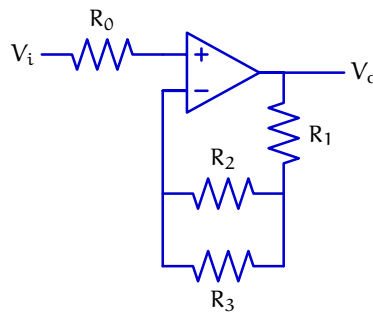
Feel free to use the symbols  $+$  and  $\parallel$  to represent series and parallel combinations of resistors, respectively, rather than solving completely.

#### Circuit 1



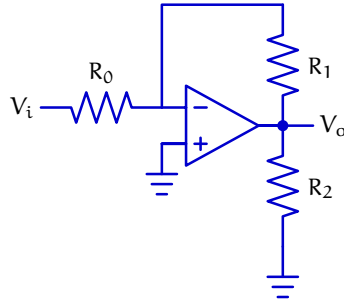
$$G = \frac{V_o}{V_i} = \boxed{\phantom{000000}}$$

#### Circuit 2



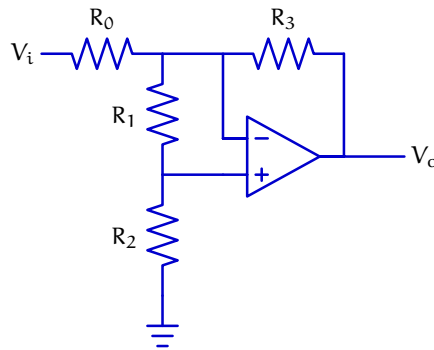
$$G = \frac{V_o}{V_i} = \boxed{\phantom{000000}}$$

**Circuit 3**



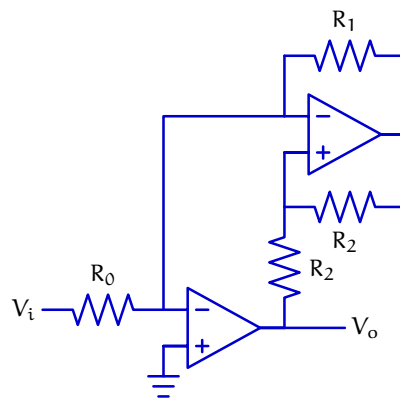
$$G = \frac{V_o}{V_i} = \boxed{\phantom{000000}}$$

**Circuit 4**



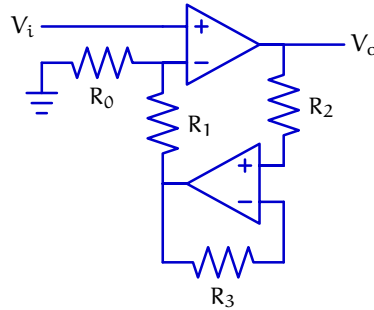
$$G = \frac{V_o}{V_i} = \boxed{\phantom{000000}}$$

**Circuit 5**



$$G = \frac{V_o}{V_i} = \boxed{\phantom{000000}}$$

## Circuit 6

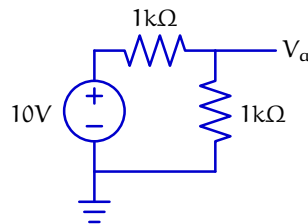


$$G = \frac{V_o}{V_i} = \boxed{\phantom{000000}}$$

## 5.2 Persistence

In the previous section, we characterized each of the op-amps by a gain  $G$  (the ratio  $V_o/V_i$  in the absence of all other connections).

Now suppose that we have a voltage  $V_a$  (measured relative to ground) created with a voltage divider, as shown below:



Imagine connecting this circuit to each of the circuits from the previous section (circuits 1-6) so that the wire labeled  $V_a$  in this diagram is connected to the wire labeled  $V_i$  in the diagrams above.

For which of these circuits, if any, will the new output voltage **be something other than**  $5V \times G$ ? Enter the numbers of those circuits in the box below, or enter None if none of the circuits have this property.

Circuit Numbers or None:

## 6 Temperature Sensor (10 Points)

Alyssa P. Hacker has built a circuit where the voltage measured at a particular point varies with the temperature, and she is interested to hook it up to her computer so that she can get a nice display of the temperature on her desktop.

However, she is faced with a challenge: the mapping from temperatures to voltages is a complicated function, and there seems to be no analytical solution.

To combat this, Alyssa comes up with the clever idea to sample some voltages at known temperatures (yielding a mapping from some fixed set of temperatures to the voltages measured at those temperatures), and to use these values to guess at the temperature based on a measured voltage.

Alyssa takes her measurements at temperatures  $t_1, t_2, \dots, t_n$ , and measures voltages  $v_1, v_2, \dots, v_n$ . She uses these to create a list of tuples:

```
MEASUREMENTS = [(t1, v1), (t2, v2), ..., (tn, vn)]
```

Alyssa's strategy is to measure a new voltage  $v$ , find the values in the MEASUREMENTS list that are closest to that measurement above and below (call them  $V_+$  and  $V_-$ ) and note their corresponding temperatures (call them  $T_+$  and  $T_-$ ), and estimate the corresponding temperature  $t$  via *linear interpolation* between  $T_+$  and  $T_-$ :

$$t = \left( \frac{w_2}{w_1 + w_2} \right) T_- + \left( \frac{w_1}{w_1 + w_2} \right) T_+$$

where  $w_1$  is the distance between  $V_-$  and  $v$ , and  $w_2$  is the distance between  $V_+$  and  $v$ .

On the following pages, implement two functions:

- The `find_surrounding_values` function should take a single argument (a measured voltage) and return a tuple of length 4 containing  $(T_-, V_-, T_+, V_+)$ , or `None` if the input voltage is lower than the smallest measured voltage or higher than the largest measured voltage. You may **not** assume that the measurements in the MEASUREMENTS list are stored in any particular order. For full credit, your code should not rely on the built-in "sorted" function, or the "sort" list method.
- The `temperature_from_voltage` function should take a single argument representing a measured voltage, and it should return a single number representing the estimated temperature (estimated using the process described above). If  $v$  is lower than the smallest measured voltage or higher than the largest measured voltage, your code should return `None` to indicate that it is not possible to make a reasonable estimate of the temperature.

Your code for `temperature_from_voltage` can assume a perfectly-working copy of `find_surrounding_values`.

Please write your code on the following two pages. These instructions have been repeated on the last page of this handout, which you may remove.

```
def find_surrounding_values(voltage):  
    # your code here
```

```
def temperature_from_voltage(voltage):  
    # your code here
```







*Worksheet (intentionally blank)*









## Repeated Instructions for Question 6

Alyssa P. Hacker has built a circuit where the voltage measured at a particular point varies with the temperature, and she is interested to hook it up to her computer so that she can get a nice display of the temperature on her desktop.

However, she is faced with a challenge: the mapping from temperatures to voltages is a complicated function, and there seems to be no analytical solution.

To combat this, Alyssa comes up with the clever idea to sample some voltages at known temperatures (yielding a mapping from some fixed set of temperatures to the voltages measured at those temperatures), and to use these values to guess at the temperature based on a measured voltage.

Alyssa takes her measurements at temperatures  $t_1, t_2, \dots, t_n$ , and measures voltages  $v_1, v_2, \dots, v_n$ . She uses these to create a list of tuples:

```
MEASUREMENTS = [(t1, v1), (t2, v2), ..., (tn, vn)]
```

Alyssa's strategy is to measure a new voltage  $v$ , find the values in the MEASUREMENTS list that are closest to that measurement above and below (call them  $V_+$  and  $V_-$ ) and note their corresponding temperatures (call them  $T_+$  and  $T_-$ ), and estimate the corresponding temperature  $t$  via *linear interpolation* between  $T_+$  and  $T_-$ :

$$t = \left( \frac{w_2}{w_1 + w_2} \right) T_- + \left( \frac{w_1}{w_1 + w_2} \right) T_+$$

where  $w_1$  is the distance between  $V_-$  and  $v$ , and  $w_2$  is the distance between  $V_+$  and  $v$ .

On the following pages, implement two functions:

- The `find_surrounding_values` function should take a single argument (a measured voltage) and return a tuple of length 4 containing  $(T_-, V_-, T_+, V_+)$ , or `None` if the input voltage is lower than the smallest measured voltage or higher than the largest measured voltage. You may **not** assume that the measurements in the MEASUREMENTS list are stored in any particular order. For full credit, your code should not rely on the built-in "sorted" function, or the "sort" list method.
- The `temperature_from_voltage` function should perform the overall operation described above. Your function should take a single argument representing a measured voltage, and it should return a single number representing the estimated temperature (estimated using the process described above). If  $v$  is lower than the smallest measured voltage or higher than the largest measured voltage, your code should return `None` to indicate that it is not possible to make a reasonable estimate of the temperature.

Your code for `temperature_from_voltage` can assume a perfectly-working copy of `find_surrounding_values`.

