6.01 Midterm 1 Fall 2016

Name: **Section Number:**

Kerberos (Athena) name:

Section	Design Lab Time
1:	Thursday 9:30am
2:	Thursday 2:00pm

Please WAIT until we tell you to begin.

During the exam, you may refer to any written or printed paper material.
You may NOT use any electronic devices (including calculators, phones, etc).

If you have questions, please **come to us at the front** to ask them.

Enter all answers in the boxes provided.

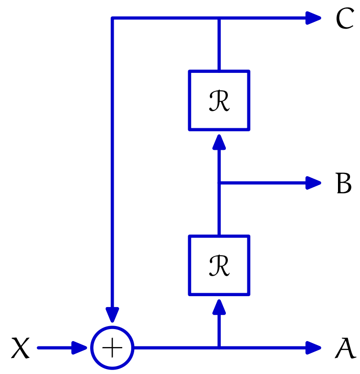
Extra work may be taken into account when assigning partial credit.

For staff use:

1.	/11
2.	/12
3.	/12
4.	/15
5.	/16
6.	/12
total:	/78

1 The Jackson Five (11 Points)

Consider the following block diagram:



1.1 Individual Signals

Solve for each of the following system functionals:

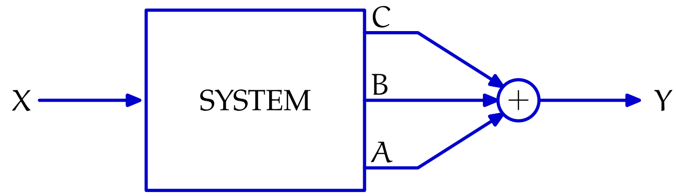
$\frac{A}{X} =$

$\frac{B}{X} =$

$\frac{C}{X} =$

1.2 Summing

Now consider the following system, which sums up A, B, and C from the previous section:

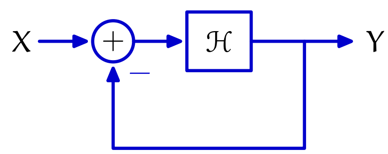


What is the system functional associated with this system?

$\frac{Y}{X} =$

1.3 Design

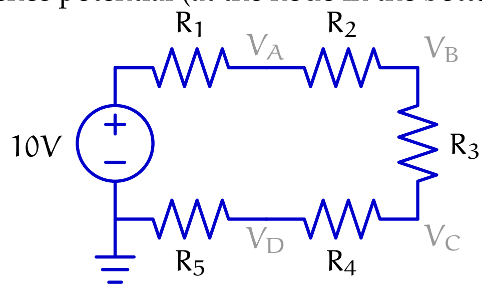
Now, let's say we wanted to make a system of the form shown below that is equivalent to the system above (in terms of input/output relationship, when started from rest). What must $\mathcal{H}$ be in order to guarantee that these two systems are equivalent?



To make the two systems equivalent, $\mathcal{H} =$

2 Virtual Reality (12 Points)

Consider the following circuit, where the labeled voltages V_A, V_B, V_C, and V_D are all measured relative to the indicated reference potential (at the node in the bottom left).



Is it possible to set the resistances R_1 through R_5 such that $V_A = 8V$, $V_B = 4V$, $V_C = 2V$, and $V_D = 1V$ using only positive, finite, nonzero resistances? If so, enter the necessary resistance values (**including units**) below. If not, put an `X` in each box and explain briefly:

$R_1 =$ ☐ $R_2 =$ ☐ $R_3 =$ ☐ $R_4 =$ ☐ $R_5 =$ ☐

If not possible, explain briefly (1-2 sentences):

Is it possible to set the resistances R_1 through R_5 such that $V_A = 3V$, $V_B = 2V$, $V_C = 1V$, and $V_D = 0V$ using only positive, finite, nonzero resistances? If so, enter the necessary resistance values (**including units**) below. If not, put an `X` in each box and explain briefly:

$R_1 =$ ☐ $R_2 =$ ☐ $R_3 =$ ☐ $R_4 =$ ☐ $R_5 =$ ☐

If not possible, explain briefly (1-2 sentences):

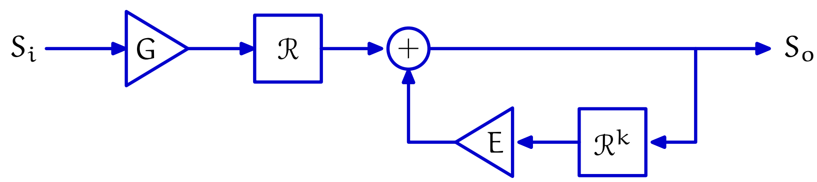
Is it possible to set the resistances R_1 through R_5 such that $V_A = 1V$, $V_B = 2V$, $V_C = 4V$, and $V_D = 8V$ using only positive, finite, nonzero resistances? If so, enter the necessary resistance values (**including units**) below. If not, put an `X` in each box and explain briefly:

$R_1 =$ ☐ $R_2 =$ ☐ $R_3 =$ ☐ $R_4 =$ ☐ $R_5 =$ ☐

If not possible, explain briefly (1-2 sentences):

3 The Dolphin (12 Points)

Your favorite band is giving a concert. Their singer speaks (or sings, screams, or mumbles) into a microphone with a signal S_i. The signal is fed into an amplifier with gain G before being played by a speaker at the audience. The output of the speaker can take two routes to the audience: directly to their ears, or via echoing off the environment. A system which models the audio system and concert venue with an echo in the "plant" is shown below:



The echo of a concert venue is characterized by two parameters: E and k. Note that applying $\mathcal{R}^k$ to a signal yields the same result as applying k right-shift operators sequentially.

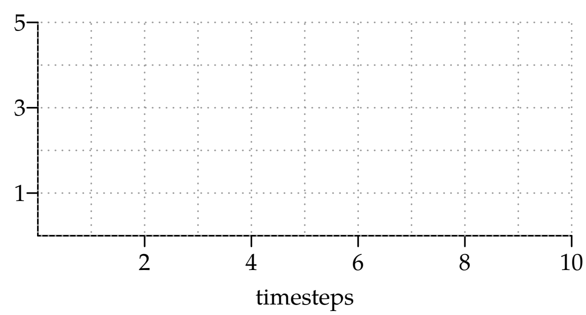
3.1 What's the Difference?

Derive a difference equation for $s_o[n]$ in terms of the signals and variables defined above.

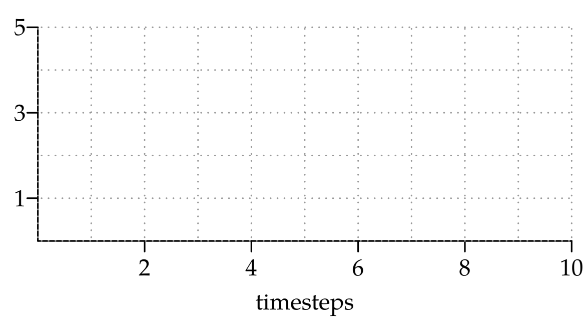
$s_o[n]$ =

3.2 Graphs

If the concert is held in a concrete storage facility, the echo can be characterized by $E = 0.8$ and $k = 3$. Plot below the response of the system to a unit sample signal when the amplifier is set to $G = 5$ (you can think of this as a brief yell into the microphone).

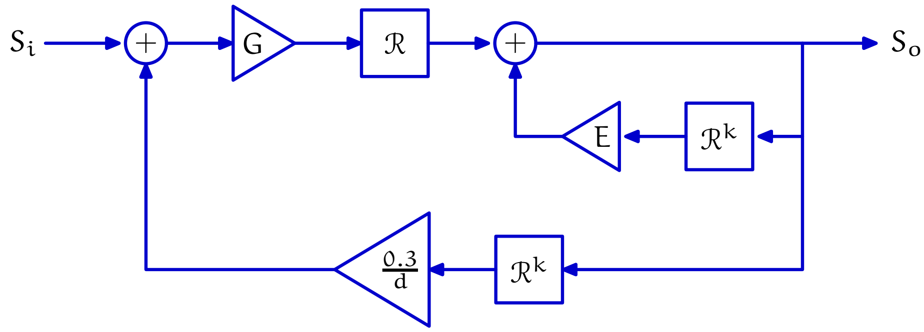


If the concert is instead held in an acoustically engineered room, the echo can be characterized by $E = 0.3$ and $k = 1$. Plot below the unit sample response of this system when the amplifier is set to $G = 5$.



3.3 Distance

In addition to affecting the audience, the echo signal can also leak back into the singer's microphone. This audio feedback can be modeled by adjusting our system from above to include a new feedback path as shown below:



The audio feedback into the microphone is dependent on how far away the singer is from the speaker (call this distance, in meters, d). If the singer gets too close to the speaker, the system will go unstable, resulting in loud howling from the speaker (perhaps you have experienced this before!).

Assume that a concert is being given with $G = 8$ at a venue described by $E = 0.2$ and $k = 1$. In the box below, write an equation (or inequality) that represents the value(s) of d for which the system is stable. Your equation (or inequality) should contain only constants and d, but does not need to be simplified.

4 Chirp, Chirp (15 Points)

In this problem, we will design a sonar emitter that acts something like the sonar on our robot. In particular, we would like to output a high-frequency oscillating "chirp" $y[n]$ that lasts for a desired amount of time, and then the output stops. Specifically, we want a sinusoidal output with constant amplitude A and constant peak-to-peak period ϕ_0 that runs for a fixed number of time steps, n_{off}:

$$y[n] = \begin{cases} A\cos(nB) & \text{if } 0 \leqslant n < n_{off} \\ 0 & \text{otherwise} \end{cases}$$

for some value of B.

Our design will use the following input signal X:

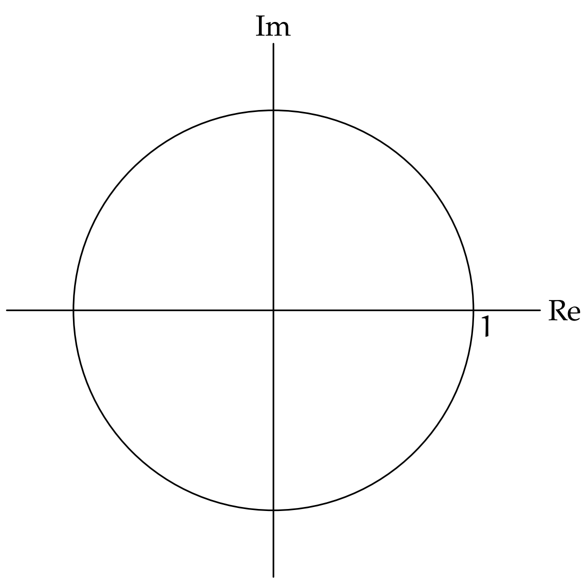
$$x[n] = \begin{cases} 1 & \text{if } n = 0 \\ K & \text{if } n = n_{off} \\ 0 & \text{otherwise} \end{cases}$$

The value $x[0] = 1$ should turn the sinusoid "on", and the value $x[n_{off}] = K$ should turn the sinusoid "off" (we will determine K later).

1. We would like to emit exactly 5 peak-to-peak cycles of our sonar output from $n = 0$ to $n = n_{off} = 100$. To generate the desired sinusoidal output, what should the poles of our sonar emitter be? Enter your answers below (they may contain constants and/or n_{off}):

Poles:

On the complex plane below, indicate the locations of the poles with "X", and clearly label the angle (with respect to the postive real axis), as well as the magnitude of each pole on the diagram.



2. If we want exactly 5 peak-to-peak cycles of our sonar output between $n = 0$ and $n = n_{off}$, what value for K should we provide for $x[n_{off}] = K$, so that the output $y[n] = 0$ for $n > n_{off}$? If no K will achieve this goal, briefly explain why not (1-2 sentences).

3. In this design, one problem is that our $x[n_{off}]$ has to be precisely valued (K) and precisely timed (at n_{off}) just right, so that we perfectly "turn off" the sonar emitter. If by some freak accident we were to get $x[n_{off}] = 1.05K$ instead of $x[n_{off}] = K$, write an expression for $y[n]$ for $n > n_{off}$.

Your expression may contain n, π, A, n_{off}, trigonometric operations, numbers, and basic arithmetic.

When $n > n_{off}$, $y[n] =$

4. Suppose we want to adjust our system so that it emits exactly 4.5 peak-to-peak cycles of our sonar output (instead of 5) from $n = 0$ to $n = n_{off}$. What value for K should we provide for $x[n_{off}] = K$, so that the output $y[n] = 0$ for $n > n_{off}$? If no K will achieve this goal, explain why not.

5. Suppose we want to adjust our system so that it emits exactly 4.25 peak-to-peak cycles of our sonar output (instead of 5) from $n = 0$ to $n = n_{off}$. What value for K should we provide for $x[n_{off}] = K$, so that the output $y[n] = 0$ for $n > n_{off}$? If no K will achieve this goal, explain why not.

5 Sixohone Systems, Ltd. (16 Points)

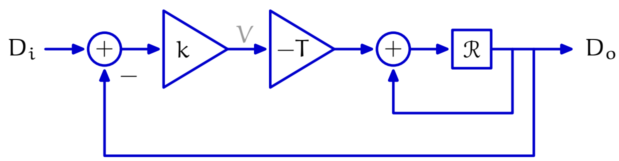
Ben Bitdiddle recently got a new internship at Sixohone Systems, Ltd, a company specializing in simulation of robots using the simulation framework developed in 6.01.

Ben sees the benefits of linear systems, but he is annoyed by the fact that the systems we have seen in 6.01 have not always perfectly accounted for the robot's behavior (in particular, he noticed that LTI systems could not properly account for the fact that the 6.01 robots have a limited velocity).

He decides to fix this by introducing a new nonlinear component into his company's simulation framework: the `LimitGain`. He designs the `LimitGain` component so that it scales its input by a constant value `k` (just as the normal `Gain` element did), but it also "clips" its output so that it never goes above $+L$ or below $-L$, where L is a nonnegative limit set at initialization time (for the 6.01 robots' forward velocities, this limit was 0.5 meters/second).

5.1 Wall Finder

Consider the following model of the "wall finder" system from lab, where the robot's velocity is represented by the signal labeled V:



5.1.1 Need for Speed

Ben runs his simulation first using a standard `Gain` element to represent the k in the diagram above, simulating the case when the robot starts 0.8 meters from the wall and wants to stop 0.5 meters from the wall, the gain k is -2, and the length of a discrete timestep is 0.1 seconds.

In this case, what are the first 3 samples of V?

$v[0] =$ ☐ $v[1] =$ ☐ $v[2] =$ ☐

5.1.2 Scenic Route

Ben then runs his simulation with a `LimitGain` element (with a limit $L = 0.5$ meters/second) to represent k, under the same conditions.

In this case, what are the first 3 samples of V?

$v[0] =$ ☐ $v[1] =$ ☐ $v[2] =$ ☐

5.2 Limit Delay Implementation

Ben's boss, Alyssa P. Hacker, says that Ben could implement this limit in a different way.

She suggests constructing a different class, called `LimitDelay`, which applies the clipping property to a *delay* element instead of a gain element. It should behave like an `R` block in most respects, except that its output should never be higher than `limit` (a nonnegative value passed in at initialization time), not lower than `-limit`.

Implement `LimitDelay` below by creating a filling in the definition below, including an appropriate `initial_state` attribute and `calculate_step` method. Recall that `calculate_step(self, state, inp)` takes a state and input as arguments, and returns a tuple containing an updated state and an output, in that order.

```
class LimitDelay:
    def __init__(self, initial_output, limit):
        # Your code here

    def calculate_step(self, state, inp):
        # Your code here
```

5.3 Part 3

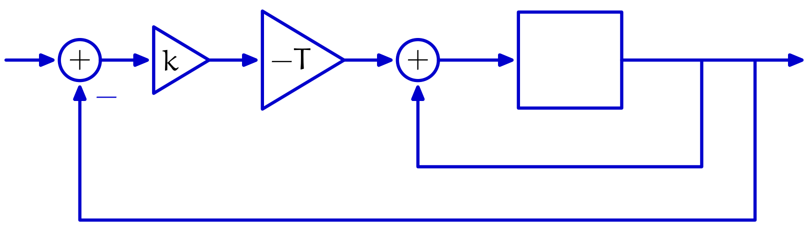
Assuming a perfectly-working copy of `LimitDelay`, Alyssa argues that it can be used instead of `LimitGain` to model the velocity clip in the wall finder.

To prove his point, Lem builds two different wall finder models. In each, he uses a regular `Gain` element for the gain k, but he uses `LimitDelay` in place of one or more of the delay elements.

For each system below, if it can be made to produce the same output as Ben's original clipped model, write "LD" **and the appropriate limit** in each box if it should be replaced with a limit delay, and write "R" in each box that should remain as a regular delay.

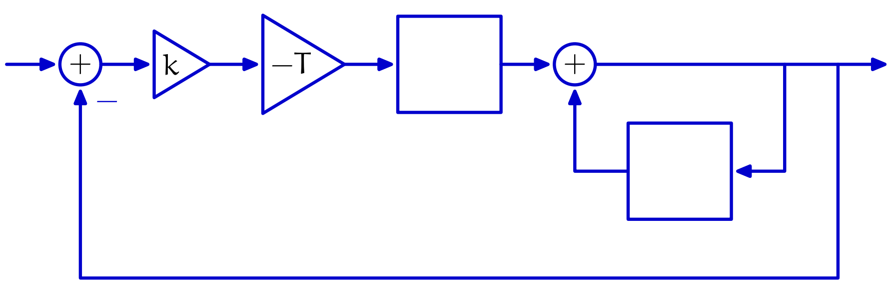
If the system cannot be made to produce the same output as Ben's original clipped model, write an "X" in every delay box in the diagram and explain briefly.

5.3.1 Model 1



If this system cannot be made equivalent, explain briefly (1-2 sentences):

5.3.2 Model 2

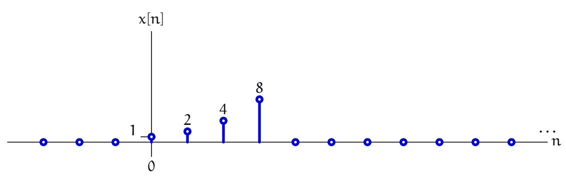


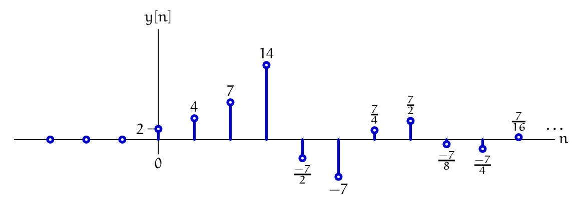
If this system cannot be made equivalent, explain briefly (1-2 sentences):

6 Mystery System (12 Points)

Consider a linear, time-invariant system $\mathcal{H}$, whose response to an input signal X is a signal Y:

$$x[n] = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n = 0 \\ 2 & \text{if } n = 1 \\ 4 & \text{if } n = 2 \\ 8 & \text{if } n = 3 \\ 0 & \text{otherwise} \end{cases} \qquad y[n] = \begin{cases} 0 & \text{if } n < 0 \\ 2 & \text{if } n = 0 \\ 4 & \text{if } n = 1 \\ 7 & \text{if } n = 2 \\ 14 & \text{if } n = 3 \\ -y[n-2]/2 & \text{otherwise} \end{cases}$$





6.1 Poles

Is it possible to represent this system with a finite number of poles?

Yes or No: [ ]

If yes, enter the number of poles and list the poles below. If a pole is repeated k times, then enter that pole k times. If there are more than 5 poles, enter any 5 poles. If there are fewer than 5 poles, write `None` in the remaining boxes.

Number of Poles: [ ]

Poles: [ ] [ ] [ ] [ ] [ ]

If no, explain briefly:

6.2 Diagram

Is it possible to represent this system using finitely-many gains, delays and adders (and no other components)?

Yes or No:

If yes, draw a block diagram of the system:

If no, explain briefly:

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