6.01 Midterm 1 Spring 2017

Name: Answers **Section Number:**

Kerberos (Athena) name:

Section	Design Lab Time
1:	Wednesday 9:30am
2:	Wednesday 2:00pm
3:	Thursday 2:00pm

Please WAIT until we tell you to begin.

During the exam, you may refer to any written or printed paper material.
You may NOT use any electronic devices (including calculators, phones, etc).

If you have questions, please **come to us at the front** to ask them.

Enter all answers in the boxes provided.

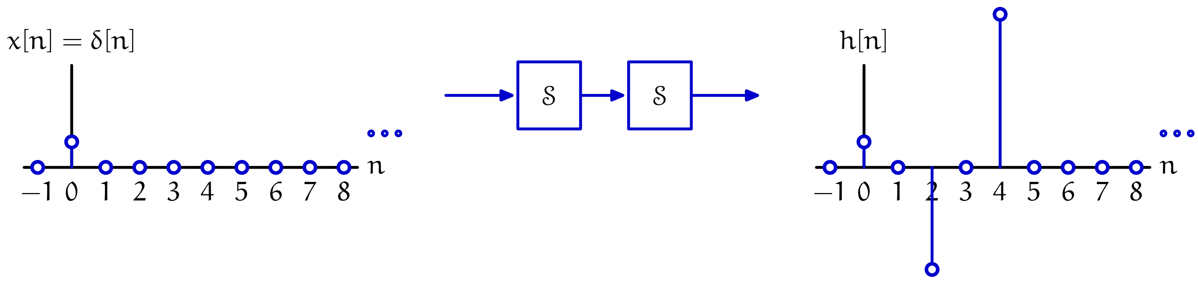
Extra work may be taken into account when assigning partial credit,
but only work shown on pages with QR codes will be considered.

Question 1: 17 Points
Question 2: 20 Points
Question 3: 15 Points
Question 4: 18 Points
Question 5: 24 Points
Question 6: 23 Points
Total: 117 Points

1 Two's Company (17 Points)

Let a system $\mathcal{H}$ represent the cascade of 2 copies of an LTI system $\mathcal{S}$. Assume that $\mathcal{H}$ produces the following as its unit-sample response:

$$h[n] = \begin{cases} 1 & \text{if } n = 0 \\ -6 & \text{if } n = 2 \\ 9 & \text{if } n = 4 \\ 0 & \text{otherwise} \end{cases}$$



1. Calculate the first 6 samples of the response of the system $\mathcal{H}$ to the following input signal:

$$x[n] = \begin{cases} 1 & \text{if } n = 0 \\ -2 & \text{if } n = 2 \\ 0 & \text{otherwise} \end{cases}$$

$y[0] =$	1	$y[1] =$	0	$y[2] =$	−8
$y[3] =$	0	$y[4] =$	21	$y[5] =$	0

2. Determine the system functional $\mathcal{S}$. Express your answer as a ratio of polynomials in $\mathcal{R}$.

$\mathcal{S} = 1 - 3\mathcal{R}^2$
or
$3\mathcal{R}^2 - 1$

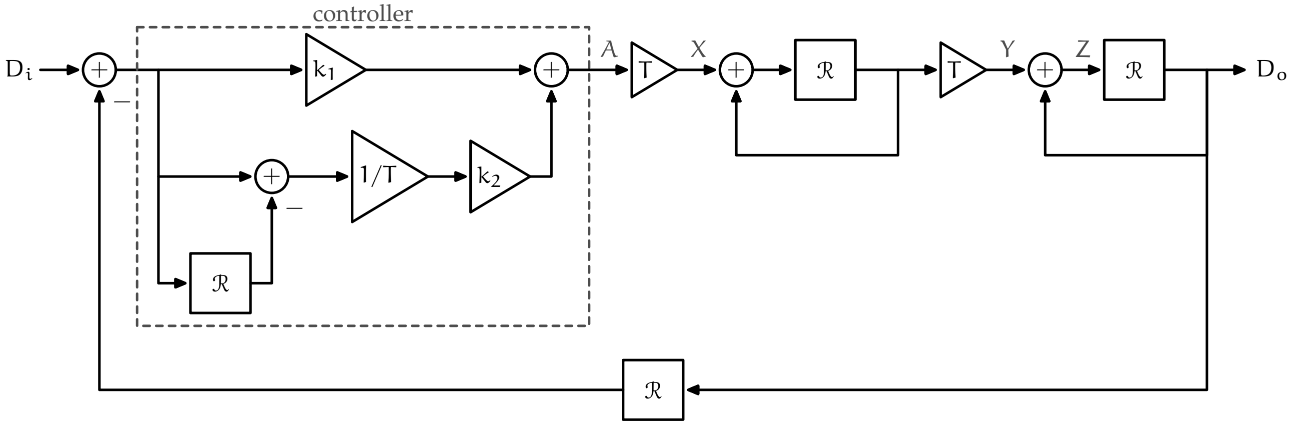
3. Let A be the signal between the two copies of $\mathcal{S}$ in the system above when the input to the whole system is the unit sample signal Δ. Enter the following values of A:

$a[0] =$ 1 or -1 $a[1] =$ 0 $a[2] =$ -3 or 3 $a[3] =$ 0

For all $n > 3$, $a[n] =$ 0

2 Cruise Control (20 Points)

Below is a block diagram representing an "adaptive" cruise-control system, based off of a car-following model. In this system, the controller sets the car's **acceleration**, the signal A, based off an error signal E, which is computed by subtracting a measured forward distance D_s from a desired following distance D_i. Assume that all distances are measured in meters, that all times are measured in seconds, and that T is the length of one timestep.



2.1 Physics

What are the units of the gain k_1?	$\frac{1}{s^2}$
What are the units of the gain k_2?	$\frac{1}{s}$
What are the units of the signal labeled X?	$\frac{m}{s}$
What are the units of the signal labeled Y?	m
What are the units of the signal labeled Z?	m

Which of the following most accurately describes the behavior of the controller?

A. The controller outputs an acceleration that is proportional to the current error.

B. The controller outputs an acceleration that is proportional to the previous error.

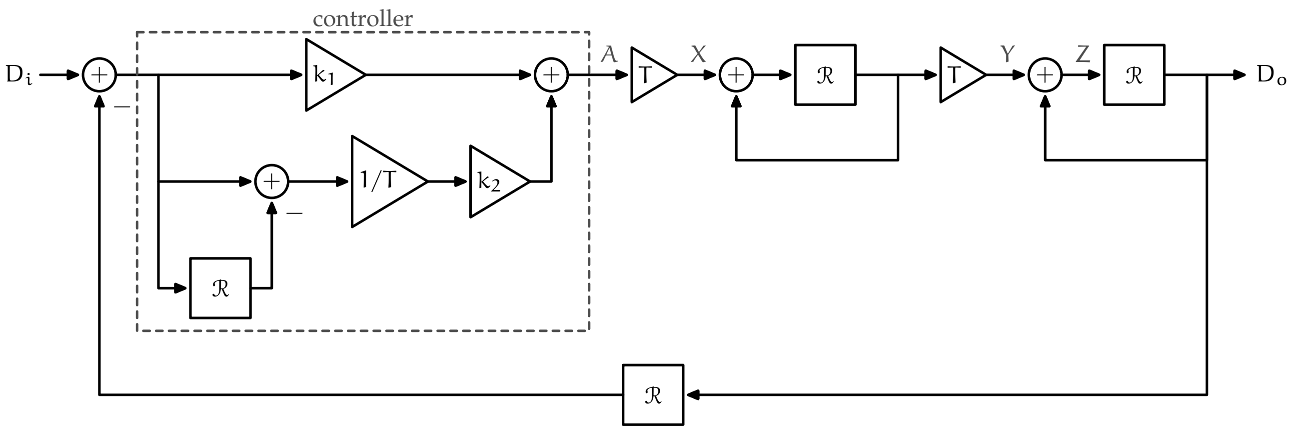
C. The controller outputs an acceleration that is the sum of the scaled error, and the scaled integral of error.

D. The controller outputs an acceleration that is the sum of the scaled error, and the scaled derivative of error.

A, B, C, or D: D

2.2 Modeling

In this section, we will consider modeling the system using 6.01's `lti` infrastructure. Assume that you do not have direct access to the `Polynomial` class, and thus must use only the primitives and combinations in the `lti` module.



What is the minimum number of instances of `FeedforwardAdd` required to model this system **as drawn**?

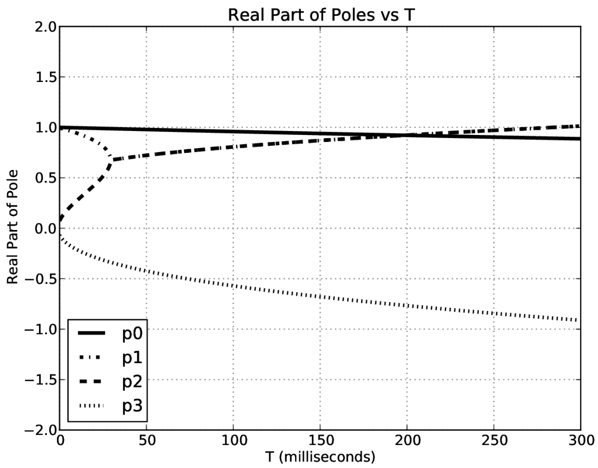
Minimum number of `Feedforward` instances: 2

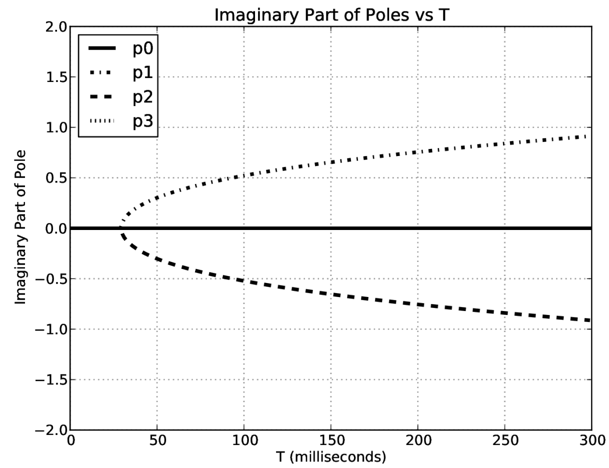
What is the minimum number of instances of `FeedbackAdd` required to model this system **as drawn**?

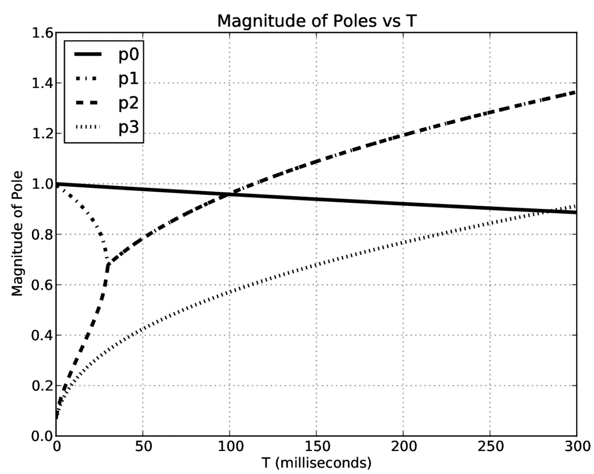
Minimum number of `Feedback` instances: 3

2.3 Poles

For some particular values of k_1 and k_2, the graphs below show the **four** poles of the system as T is varied linearly from $T = 10^{-3}$ seconds to $T = 3 \cdot 10^{-1}$ seconds.







Note: The dash-dotted (p1) line and the long-dashed (p2) line overlap in part of the real and magnitude plots. The solid (p0) and the short-dashed (p3) lines overlap in the imaginary part plot.

For each of the following values of T, state whether the system is stable, and whether the system is oscillatory in the long term (as $n \to \infty$), with that value of T and the k_1 and k_2 values that were used to generate the graphs above. Enter `Yes` or `No` in each box, or enter `NEI` (for "not enough information") if not enough information was given to answer that question.

T	**Stable?**	**Oscillatory?**
$25ms$	Yes	No
$50ms$	Yes	No
$105ms$	Yes	Yes
$150ms$	No	Yes

For which value of T (approximately) does the system converge most quickly? Consider all possible values of T, not just those shown in the table on the previous page.

T for fastest convergence: $\sim 100ms$

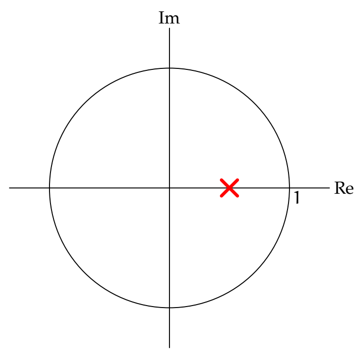
3 Conga Line (15 Points)

3.1 Part 1

Consider a linear, time-invariant system $\mathcal{H}_1$, whose unit **step** response (when started from rest) is given by:

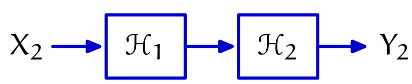
$$y_1[n] = \begin{cases} 0 & \text{if } n < 0 \\ -\frac{1}{2} + \frac{1}{8}\left(\frac{1}{2}\right)^n & \text{if } n \geqslant 0 \end{cases}$$

Sketch the location(s) of $\mathcal{H}_1$'s pole(s) on the complex plane below. If there are no poles, write "`None`".



3.2 Part 2

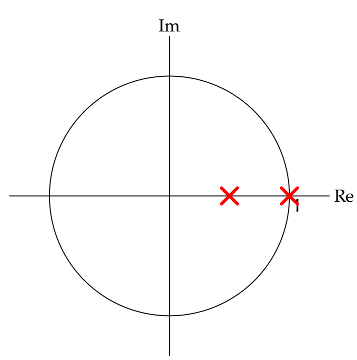
Now consider connecting a second system $\mathcal{H}_2$ in cascade with $\mathcal{H}_1$, as shown below:



If, when started from rest, the unit **sample** response of the entire composite system $\frac{Y_2}{X_2}$ is the same as $\mathcal{H}_1$'s unit **step** response, what is the system functional form of $\mathcal{H}_2$?

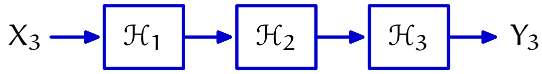
$$\mathcal{H}_2 = \frac{1}{1 - \mathcal{R}}$$

Sketch the location(s) of the pole(s) of the composite system $\frac{Y_2}{X_2}$ on the complex plane below. If there are no poles, write "`None`".



3.3 Part 3

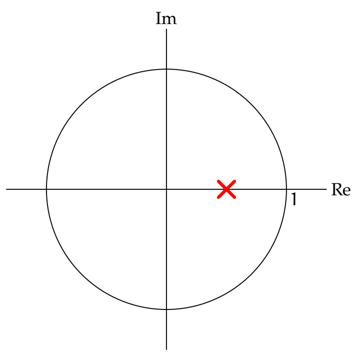
Now consider connecting a third system, $\mathcal{H}_3$ in cascade with the others, as shown below:



If, when started from rest, the unit **sample** response of this composite system $\frac{Y_3}{X_3}$ is the same as the unit **sample** response of $\mathcal{H}_1$, and $\mathcal{H}_2$ is defined according to the specification from the previous section, what is the system functional form of $\mathcal{H}_3$?

$\mathcal{H}_3 = 1 - \mathcal{R}$

Sketch the location(s) of the pole(s) of the composite system $\frac{Y_3}{X_3}$ on the complex plane below. If there are no poles, write "`None`".

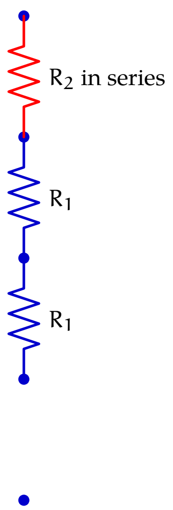


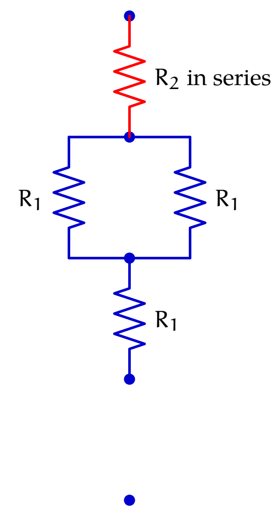
4 Changing Resistance (18 Points)

In each of the resistive networks below, assume that all of the resistors have equal positive, finite, nonzero resistance R_1.

4.1 Part 1

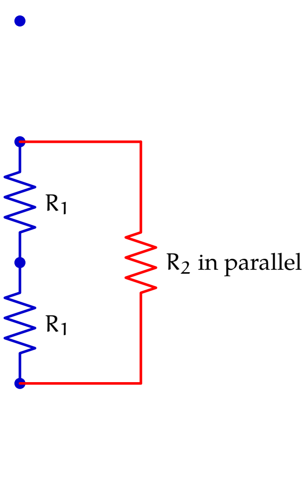
For each circuit below, draw one additional resistor of positive, finite, nonzero resistance R_2 (not necessarily the same as R_1) between any two nodes shown, so that the overall resistance of the network is maximally *increased*, regardless of the value of R_2. At least one terminal of the resistor must be connected to the existing network. If your answer depends on the values of R_1 and/or R_2, write NEI (for "not enough information") instead of drawing a resistor.

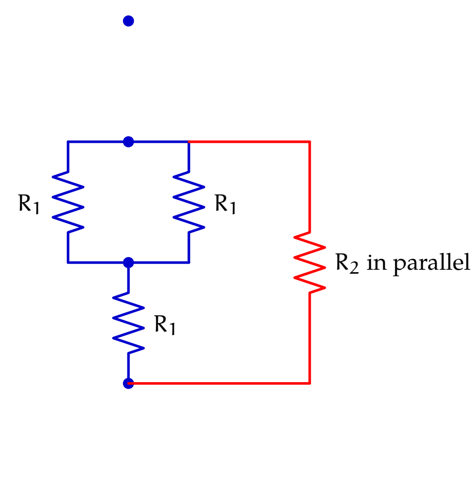




4.2 Part 2

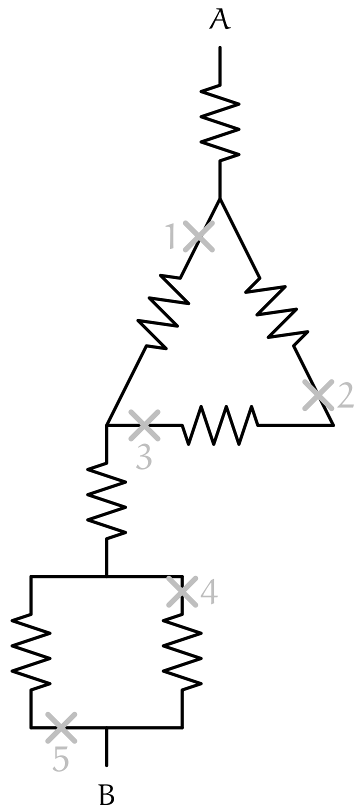
For each circuit below, draw one additional resistor of positive, finite, nonzero resistance R_2 (not necessarily the same as R_1) between any two nodes shown, so that the overall resistance of the network is maximally *decreased*, regardless of the value of R_2. Note that at least one terminal of the resistor must be connected to the existing network. If your answer depends on the values of R_1 and/or R_2, write NEI (for "not enough information") instead of drawing a resistor.





4.3 Part 3

Consider the following resistive network, where the locations labeled 1 through 5 correspond to wires that could potentially be cut. Again, assume that all resistors have resistance R_1.



For each of the five locations, state whether cutting that wire would *increase* or *decrease* the resistance between the nodes labeled A and B, and by how much (in terms of R_1 and/or constants). In each scenario, consider cutting the wire at that location *only* (in the absence of other cuts).

Location	Increase/Decrease?	By How Much?
1	Increase	$4R_1/3$
2	Increase	$R_1/3$
3	Increase	$R_1/3$
4	Increase	$R_1/2$
5	Increase	$R_1/2$

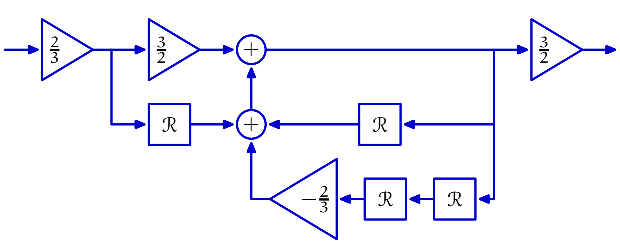
5 Mixed Up Systems (24 Points)

Professor Alyssa P. Hacker spent all night preparing a lecture about four LTI systems. She represented each in four different ways: as a difference equation, a block diagram, a system functional, and a unit sample response. However, while Alyssa was getting dressed, her cat ripped her notes to pieces and ate some of the pieces. Help Alyssa repair her notes and replace the pieces her cat ate.

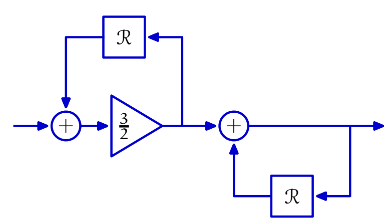
Complete the table at the bottom of the facing page by matching the representations below. If a system is missing a particular representation, generate one. When generating a block diagram, you may only use gains, delays, and adders.

5.1 Block Diagrams

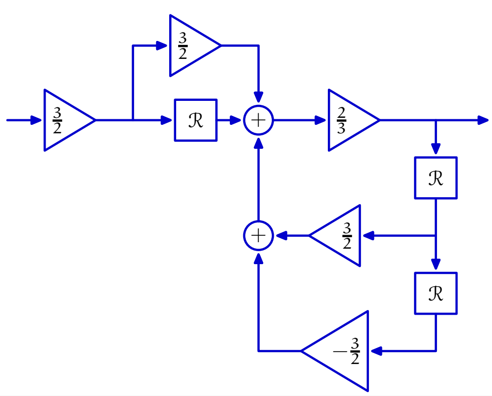
A:



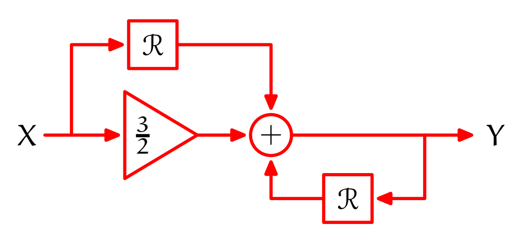
B:



C:



D:



5.2 System Functionals

i:

$$\frac{Y}{X} = \frac{\frac{3}{2} + \mathcal{R}}{1 - \mathcal{R}}$$

ii:

$$\frac{Y}{X} = \frac{\frac{3}{2} + \mathcal{R}}{1 - \mathcal{R} + \mathcal{R}^2}$$

iii:

$$\frac{Y}{X} = \frac{\frac{3}{2} + \mathcal{R}}{1 - \mathcal{R} + \frac{2}{3}\mathcal{R}^2}$$

iv:

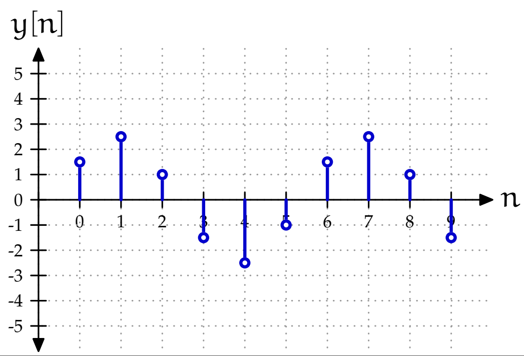
$$\frac{Y}{X} = \frac{\frac{3}{2}}{1 - \frac{5}{2}\mathcal{R} + \frac{3}{2}\mathcal{R}^2}$$

5.3 Difference Equations

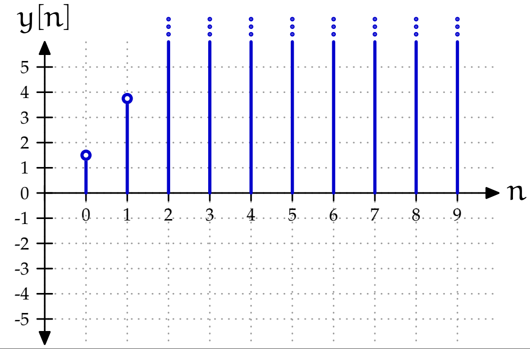
1: $y[n] = y[n-1] + \frac{3}{2}x[n] + x[n-1]$	**2:** $y[n] = y[n-1] - \frac{2}{3}y[n-2] + \frac{3}{2}x[n] + x[n-1]$
3: $y[n] = \frac{5}{2}y[n-1] - \frac{3}{2}y[n-2] + \frac{3}{2}x[n]$	**4:** $y[n] = y[n-1] - y[n-2] + \frac{3}{2}x[n] + x[n-1]$

5.4 Unit Sample Responses

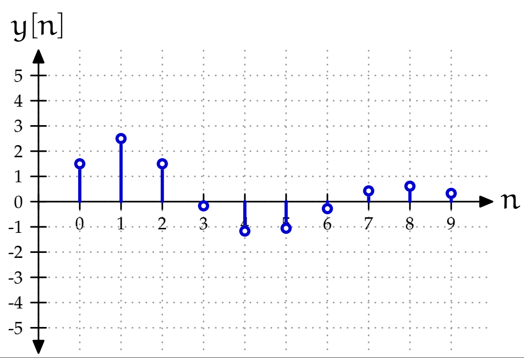
a:



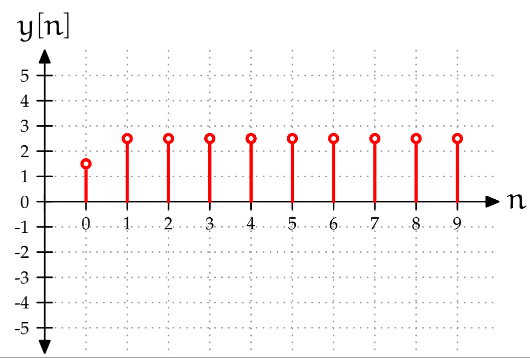
b:



c:



d:



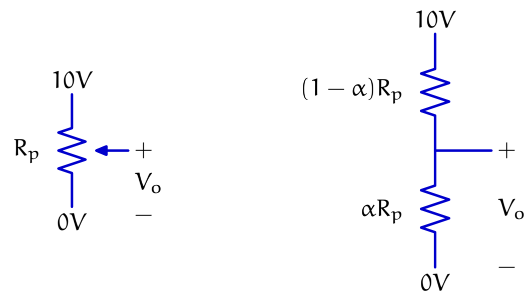
5.5 Putting It Together

For each block diagram, circle the corresponding system functional, difference equation, and unit sample response.

Block Diagram	System Functional	Difference Equation	Unit Sample Response
A	(i) (ii) (iii) (iv)	(1) (2) (3) (4)	(a) (b) (c) (d)
B	(i) (ii) (iii) (iv)	(1) (2) (3) (4)	(a) (b) (c) (d)
C	(i) (ii) (iii) (iv)	(1) (2) (3) (4)	(a) (b) (c) (d)
D	(i) (ii) (iii) (iv)	(1) (2) (3) (4)	(a) (b) (c) (d)

6 The Long and Winding Robot (23 Points)

In Design Lab 5, we constructed a variable voltage divider from a potentiometer to create a voltage V_o that varied with the angle of the potentiometer shaft:



We then connected this potentiometer to the robot and used it to control the robot's velocities remotely.

Consider the following SoaR brain, which implements a similar controller:

```
K = 1
def on_step():
    ai1, ai2, ai3, ai4 = io.getAnalogInputs() # ai1 is the voltage from the pot, in Volts
    alpha = ai1 / 10
    error = alpha - 0.5
    io.set_rotational(K * error)
    io.set_forward(0.1)
```

6.1 Match Game

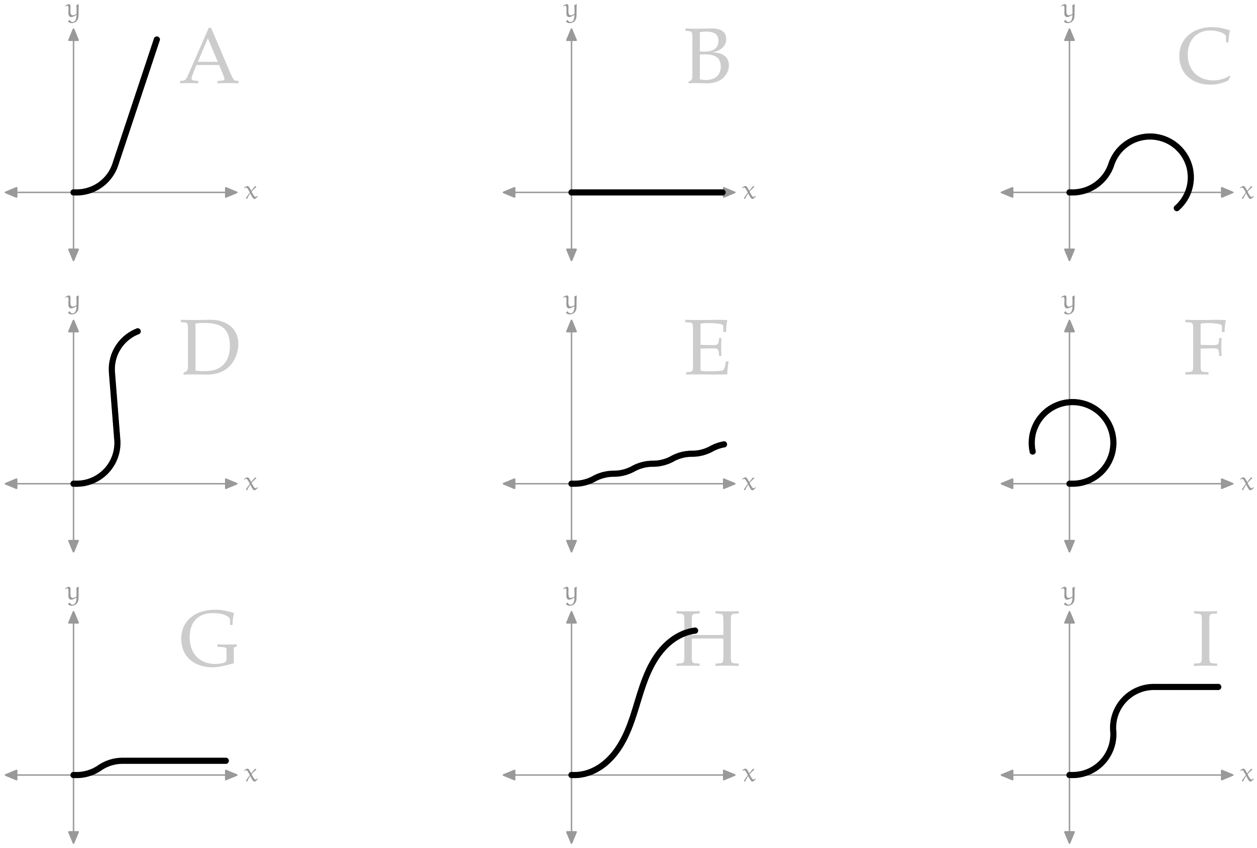
Ben Bitdiddle decided to spend his weekend trying to draw pictures using this robot, and so he ran a few experiments. On the facing page are two sets of graphs: the first represents the robot's position over time (on each time step, the robot marks its position with a dot), and the second represents the voltage from the potentiometer divider versus time.

Note that a positive rotational velocity causes the robot to turn counter-clockwise, and that the robot was started at position $(0, 0)$ in each case, facing straight along the "x" axis ($\theta = 0$).

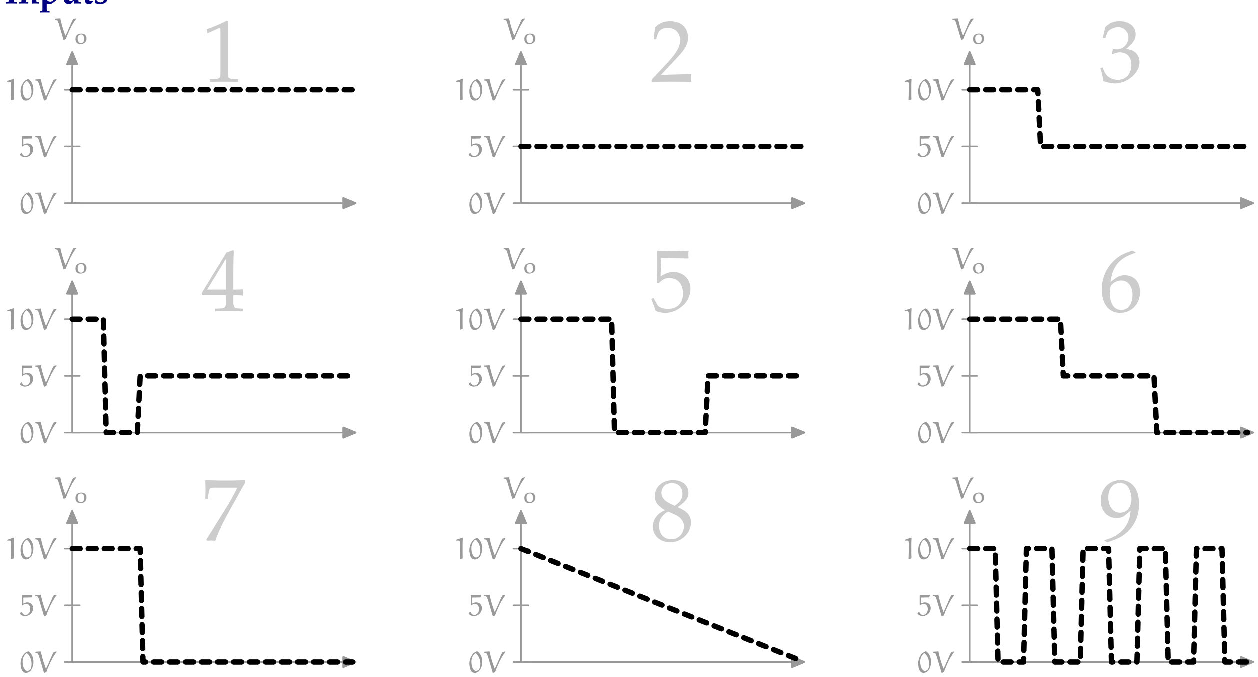
However, Ben forgot which input graph corresponds to which robot path! For each robot plot below, state which, if any, of the inputs could have produced the graph.

Response A:	3	Response B:	2	Response C:	7
Response D:	6	Response E:	9	Response F:	1
Response G:	4	Response H:	8	Response I:	5

6.2 Responses



6.3 Inputs



6.4 Simulation

Ben learned something about the value of computational simulation during the first few weeks of 6.01, and so he decides that it is too much trouble to wait for the robot to move around to get his drawings. He decides it would be faster to write a computer simulation that, given a list of voltages representing potentiometer voltages at each timestep, returns a list of tuples representing the robot's (x, y) position at each timestep, in meters. However, he is a little out of practice with Python, and so asks you to implement the function for him.

Assume that the robot starts at position (0,0), facing at $\theta = 0$ radians. Keep in mind that the robot has a maximum angular velocity of 0.5 rad/second in either direction.

Assume that variables `V`, `T`, and `k` hold the robot's forward velocity in meters per second, the length of a timestep in seconds, and the gain used in the proportional controller, respectively.

You may also make use of a function `get_updated_position(V, T, x, y, theta)`, which takes the robot's forward velocity V, the length of a timestep T, and the robot's position x, y, and θ, and returns a tuple `(x,y)`, representing the robot's new position. For example, the following computes the robot's updated position when it is at $(1, 2)$ facing along the x axis, and moving forward with a velocity of $0.5 m/s$ for 0.1 seconds.

```
>>> print(get_updated_position(0.5, 0.1, 1, 2, 0))
(1.05, 2)
```

Write your function definition in the box below:

```
def recreate_path(pot_voltages):
    positions = [(0,0)]
    theta = 0
    for pv in pot_voltages:
        x,y = pos[-1]
        alpha = pv / 10
        omega = k*(alpha - 0.5)
        omega = min(0.5, max(-0.5, omega))
        nx,ny = get_updated_position(V, T, x, y, theta)
        positions.append((nx, ny))
        theta += omega*T
    return positions

# it would also be fine to update theta before updating the position, since
# this ordering wasn't specified in the problem.
```

6.5 Get Updated Position

Now, implement the function `get_updated_position` described in the previous section. Assume that functions `sin`, `cos`, and `tan` exist to perform trig functions. Write your code in the box below:

```
def get_updated_position(V, T, x, y, theta):
    return (x + V * T * cos(theta), y + V * T * sin(theta))
```

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