Signals and Systems Review

Signals are functions of a single parameter (time for our purposes)

Systems transform signals

signal in → | system | → Signal out

In 6.01, we deal with a class of systems known as linear, time-invariant (LTI) systems.

What does it mean for a system to be LTI?

linear: the output of the system at a particular time is a linear combination of previous inputs and outputs (no constants, no conditionals, etc). The system is composed only of delays, gains, and adders.

time-invariant: the output of the system does not depend on the absolute time at which the system was started.

Representing Systems

We can represent these systems in many ways.

Difference eq'n: $y[n] = x[n] + p\,y[n-1]$

good for detailed analysis (exact outputs at specific times).

Operator Expression: $Y = X + pRY$

good for manipulation (simple algebra!)
represents a higher-level view (entire signal X instead of one specific sample $x[n]$)

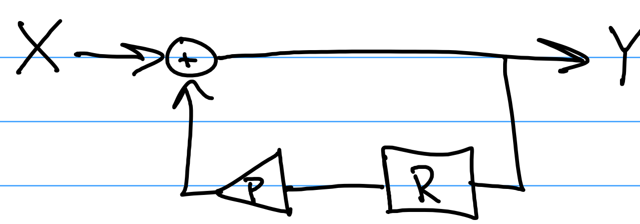
System Functional: $H = \frac{Y}{X} = \frac{1}{1-pR}$

compact system representation,
simplifies combining systems (more later)

Poles: p

good for analysis of long-term system behavior, regardless of input.

Block Diagram:



nice visual representation; easy to see what is "going on"

Moving between representations

DE $\longleftrightarrow$ operator

replace $[n-K]$ with K right-shifts (R^K)

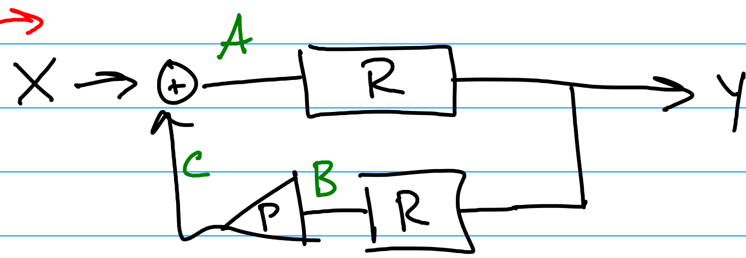
$$y[n] = x[n] + p\,y[n-1]$$

$$Y = X + pRY$$

Block Diagram $\rightarrow$ operator

give each distinct signal a name;
write simple expressions and combine

different system!



$A = X + C$
$Y = RA$
$B = RY$
$C = pB$

$$\begin{aligned} Y &= RA \\ &= R(X+C) \\ &= R(X+pB) \\ &= R(X+pRY) \end{aligned}$$

$$Y = RX + pR^2Y$$

State Machine:

trace inputs and outputs as they move through the state to determine when they contribute to the output

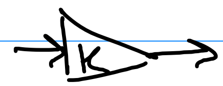
do example here

Moving between operators cont'd

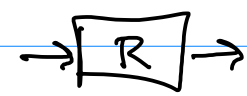
Block diagram → System Functional

PCAP! Break down into delays, gains, adders, feedback adders, and cascades

recall primitives



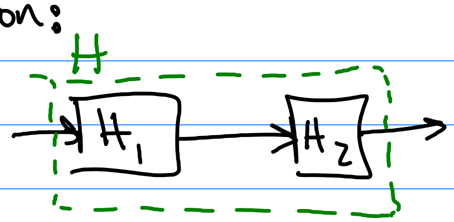
SF: $\frac{K}{1}$



SF: $\frac{R}{1}$

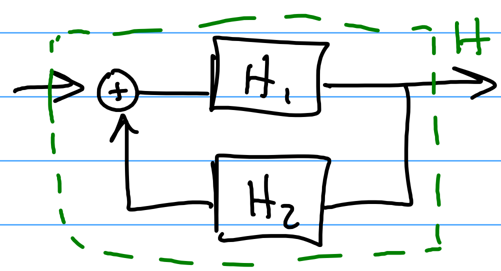
means of combination:

Cascade:



$$H = H_1 \cdot H_2$$

Feedback:



$$H = \frac{H_1}{1 - H_1 H_2}$$

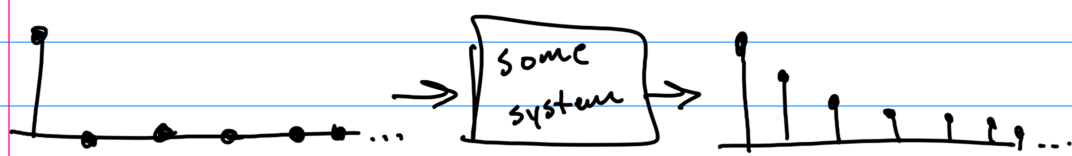
ANALYZING SYSTEM BEHAVOR: USR

Unit Sample Response: system's response to transient input; the output when feeding Δ through the system, where:

$$\delta[n] = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{otherwise} \end{cases}$$

easily calculable from DE
also possible from block dia.

to calculate: table of $n, x[n], y[n]$



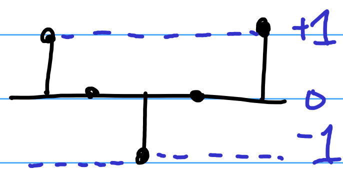
exponential! will look into that more when talking about poles.

SUPERPOSITION

Let $h[n]$ be the USR of some LTI system. The LTI nature of the system lets us calculate the output of the system in response to some arbitrary (finite) input as a weighted sum of $h[\cdot]$.

We see this by representing the input as a weighted sum of $\delta[n]$.

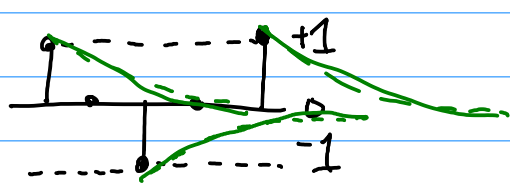
$x[n] =$



$= \delta[n] - \delta[n-2] + \delta[n-4]$

separate into pieces graphically on board

$y[n] =$



$h[n] - h[n-2] + h[n-4]$

ANALYZING SYSTEM BEHAVIOR: POLES

Poles: looking at long-term behavior for arbitrary input rather than actual output for specific input.

Poles are the roots of the denominator of the system functional represented as a polynomial in $z = \frac{1}{R}$

$$\frac{1}{1-pR} \xrightarrow{\text{sub } \frac{1}{z}} \frac{1}{1-\frac{p}{z}} \xrightarrow[\text{polynomials in } z]{\text{represent as}} \frac{z}{z-p}$$, so p is a pole!

for now this is mathematical convenience; has more of an interpretation if you carry on in sig/sys

Why do they matter? A system's output $y[n]$ is $\sum_i c_i p_i^n$ where p_i are the poles and c_i are some constants.

We are worried about long-term behavior. Note that as n gets large, the c_i's disappear, as do all the p_i^n's except the p_i with the largest magnitude (dominant pole)

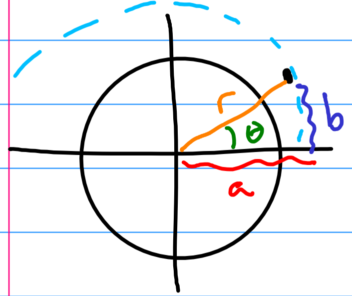
So we have $\boxed{y[n] \sim p_0^n}$ where p_0 is the dominant pole

crucial observation

We can get a good idea of how the system behaves in the long term by seeing how p_0^n changes as n increases.

Poles and Long-term behavior

(geometric view)



Say we have a dominant pole $a + bj$

We are concerned about $(a+bj)^n$

Actually much easier to see in polar form: represent as magnitude r and angle θ.

Then we have $p_0^n = (re^{j\theta})^n = r^n e^{j\theta n}$

Notice that our $y[n]$ then has a magnitude that is like r^n and an angle that is like $n\theta$. We have a magnitude that is changing exponentially, and an angle that is changing linearly.

From this, we can see:

If $r > 1$: the system diverges

If $r < 1$: the system converges

$r=1$ is interesting.. other poles take over sort of

If $b = 0$ and $a > 0$ $(\theta = 0)$: monotonic change

If $b = 0$ and $a < 0$ $(\theta = \pi)$: "alternation"

If $b \neq 0$, oscillates with period $\frac{2\pi}{\theta}$