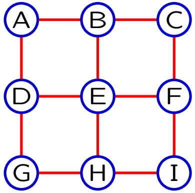
6.01

Lecture 13: Uniform-Cost Search

Last Time: Graph Search

Find path between 2 points in an arbitrary graph.



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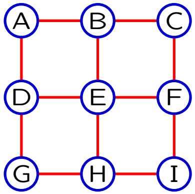
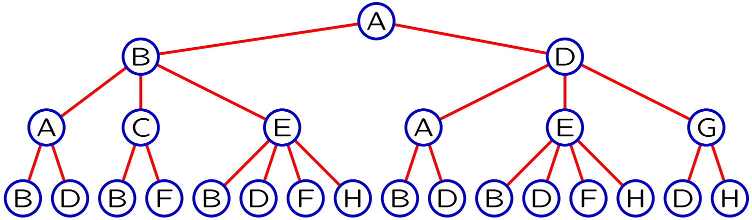
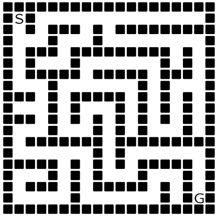


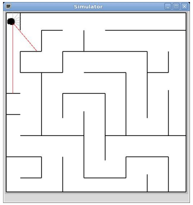
Represent **all possible paths** from A with a **tree**:



Labs

Last Week: Robots in Mazes
This Week: Uniform Cost Search, MapQuest





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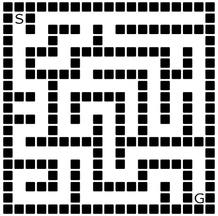
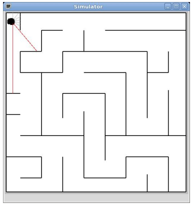
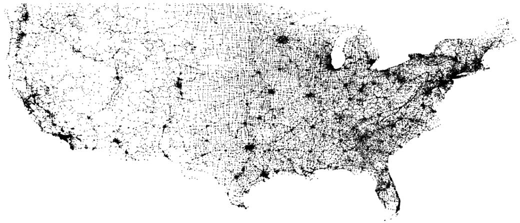






Graph Search Algorithm

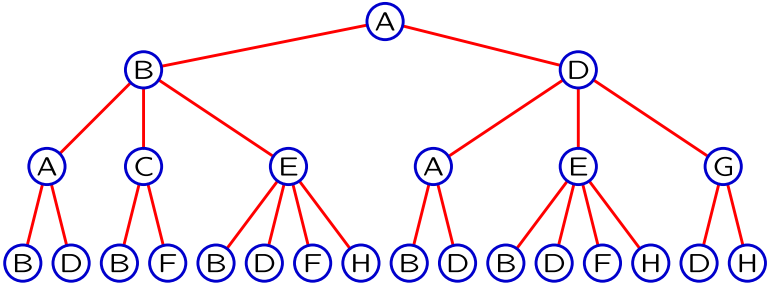
Basic Algorithm:

- Initialize **agenda** (list of nodes to consider)
- Repeat the following:
  - Remove one node from the agenda
  - Add its children to the agenda

  until **goal is found** or **agenda is empty**
- Return resulting path

Order Matters!

Strategy: Replace last node in agenda by its successors



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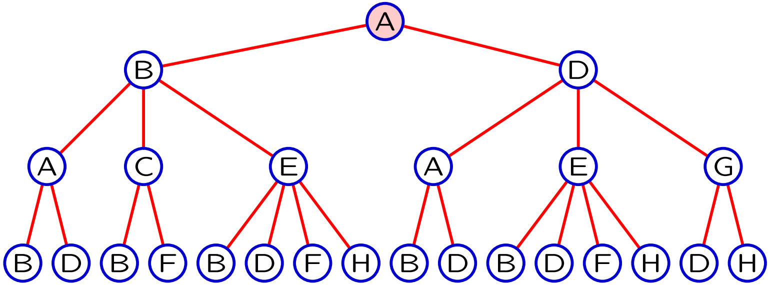

Agenda: A

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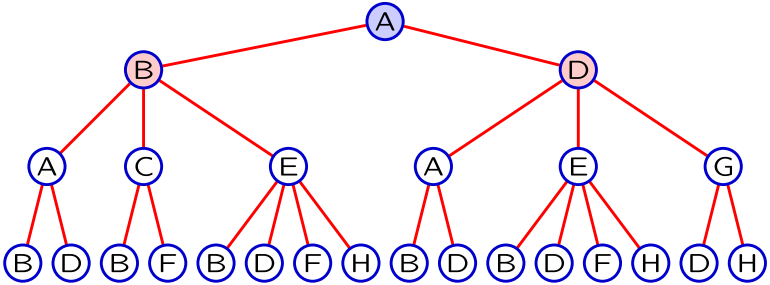

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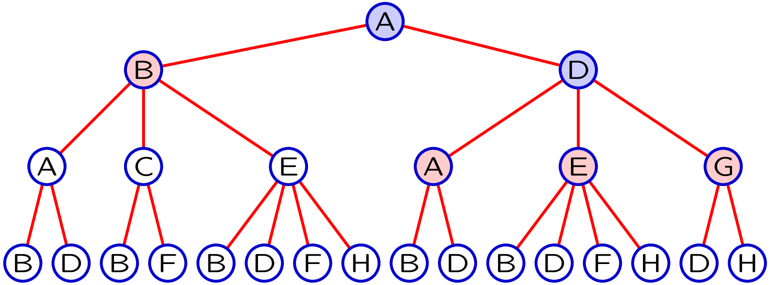

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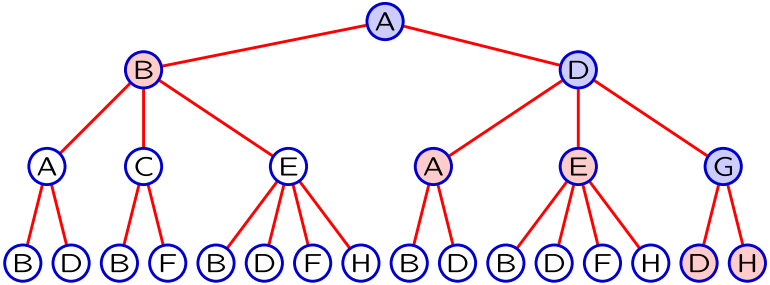

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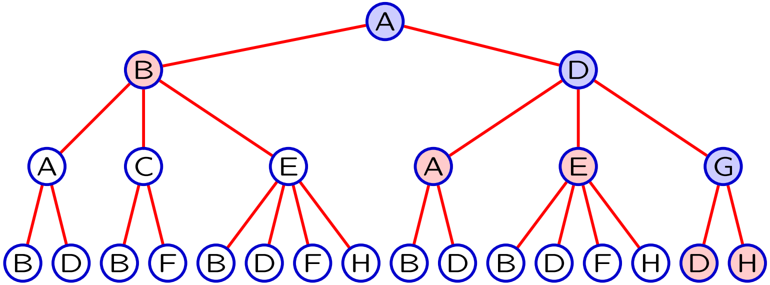

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Depth-first Search

Order Matters!

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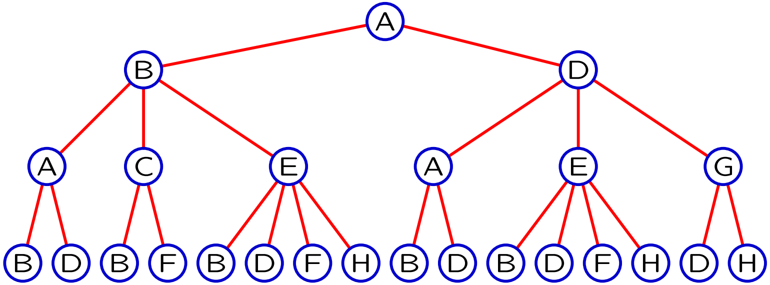

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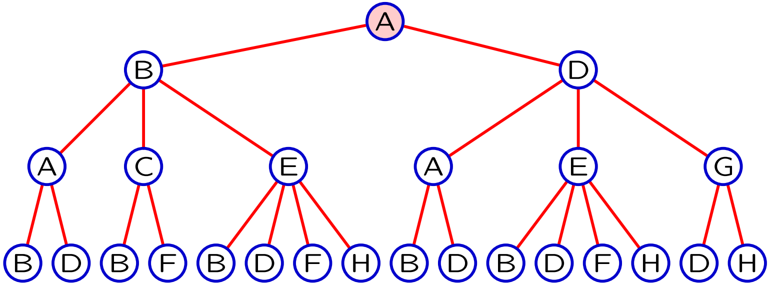

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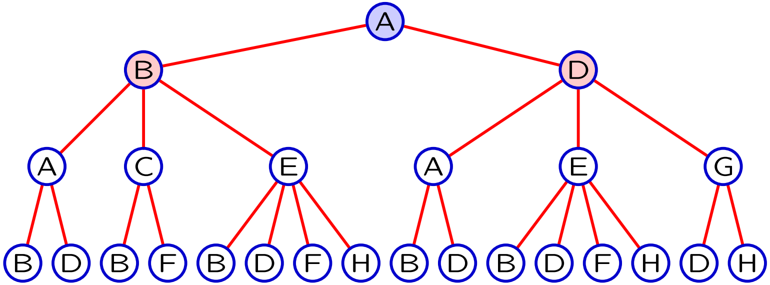

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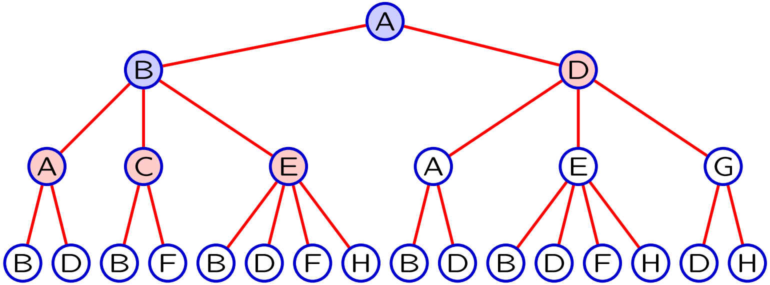

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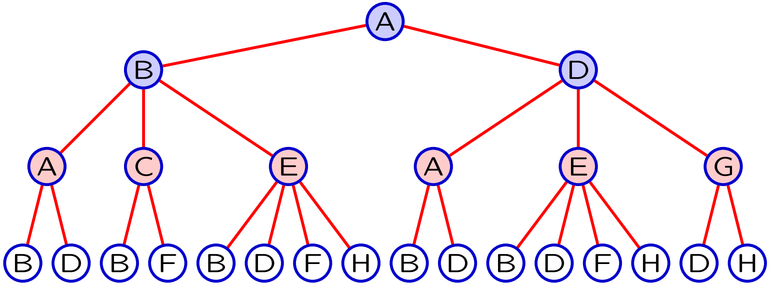

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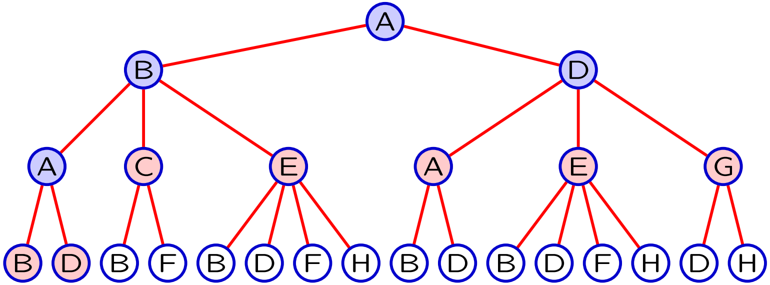

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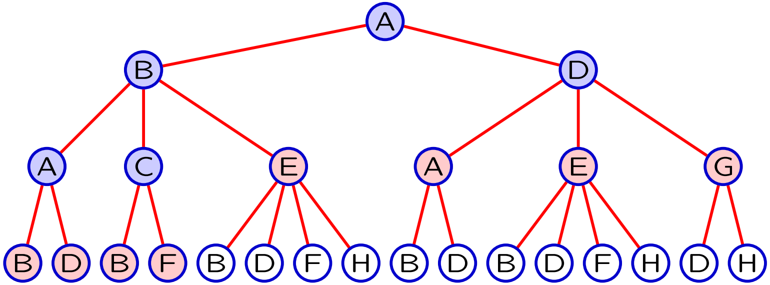

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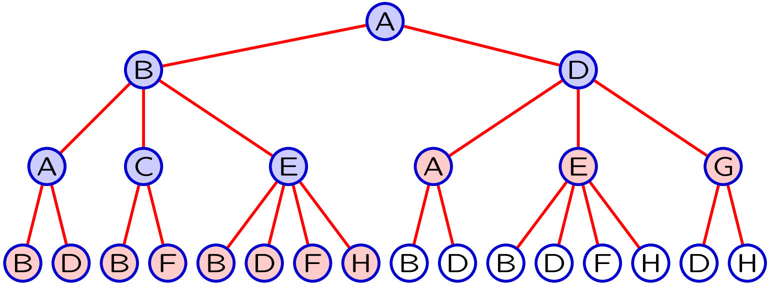

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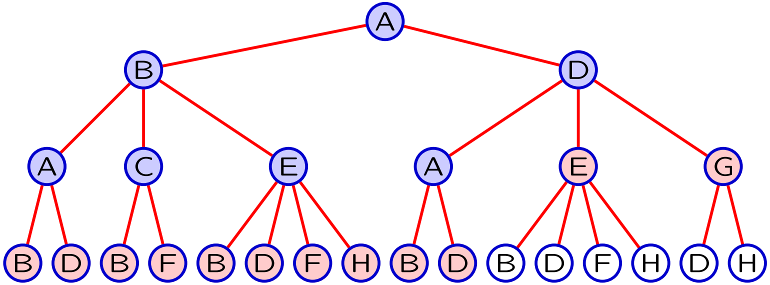

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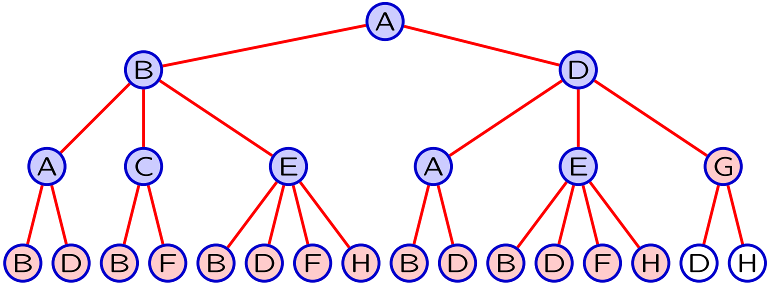

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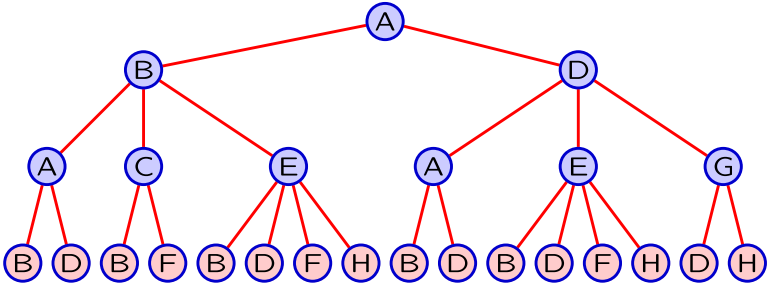

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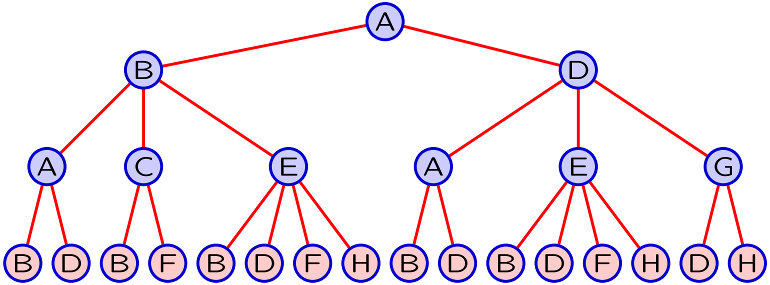

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Breadth-first Search

Dynamic Programming

As applies to search:
(Depends slightly on which algorithm we're using)

BFS: The shortest path $S \to X \to G$ is made up of the shortest path $S \to X$ and the shortest path $X \to G$.

DFS: A path $S \to X \to G$ is made up of a path $S \to X$ and a path $X \to G$.

The moral: once we have found a path $S \to X$, we don't need to spend time looking for other paths through X.

Dynamic Programming

Algorithm (including dynamic programming):

- Initialize **agenda** (list of nodes to consider)
- Intialize visited set (set of states visited)
- Repeat the following:
  - Remove one node from the agenda
  - For each of that node's children:
    - If its state is in the visited list, skip it
    - Otherwise, add it to agenda and add its state to visited list

  until **goal is found** or **agenda is empty**
- Return resulting path

Python Framework

- `SearchNode` class:
  - Attributes:
    - state (arbitrary)
    - parent (instance of `SearchNode`, or `None`)
  - Methods:
    - path (returns list of states representing path from root)

- `search` function:
  - Arguments:
    - successor function (function state$\rightarrow$list of states)
    - starting state
    - goal test (function state$\rightarrow$`bool`)
    - dfs (`True` for DFS, `False` for BFS)

16 Lines!

```
def search(successors, start_state, goal_test, dfs = False):
    if goal_test(start_state):
        return [start_state]
    else:
        agenda = [SearchNode(start_state, None)]
        visited = {start_state}
        while len(agenda) > 0:
            parent = agenda.pop(-1 if dfs else 0)
            for child_state in successors(parent.state):
                child = SearchNode(child_state, parent)
                if goal_test(child_state):
                    return child.path()
                if child_state not in visited:
                    agenda.append(child)
                    visited.add(child_state)
        return None
```

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        while len(agenda) > 0:
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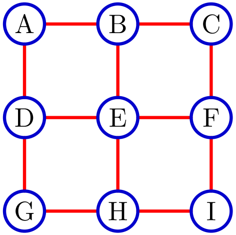
            for child_state in successors(parent.state):
                child = SearchNode(child_state, parent)
                if goal_test(child_state):
                    return child.path()
                if child_state not in visited:
                    agenda.append(child)
                    visited.add(child_state)
        return None
```

Example

Find path $A \rightarrow I$, BFS w/ DP

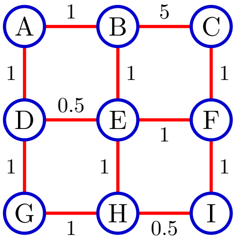


What is a Graph?

Set V of vertices
Set E of edges connecting vertices
Set W of edge costs (or “weights”)

Example

Find path $A \rightarrow I$, *minimizing total cost*



Uniform-Cost Search

Consider searching for **least-cost** paths instead of *shortest* paths.
Instead of popping from agenda based on when nodes were added, pop based on of the cost of the paths they represent.

Slight change to framework:

- `SearchNode` class:
  - Attributes:
    - state (arbitrary)
    - parent (instance of `SearchNode`, or `None`)
    - cost of whole path from start
  - Methods:
    - path (returns list of states representing path from root)

- `uniform_cost_search` function:
  - Arguments:
    - successor function (state$\rightarrow$list of (state,cost) tuples)
    - starting state
    - goal test (function state$\rightarrow$`bool`)

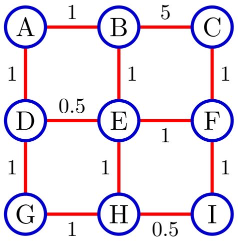
UC Search

```
def uniform_cost_search(successors, start_state, goal_test):
    if goal_test(start_state):
        return [start_state]
    agenda = [(0, SearchNode(start_state, None, cost=0))]
    expanded = set()
    while len(agenda) > 0:
        agenda.sort()
        priority, parent = agenda.pop(0)
        if parent.state not in expanded:
            expanded.add(parent.state)
            if goal_test(parent.state):
                return parent.path()
            for child_state, cost in successors(parent.state):
                child = SearchNode(child_state, parent, parent.cost+cost)
                if child_state not in expanded:
                    agenda.append((child.cost,child))
    return None
```

Testing for goal condition must be done at **expansion** time, not at visit time. Similarly for dynamic programming.

Example

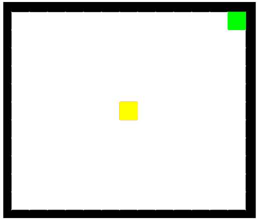
Find path $A \rightarrow I$, Uniform Cost Search



Problem?

So far, searches have radiated outward from the starting point.

We only notice the goal when we stumble upon it.



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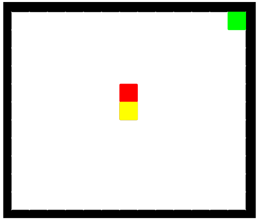

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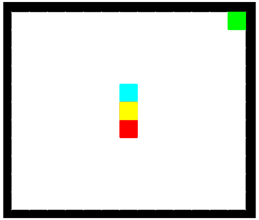

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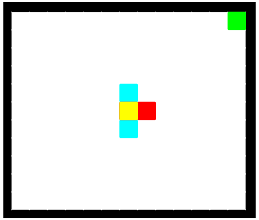

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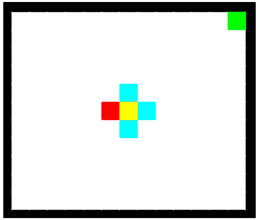

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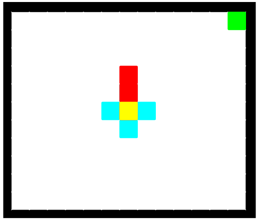

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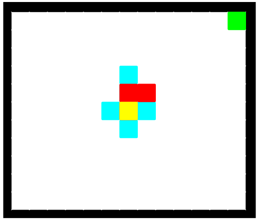

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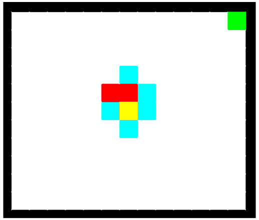

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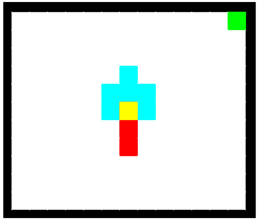

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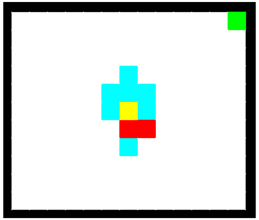

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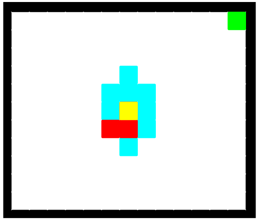

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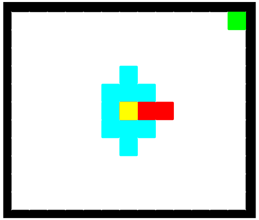

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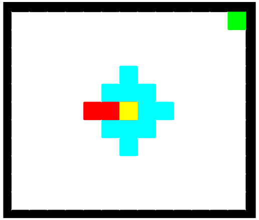

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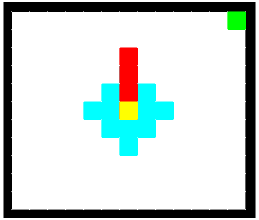

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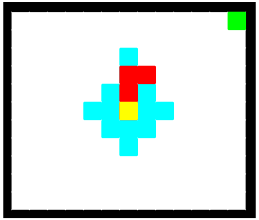

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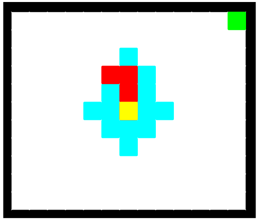

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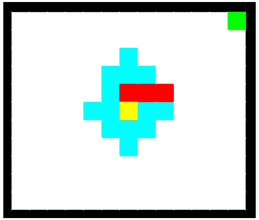

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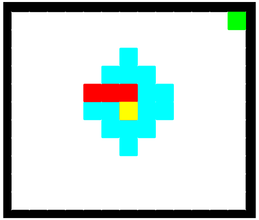

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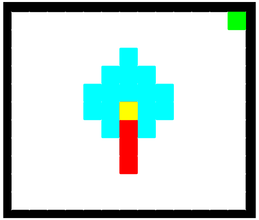

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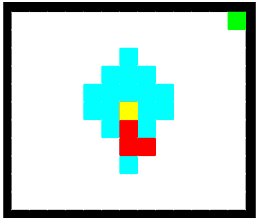

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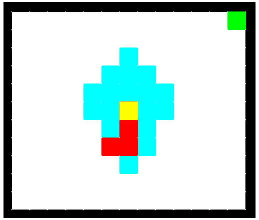

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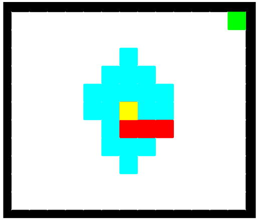

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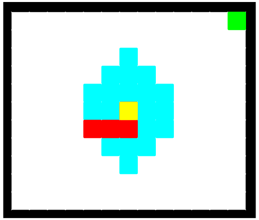

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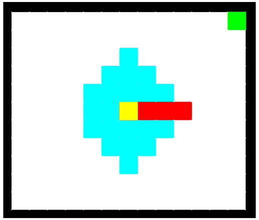

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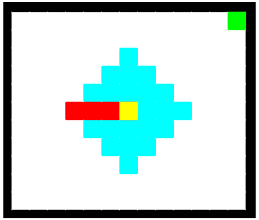

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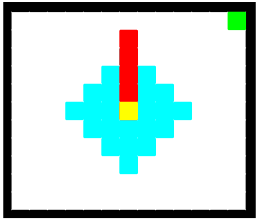

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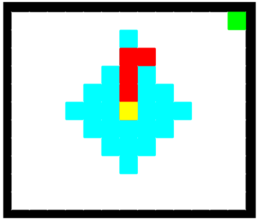

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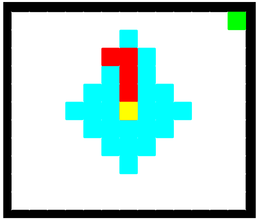

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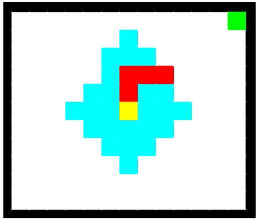

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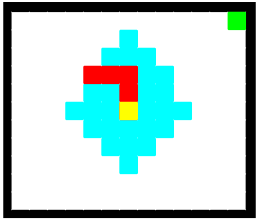

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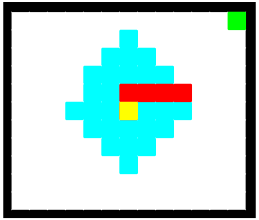

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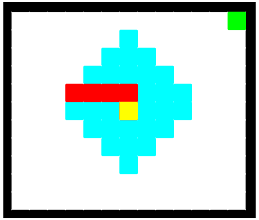

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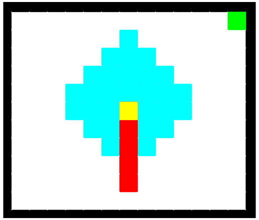

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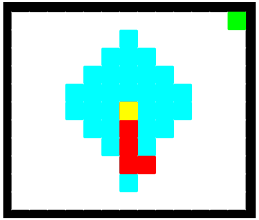

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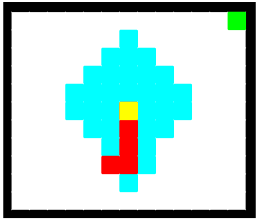

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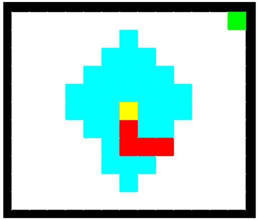

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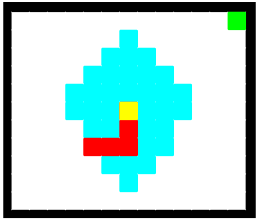

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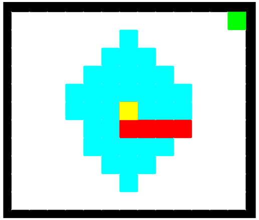

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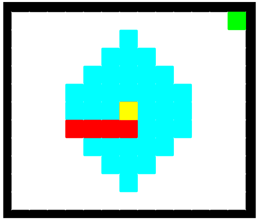

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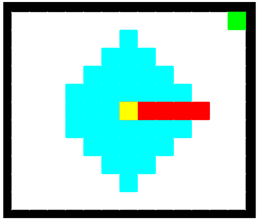

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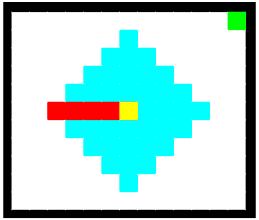

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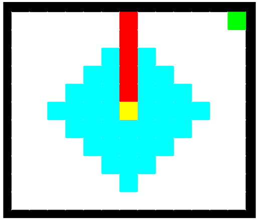

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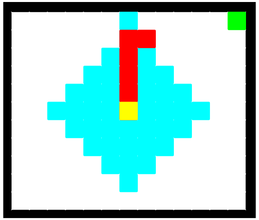

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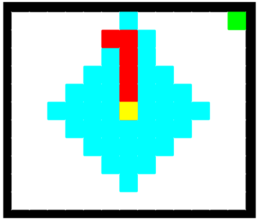

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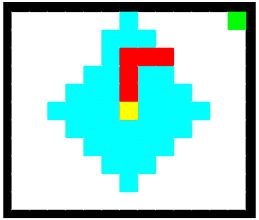

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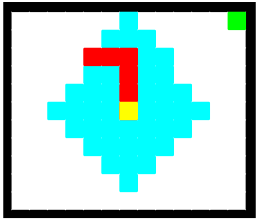

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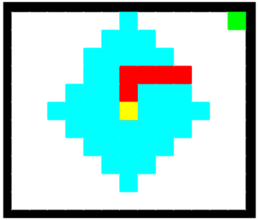

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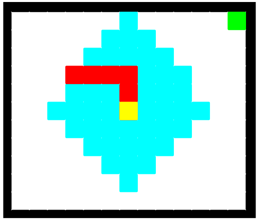

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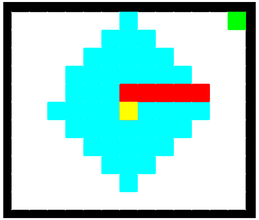

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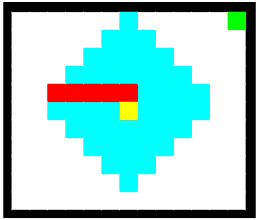

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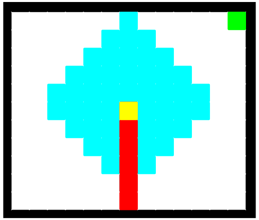

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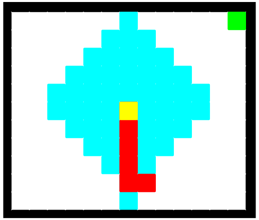

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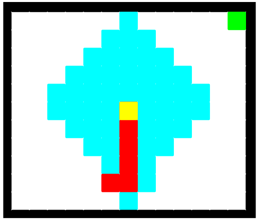

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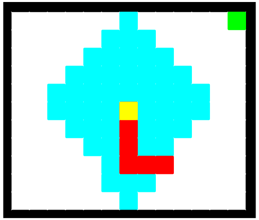

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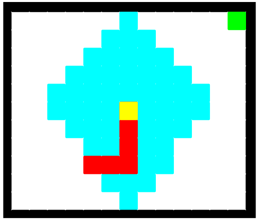

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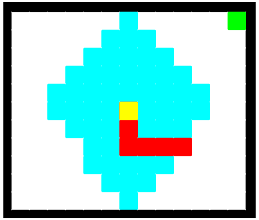

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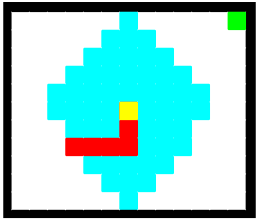

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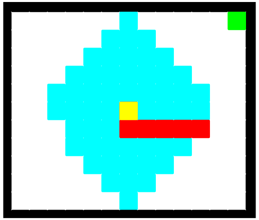

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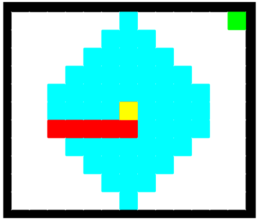

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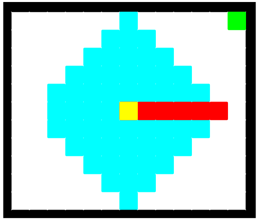

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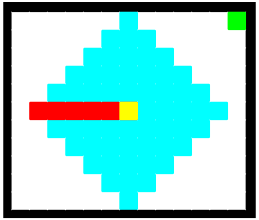

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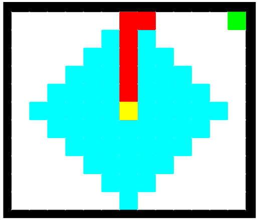

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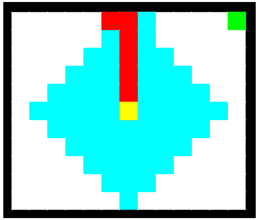

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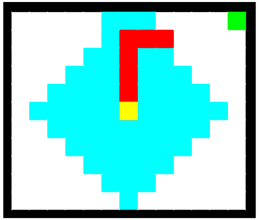

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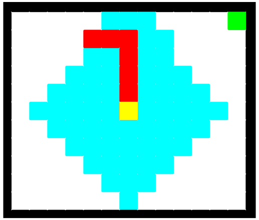

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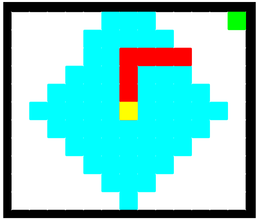

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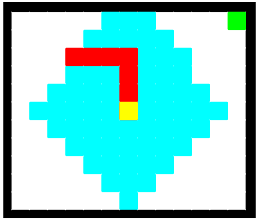

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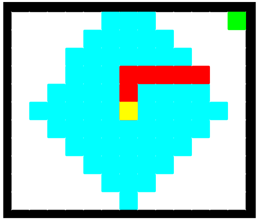

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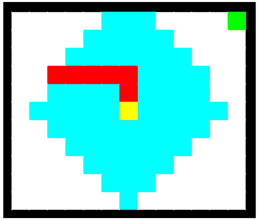

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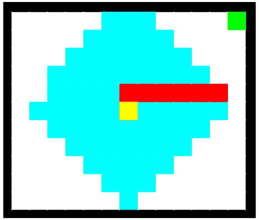

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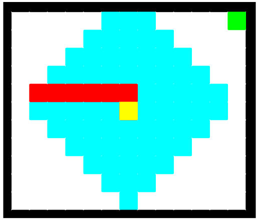

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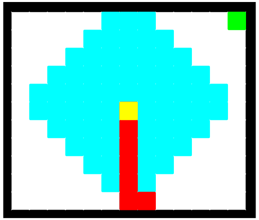

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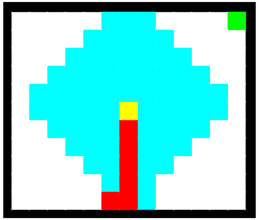

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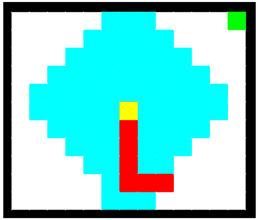

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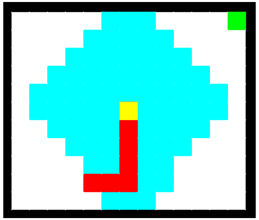

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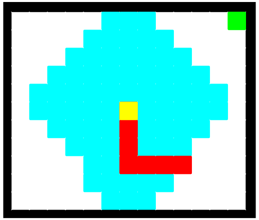

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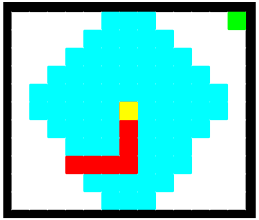

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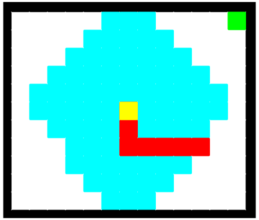

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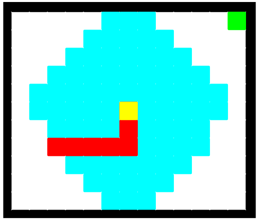

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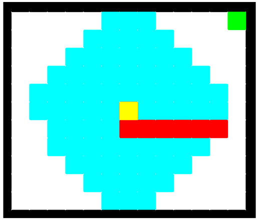

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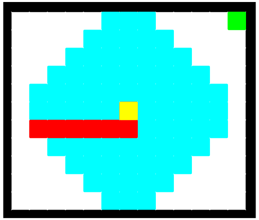

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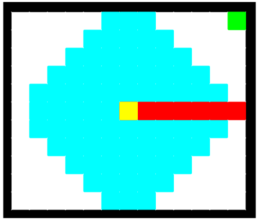

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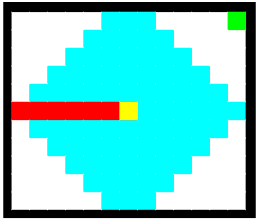

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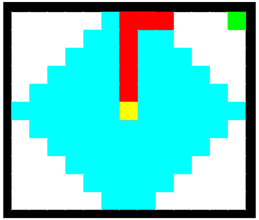

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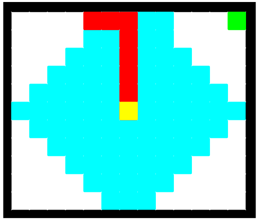

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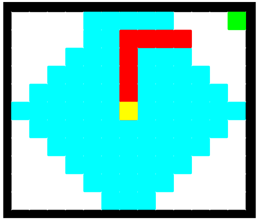

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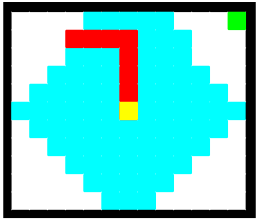

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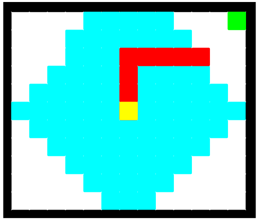

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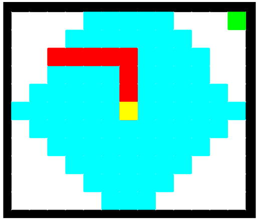

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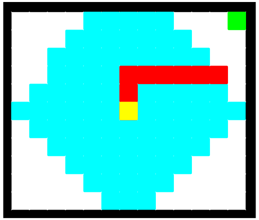

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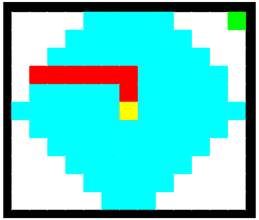

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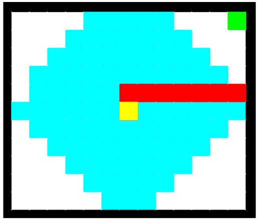

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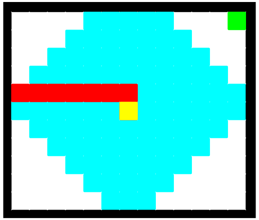

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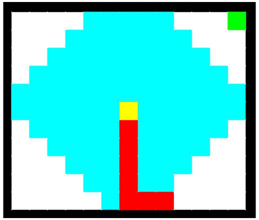

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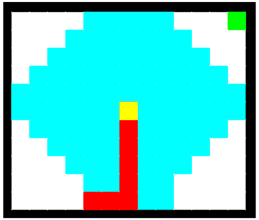

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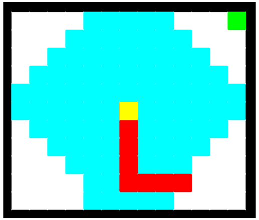

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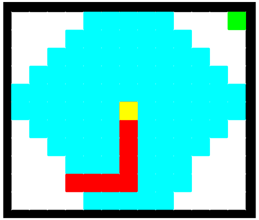

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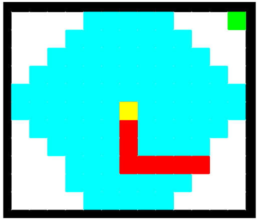

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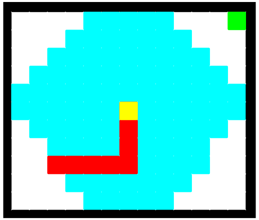

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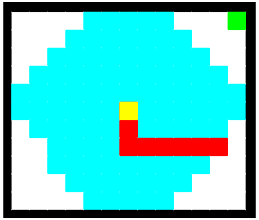

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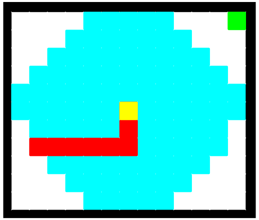

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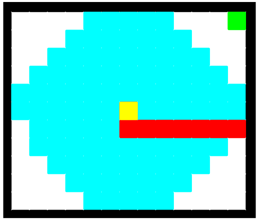

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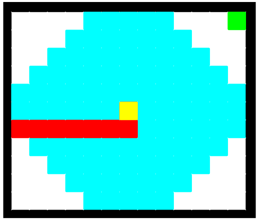

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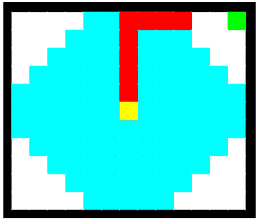

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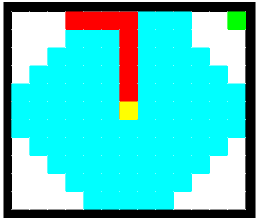

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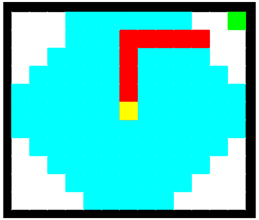

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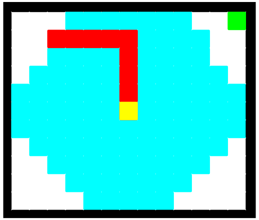

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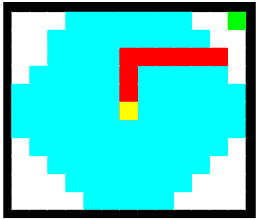

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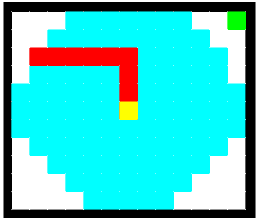

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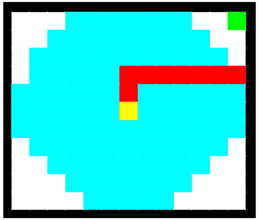

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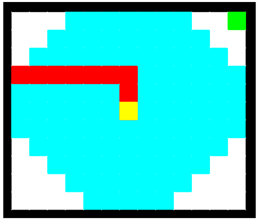

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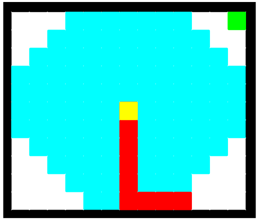

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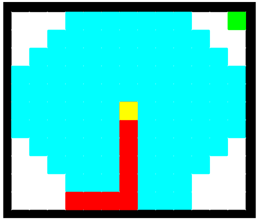

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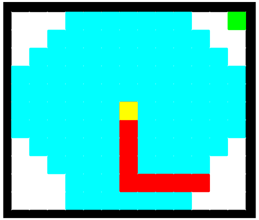

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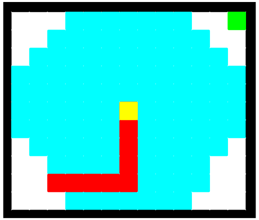

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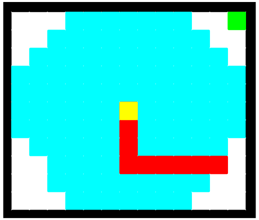

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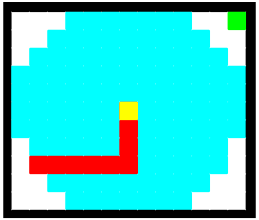

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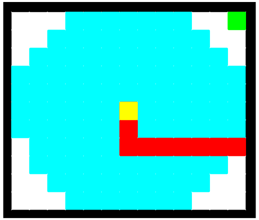

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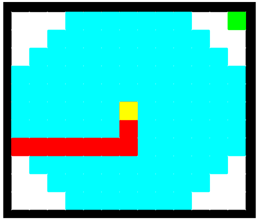

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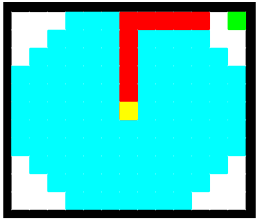

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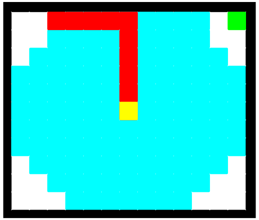

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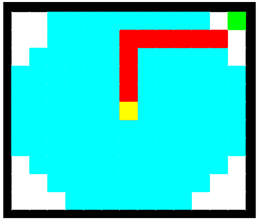

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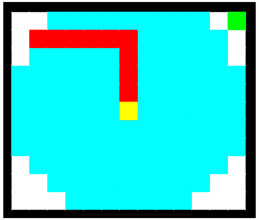

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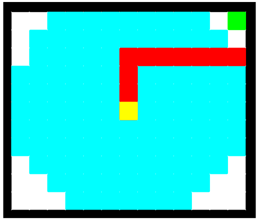

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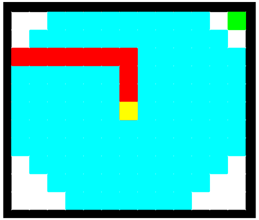

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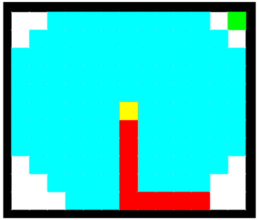

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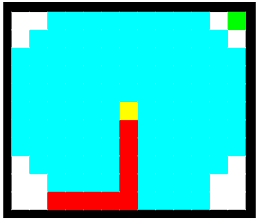

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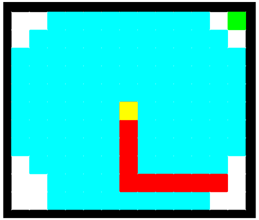

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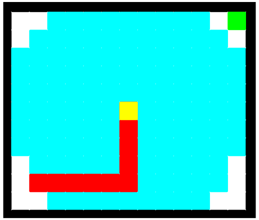

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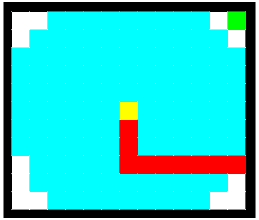

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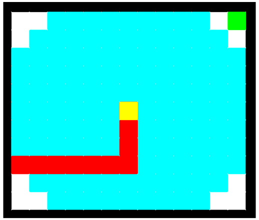

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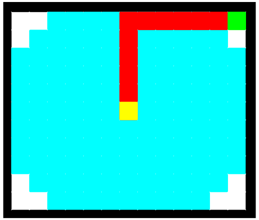

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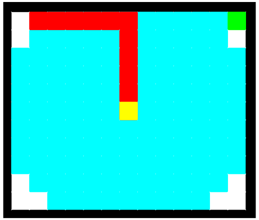

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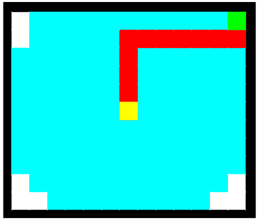

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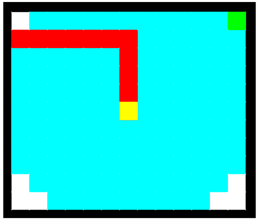

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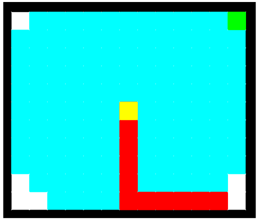

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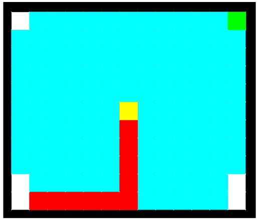

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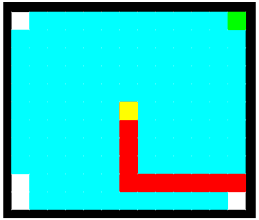

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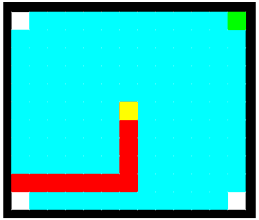

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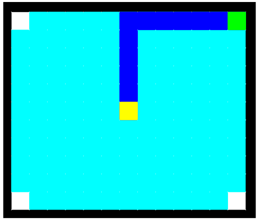

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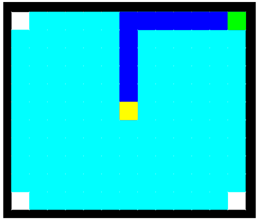
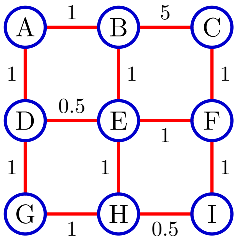


Too much time spent searching on the **wrong side of the goal**.

Example

Find path $E \to I$, Uniform Cost Search



Heuristics

So far, our searches only consider **start-to-current**. We can add **heuristics** to consider an estimate of **current-to-goal** as well.

$h(x)$ estimate of cost of lowest-cost path $X \rightarrow$ goal

- `SearchNode` class:
  - Attributes:
    - state (arbitrary)
    - parent (instance of `SearchNode`, or `None`)
    - cost of whole path from start
  - Methods:
    - path (returns list of states representing path from root)

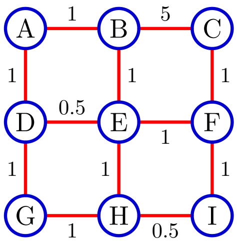
- `uniform_cost_search` function:
  - Arguments:
    - successor function (state$\rightarrow$list of (state,cost) tuples)
    - starting state
    - goal test (function state$\rightarrow$`bool`)
    - heuristic (function state$\rightarrow$estimated cost)

UC Search with Heuristic (A*)

```
def uniform_cost_search(successors, start_state, goal_test, heuristic=lambda s: 0):
    if goal_test(start_state):
        return [start_state]
    agenda = [(heuristic(start_state), SearchNode(start_state, None, cost=0))]
    expanded = set()
    while len(agenda) > 0:
        agenda.sort()
        priority, parent = agenda.pop(0)
        if parent.state not in expanded:
            expanded.add(parent.state)
            if goal_test(parent.state):
                return parent.path()
            for child_state, cost in successors(parent.state):
                child = SearchNode(child_state, parent, parent.cost+cost)
                if child_state not in expanded:
                    agenda.append((child.cost+heuristic(child_state),child))
    return None
```

Example

Find path $E \rightarrow I$, A*, heuristic: $h(s) = M(s, I)/2$



Admissible Heuristics

An **admissible heuristic** does not overestimate the actual cost of the shortest cost path.

If the heuristic $h(s)$ is larger than the actual cost from s to goal, then the "best" solution may be missed!

If the heuristic is an underestimate, the search space will be larger than necessary, but we are guaranteed the shortest path.

The ideal heuristic should be:

- as close as possible to actual cost (without overestimating)
- easy to calculate

A* (without DP) is guaranteed to find least-cost path if heuristic is admissible.
With DP, heuristic must also be *consistent*.

Check Yourself!

Consider searching in a four-action grid (up, down, left, right), where all actions have cost 1. Let `(r0, c0)` represent the current location, and `(r1, c1)` represent the goal.

Which of the following heuristics are **admissible**?

1. `abs(r0-r1) + abs(c0-c1)`
2. `min(abs(r0-r1), abs(c0-c1))`
3. `max(abs(r0-r1), abs(c0-c1))`
4. `2*min(abs(r0-r1), abs(c0-c1))`
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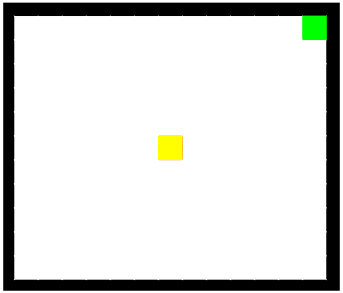

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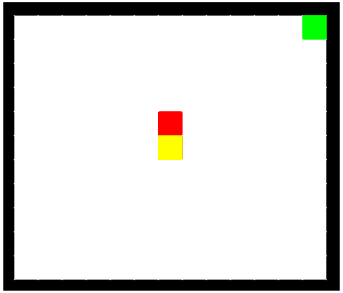

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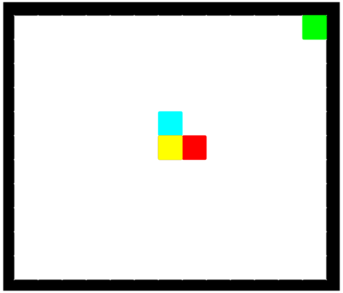

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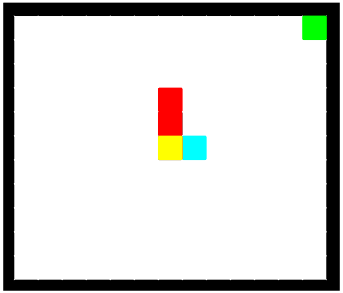

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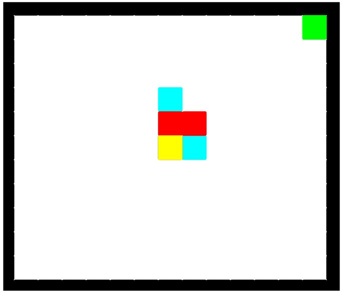

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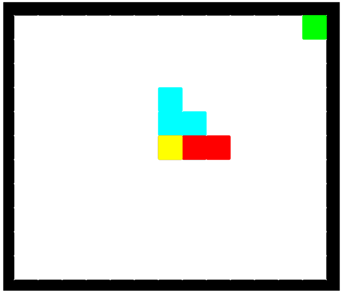

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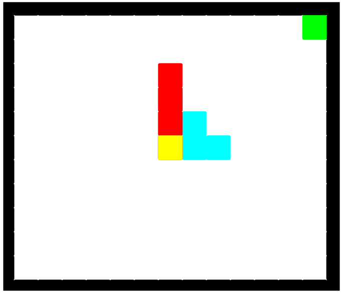

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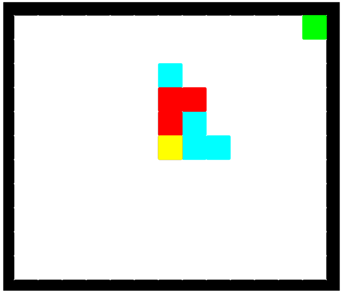

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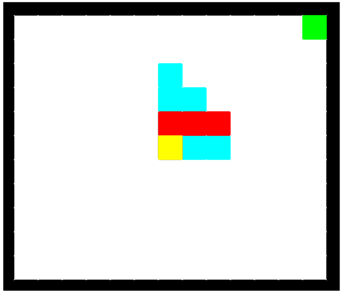

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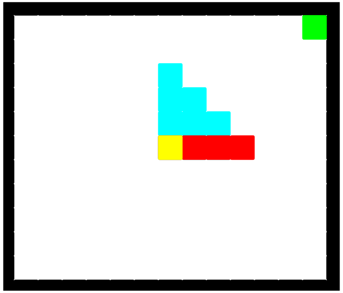

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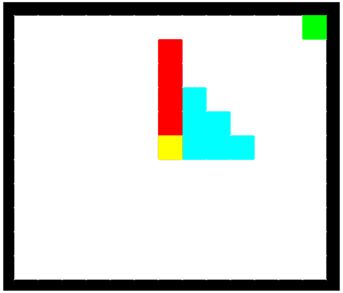

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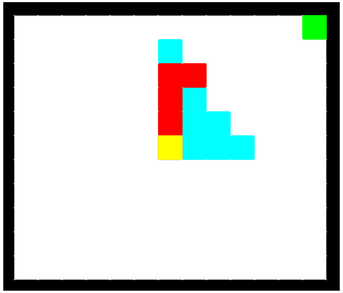

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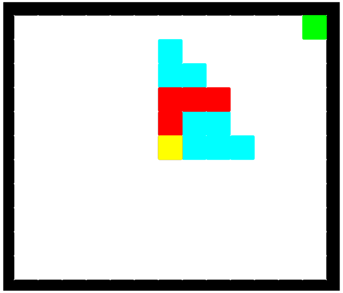

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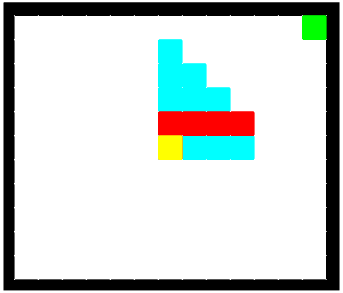

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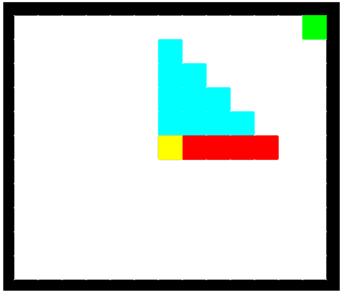

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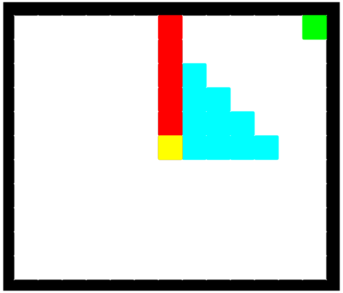

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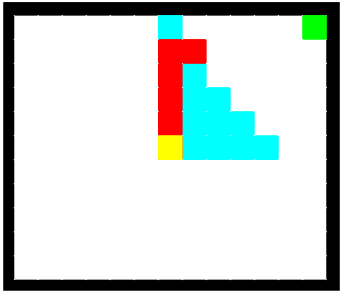

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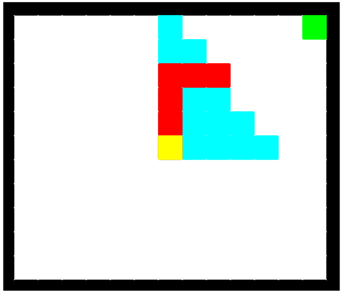

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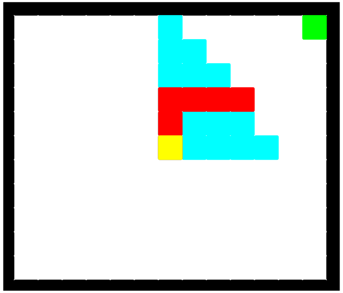

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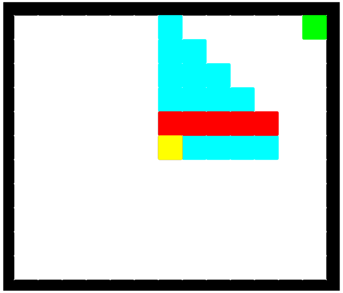

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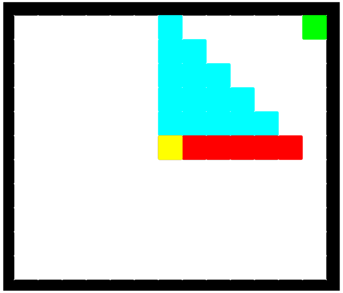

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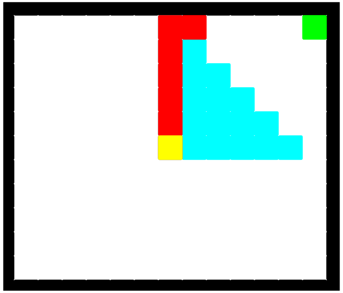

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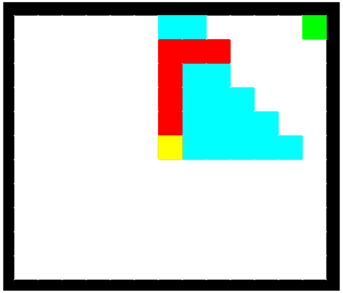

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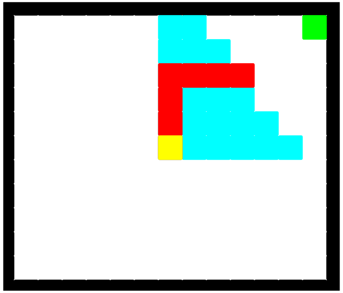

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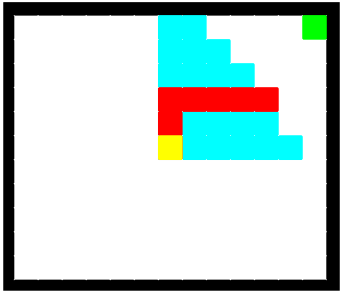

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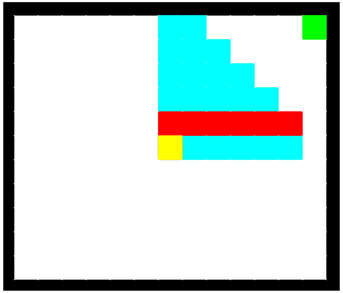

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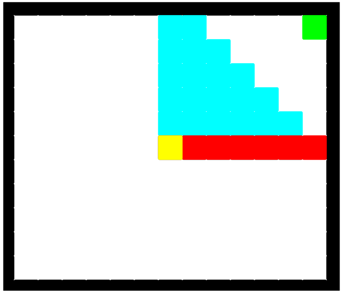

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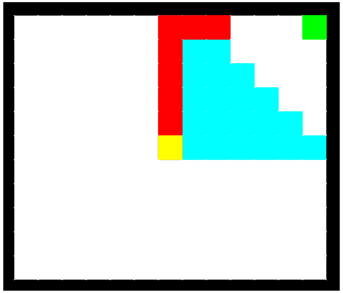

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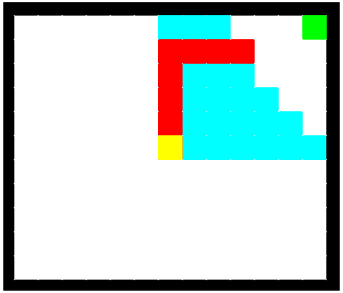

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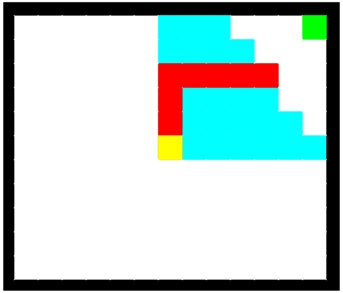

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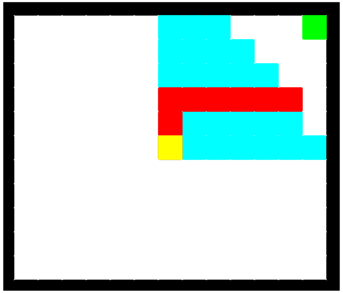

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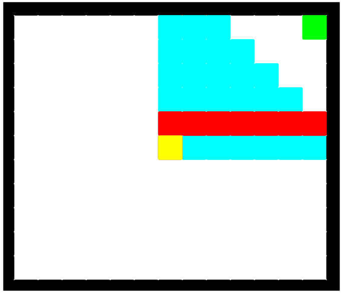

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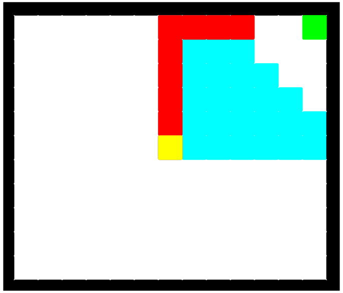

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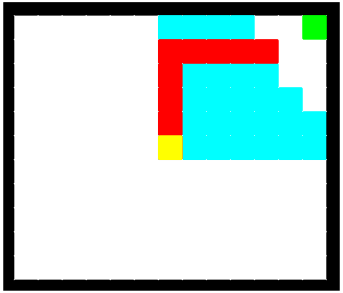

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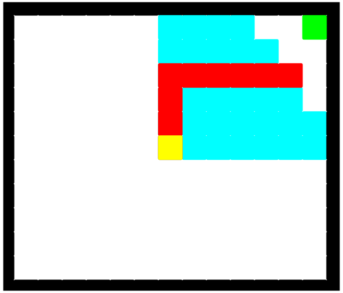

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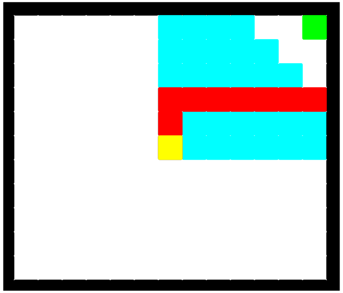

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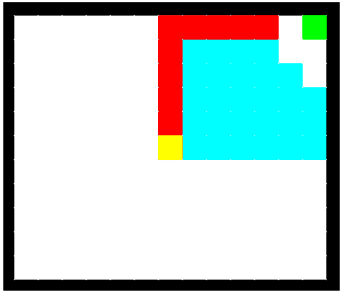

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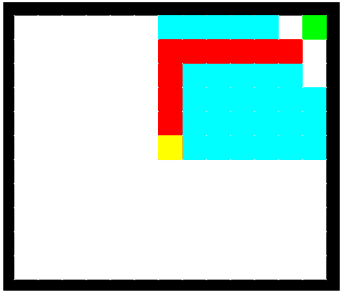

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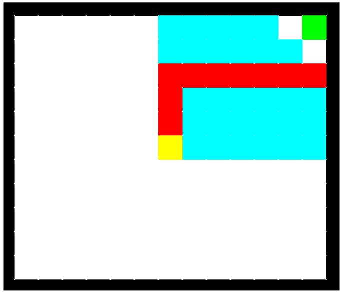

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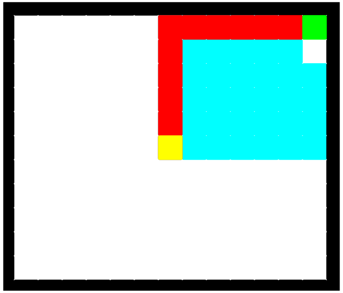

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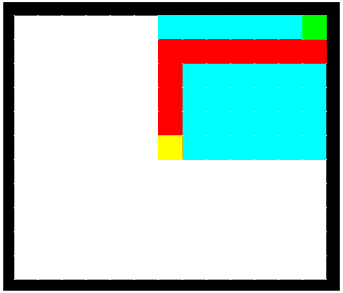

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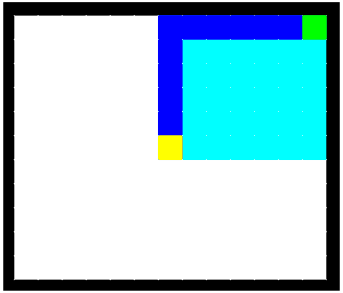

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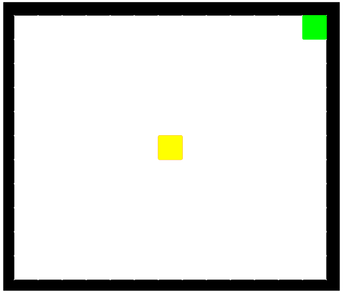

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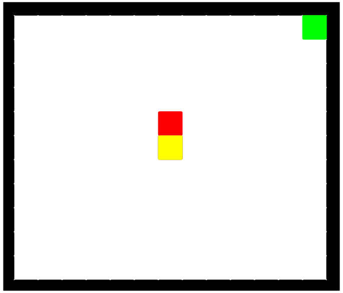

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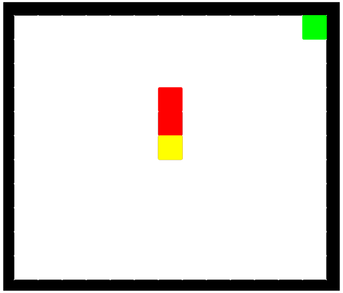

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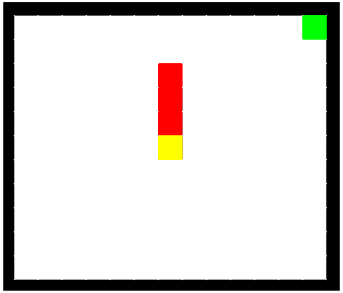

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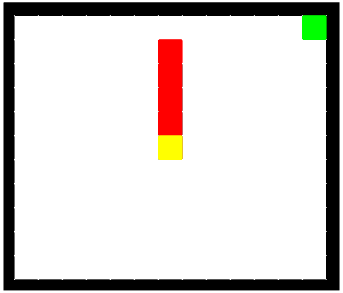
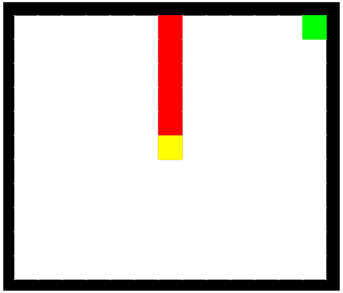


`min(abs(r0-r1), abs(c0-c1))`



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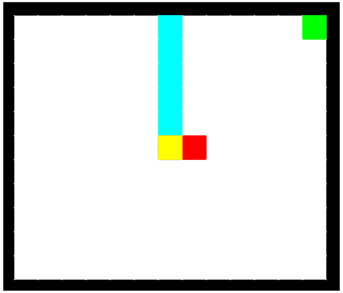

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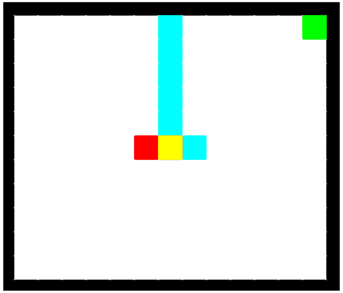

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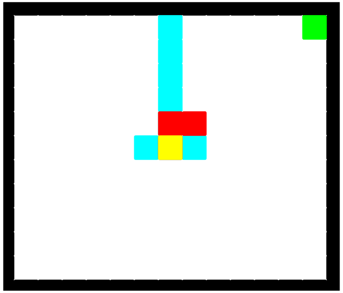

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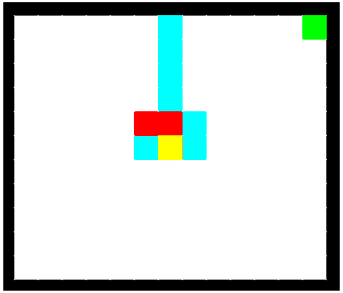

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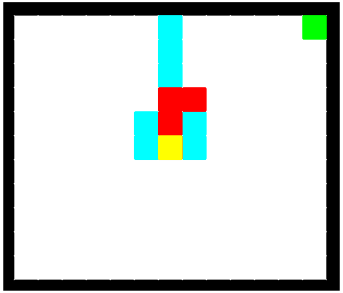

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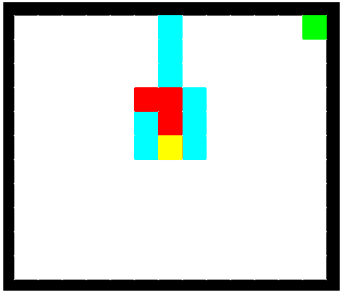

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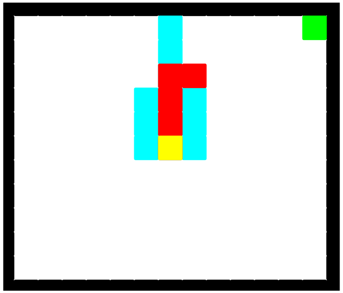

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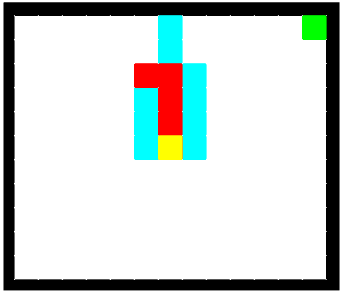

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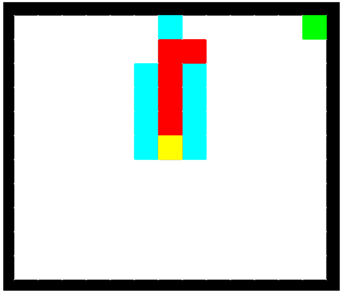

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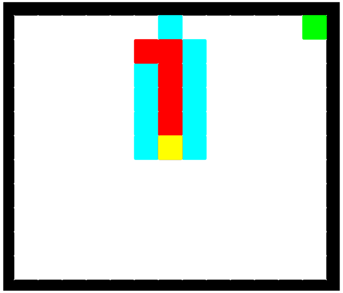

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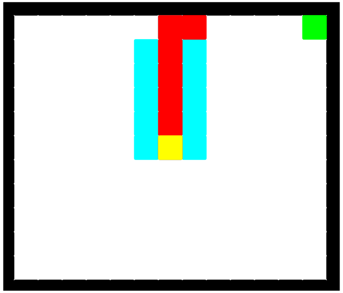

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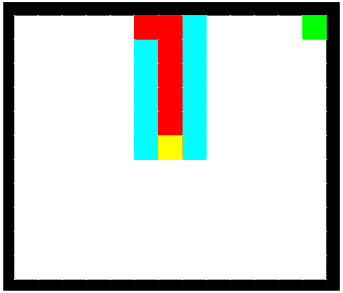

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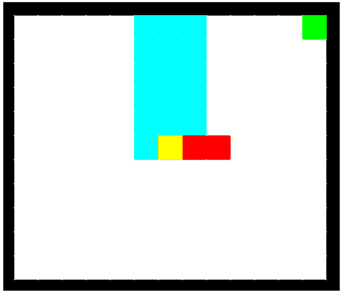

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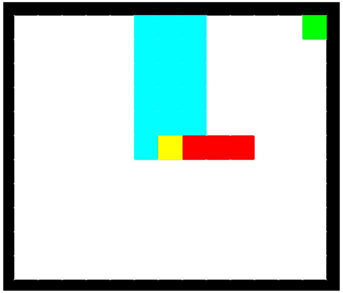

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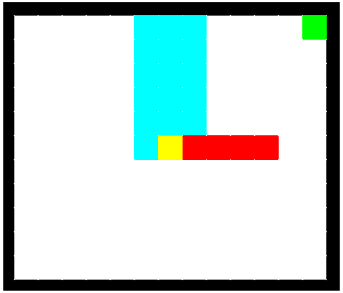

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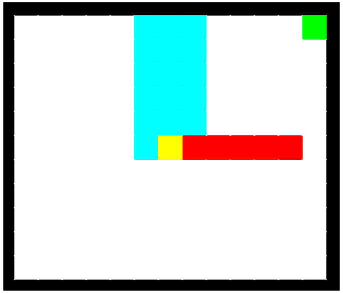

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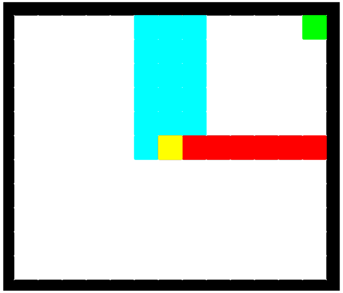

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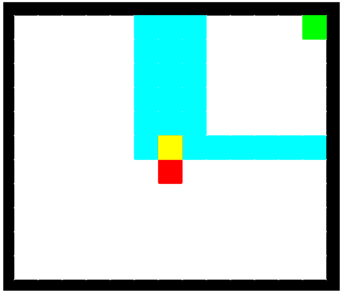

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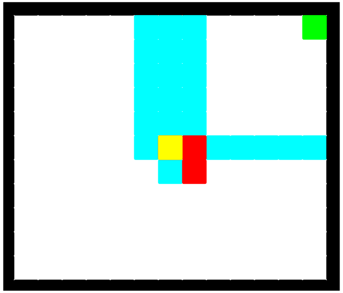

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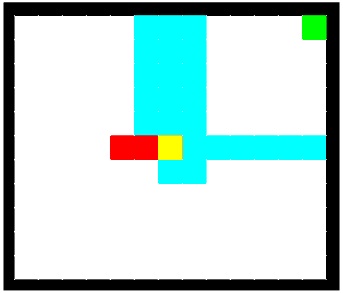

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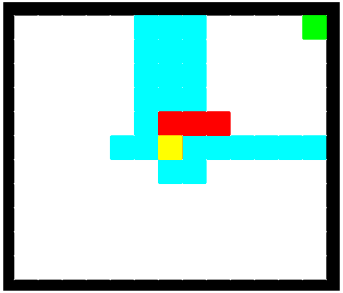

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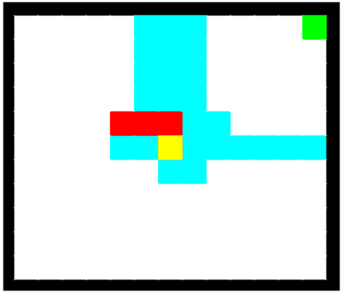

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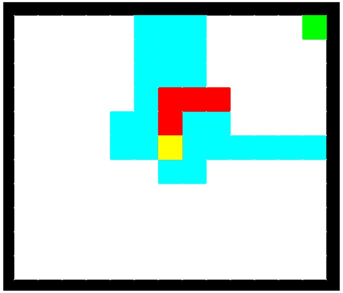

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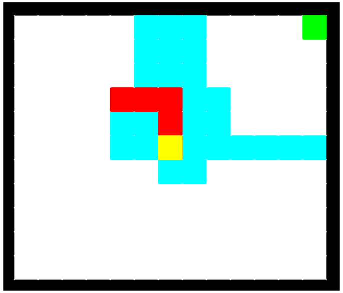

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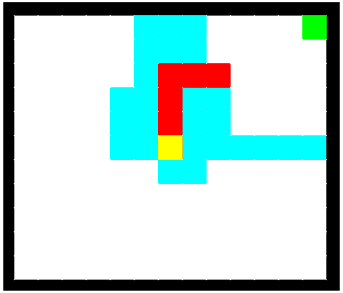

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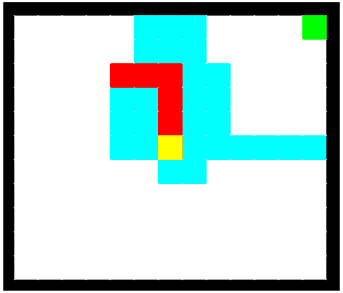

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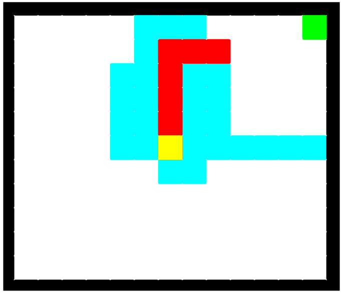

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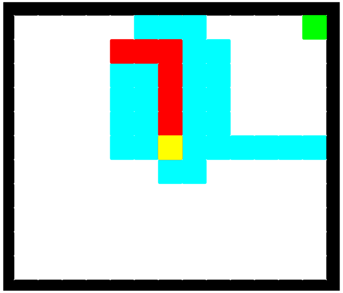

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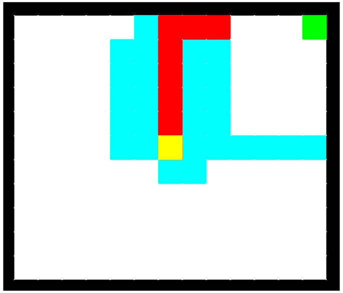

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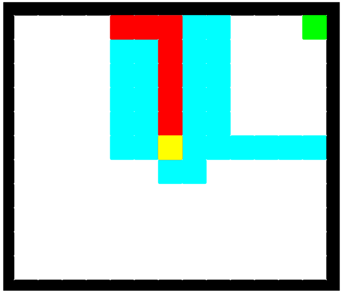

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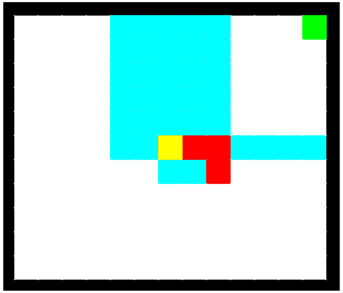

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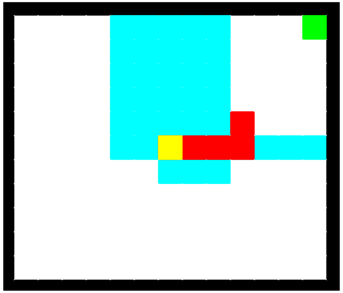

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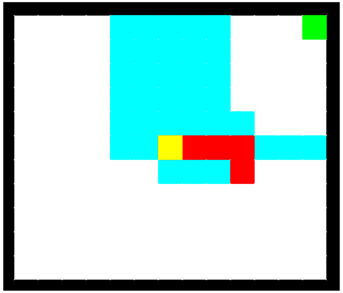

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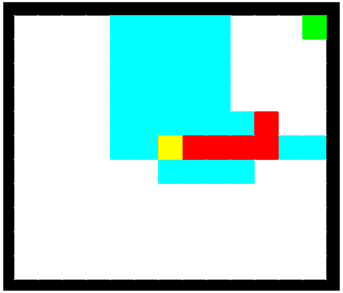

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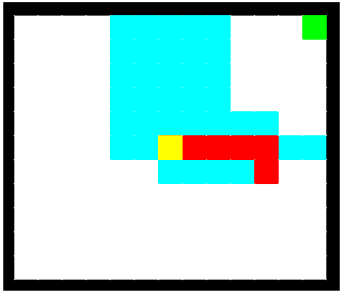

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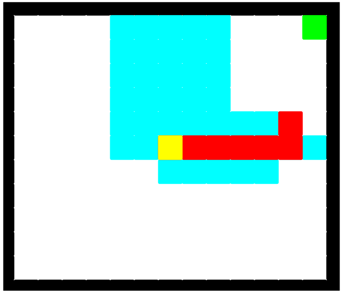

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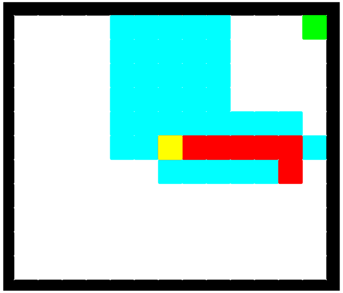

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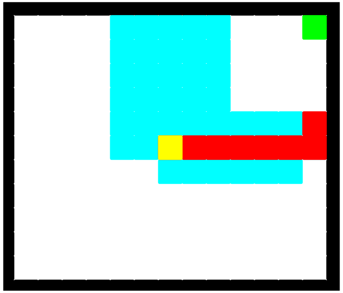

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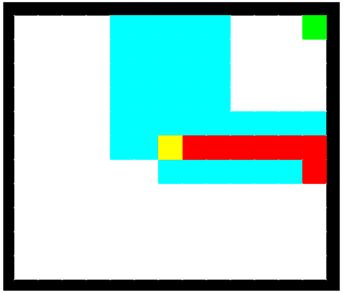

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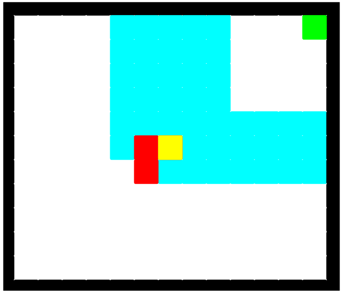

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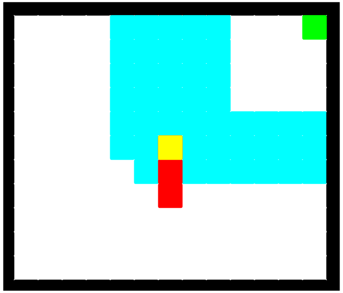

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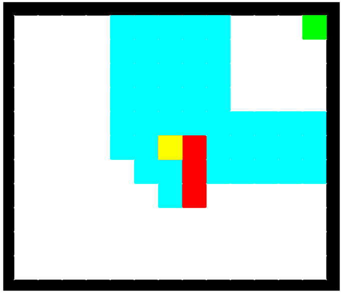

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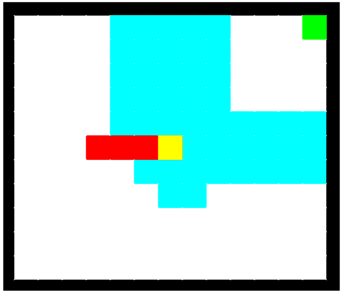

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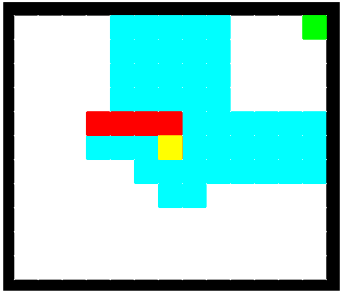

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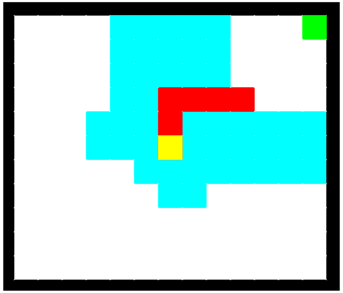

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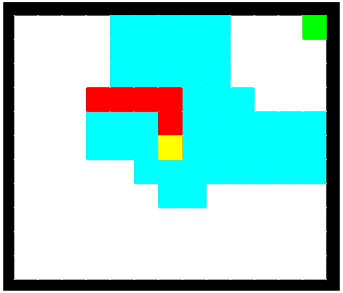

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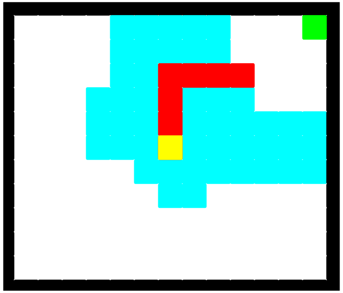

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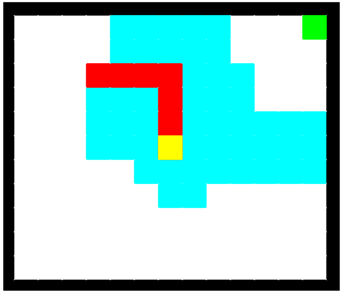

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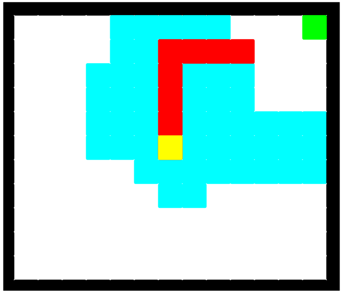

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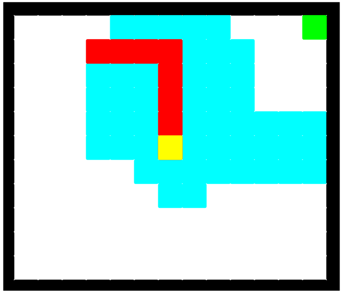

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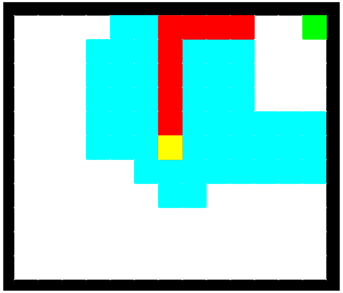

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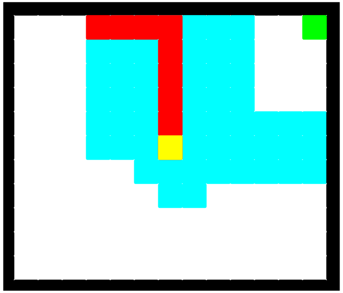

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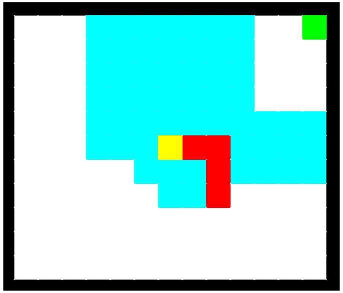

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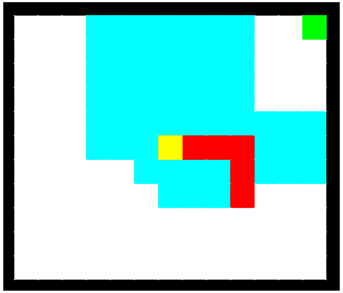

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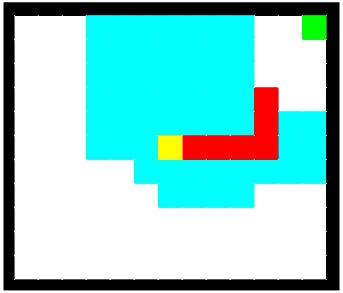

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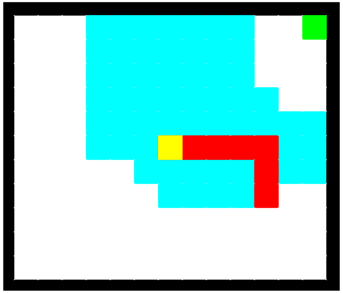

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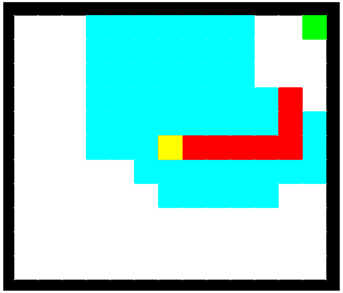

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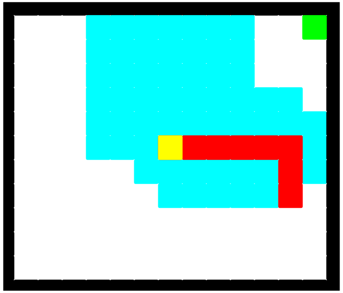

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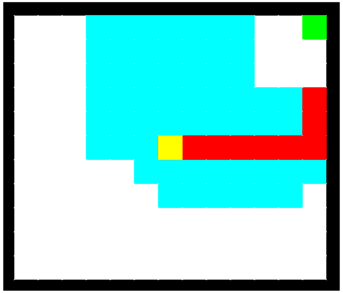

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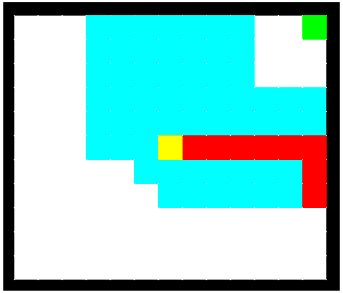

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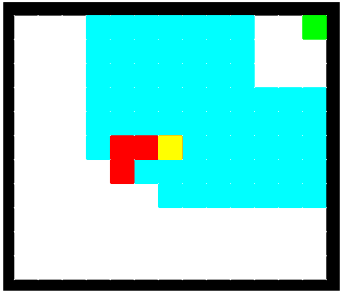

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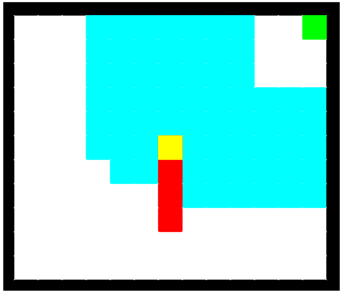

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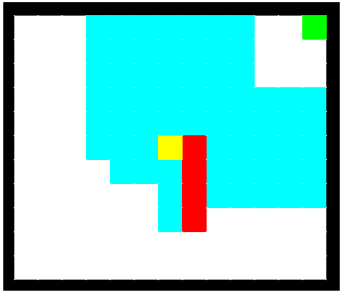

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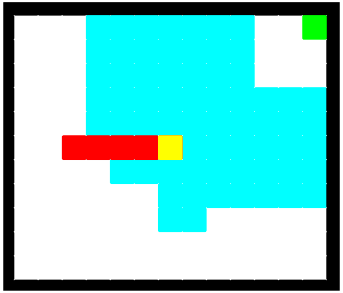

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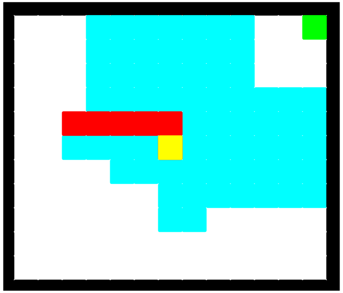

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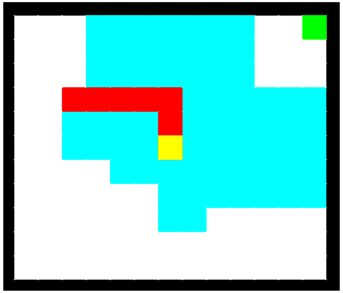

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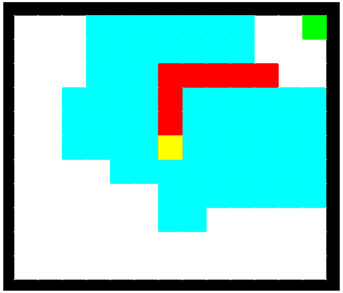

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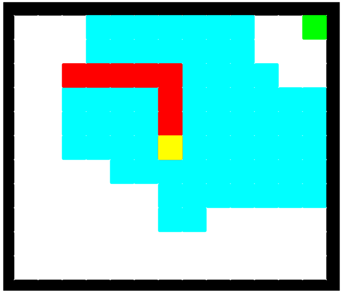

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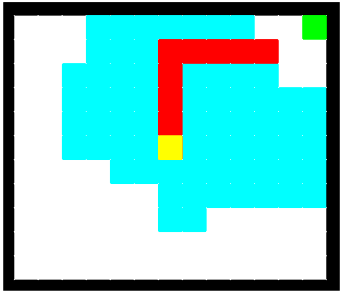

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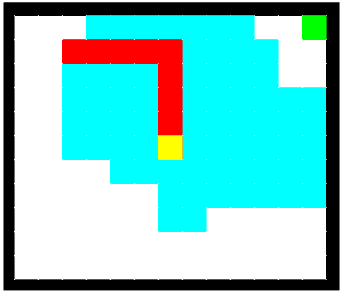

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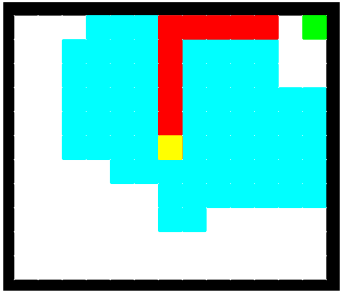

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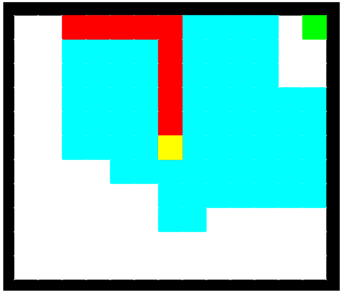

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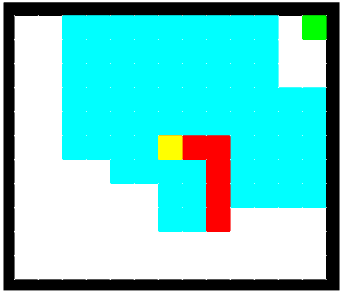

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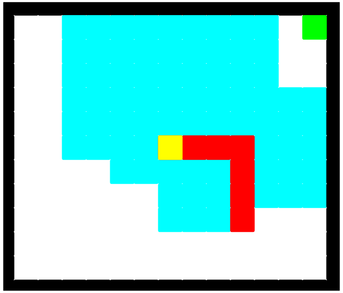

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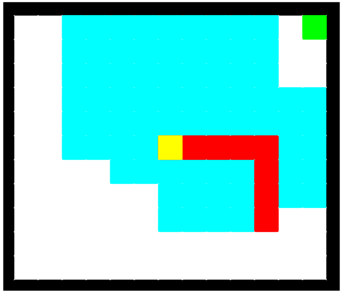

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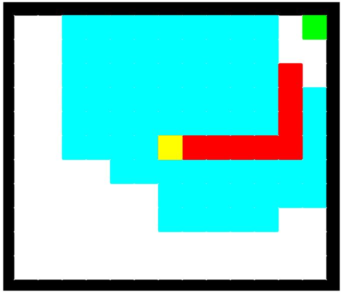

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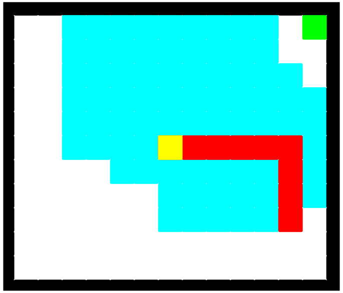

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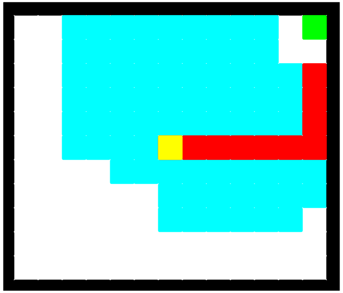

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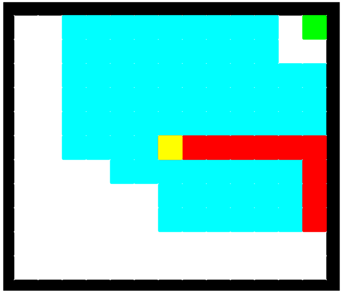

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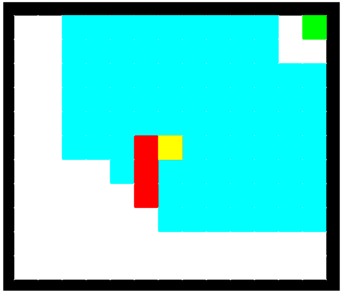

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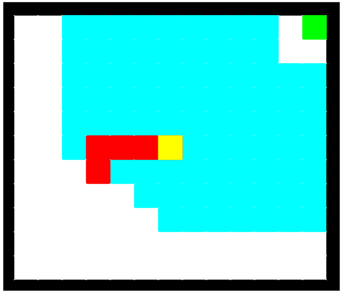

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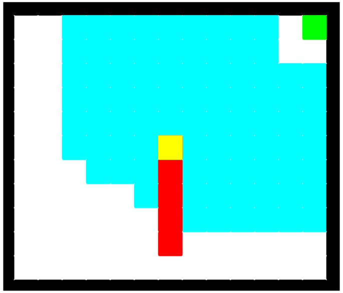

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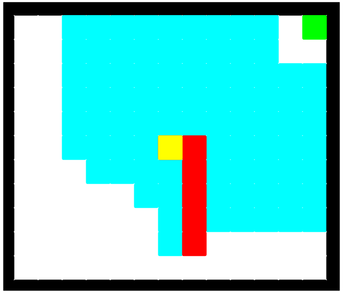

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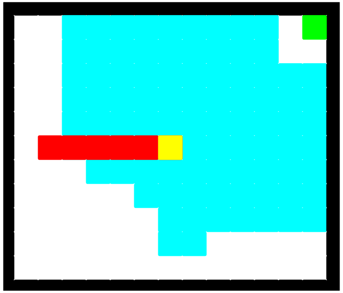

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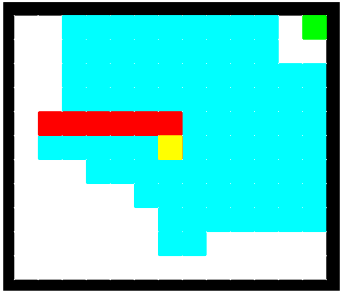

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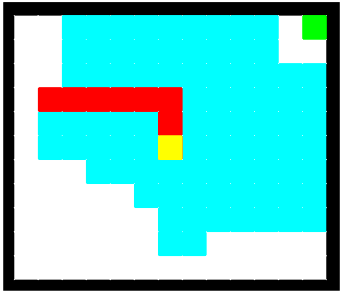

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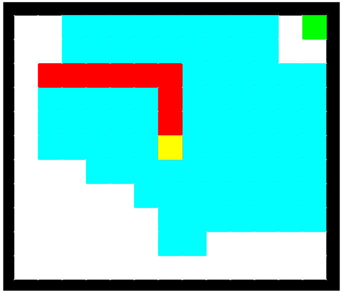

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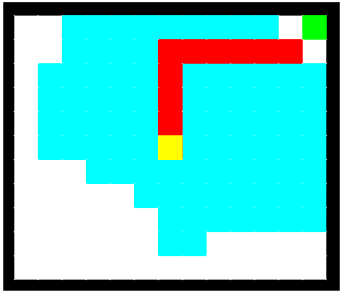

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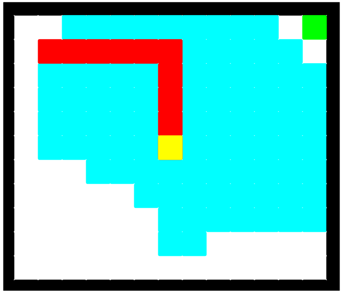

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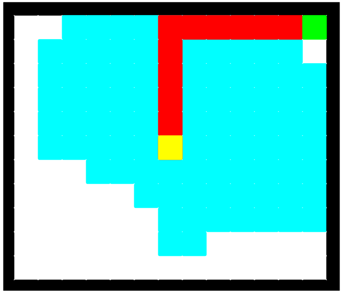

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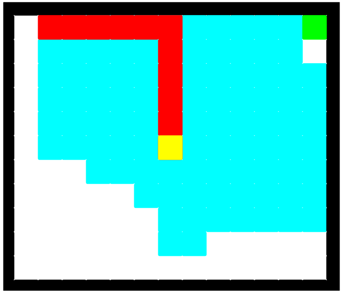

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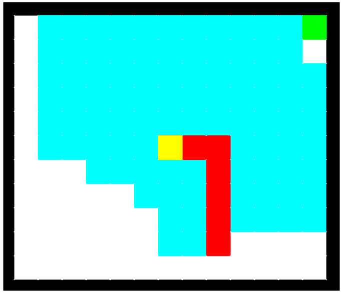

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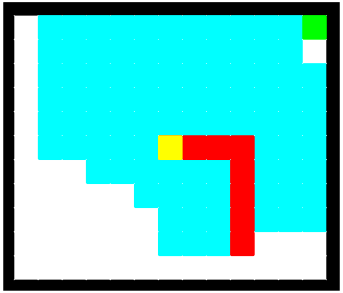

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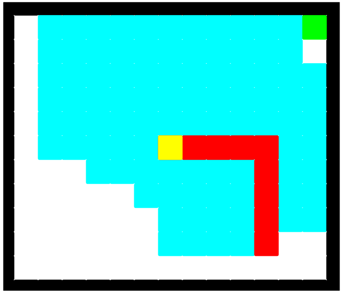

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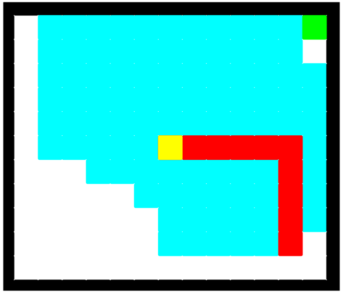

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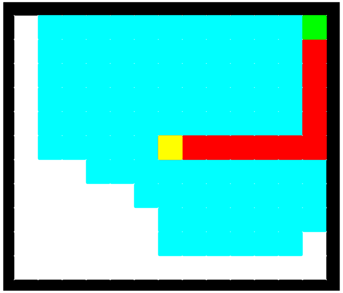

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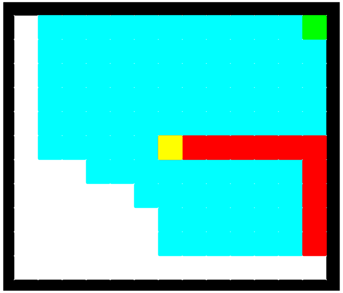

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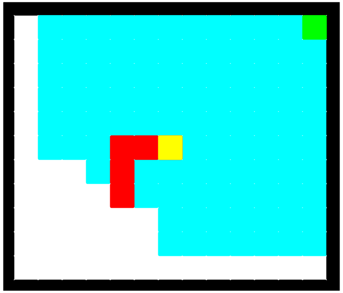

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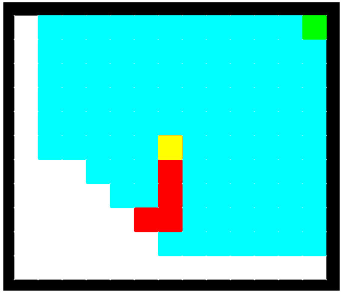

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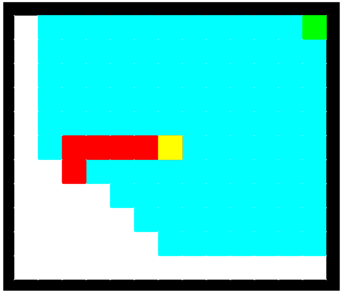

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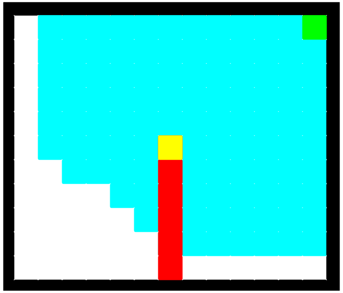

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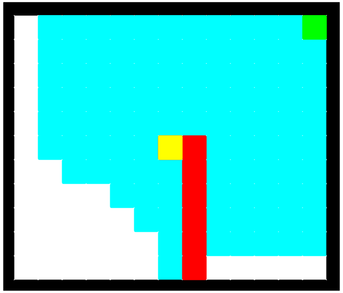

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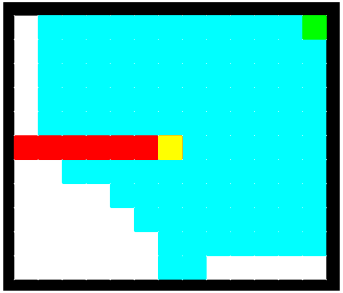

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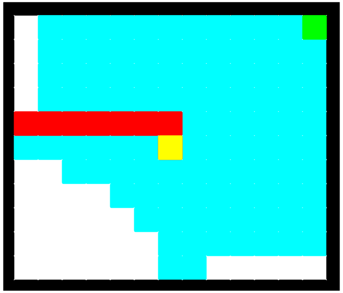

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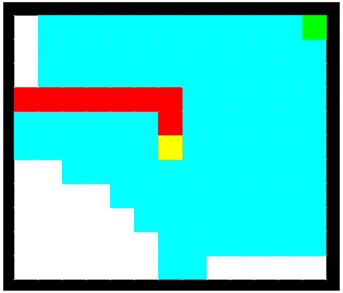

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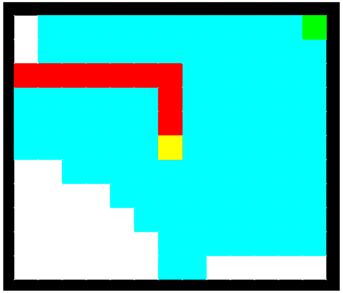

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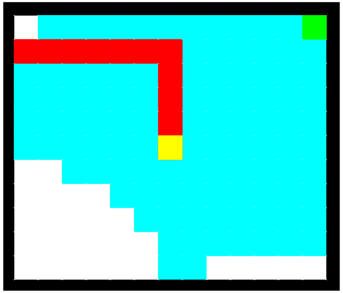

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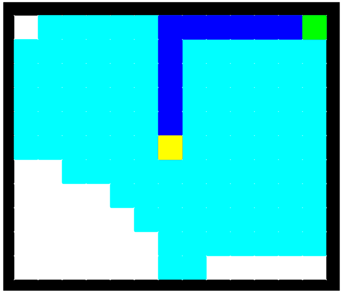

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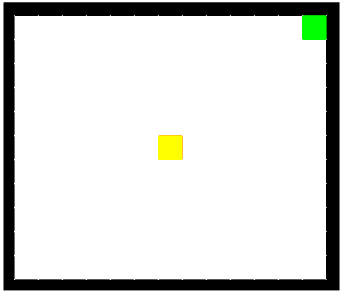

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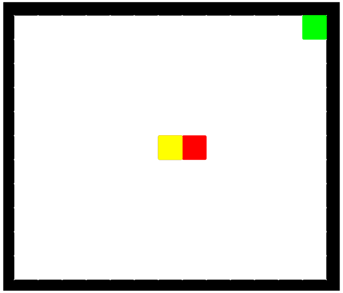

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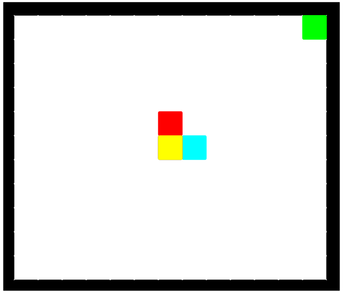

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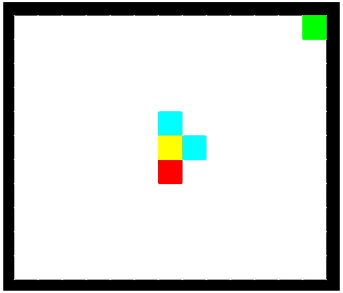

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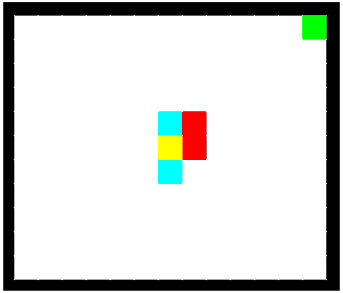

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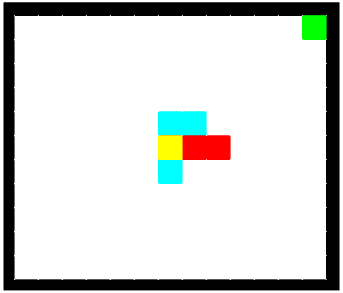

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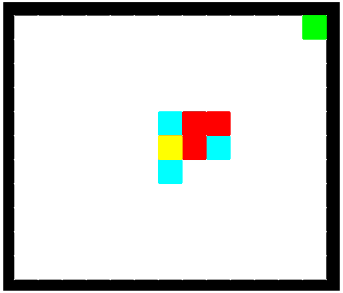

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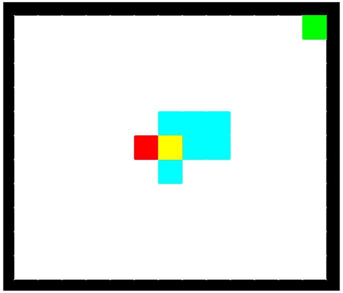

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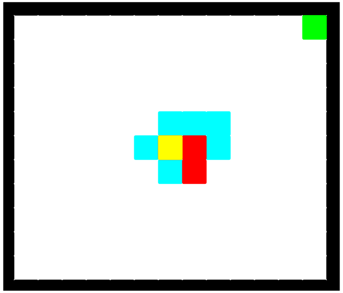

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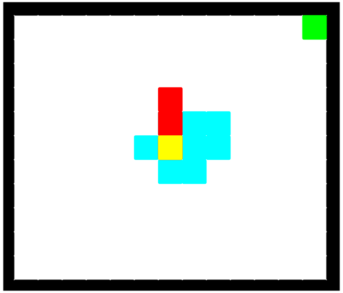

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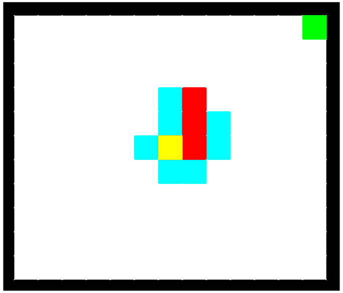

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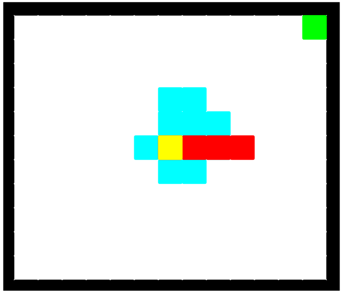

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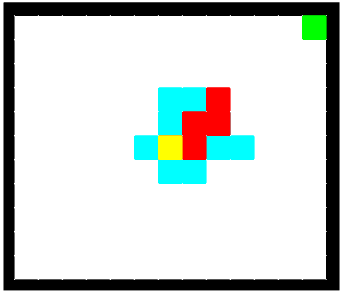

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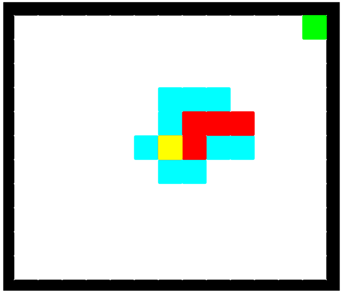

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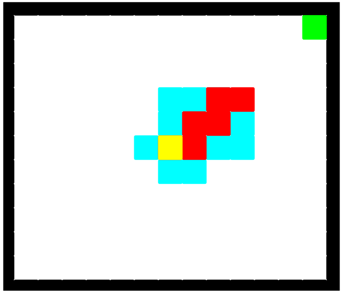

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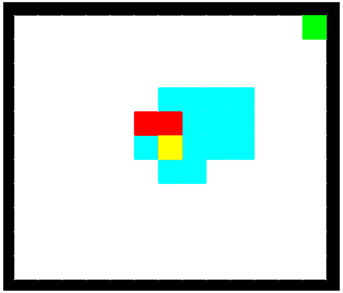

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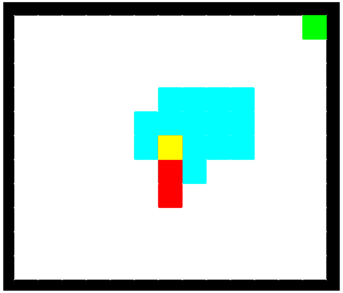

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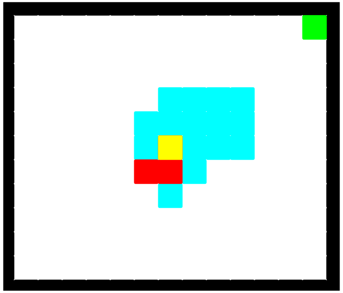

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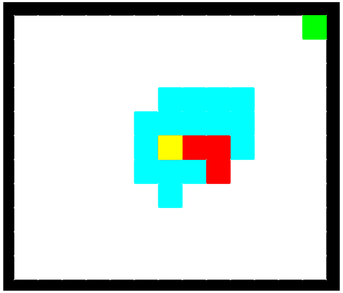

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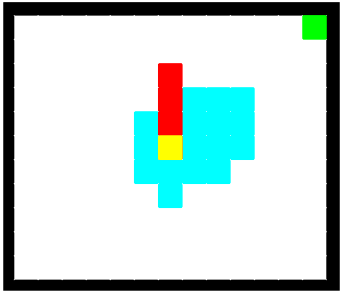

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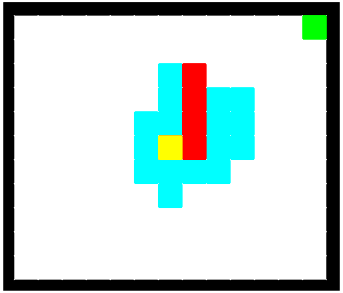

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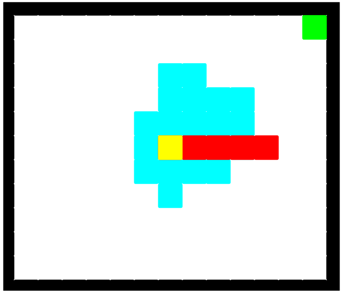

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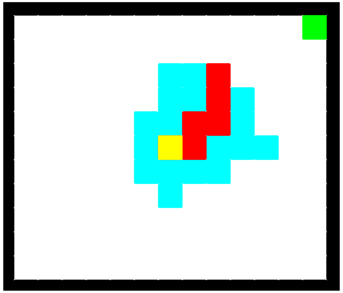

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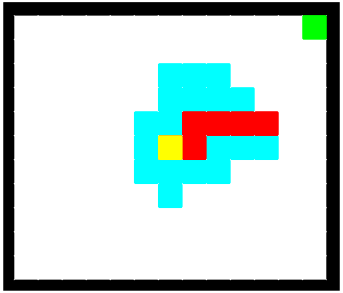

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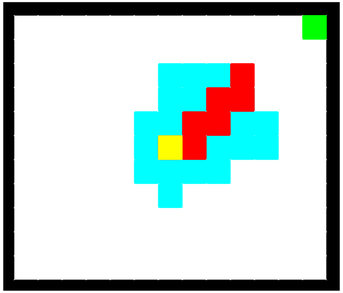

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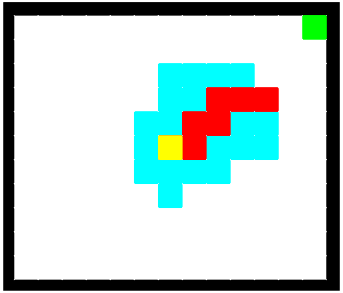

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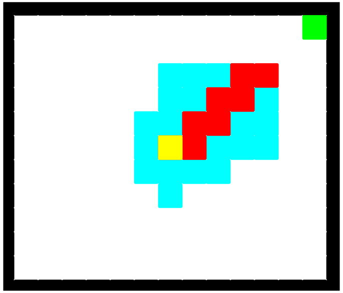

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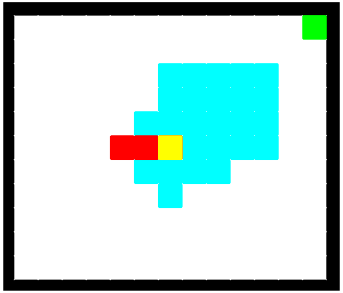

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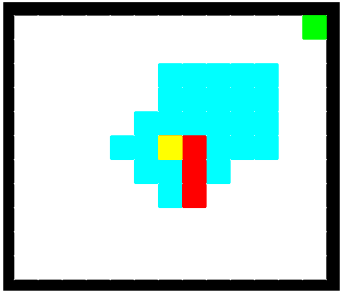

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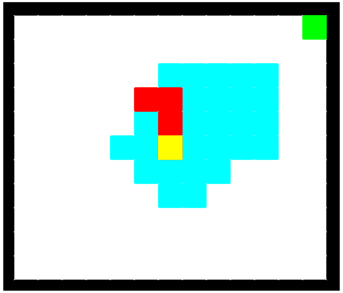

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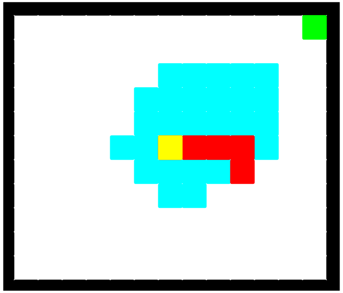

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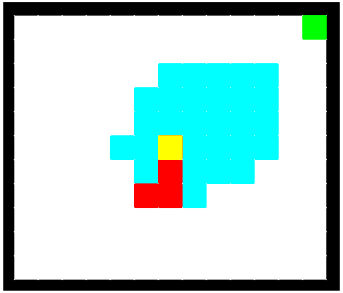

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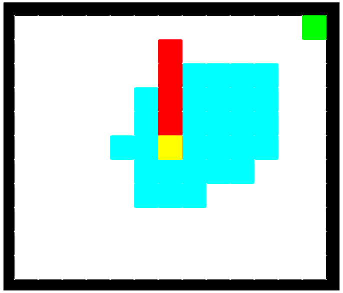

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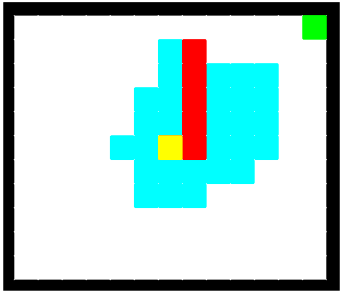

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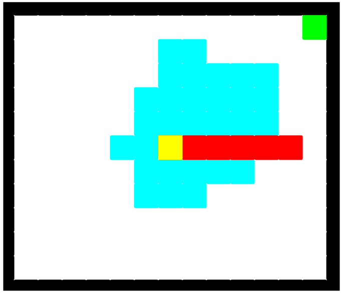

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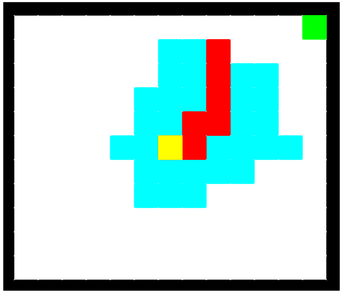

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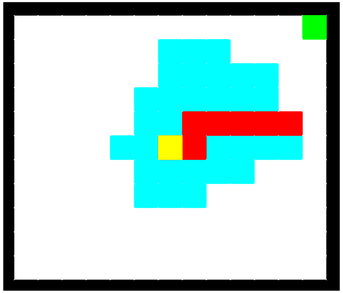

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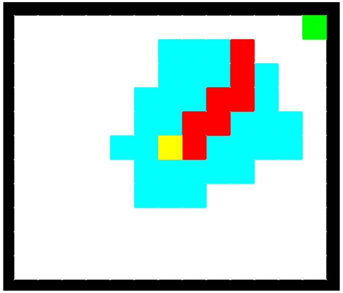

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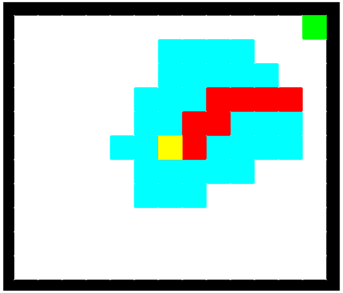

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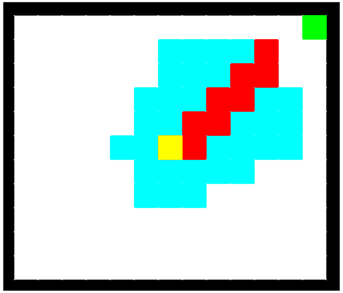

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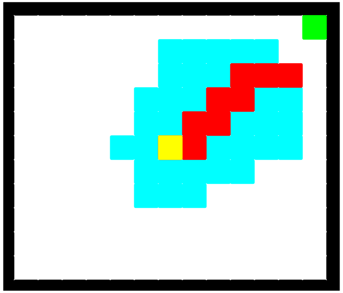

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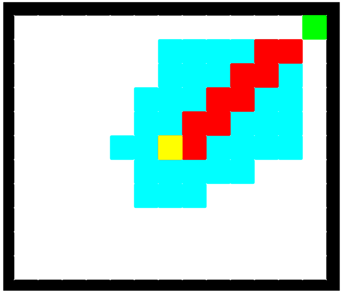

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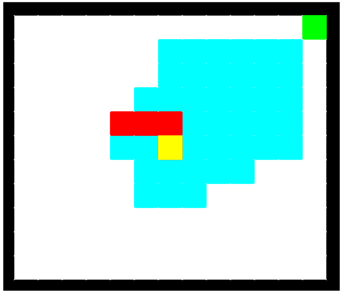

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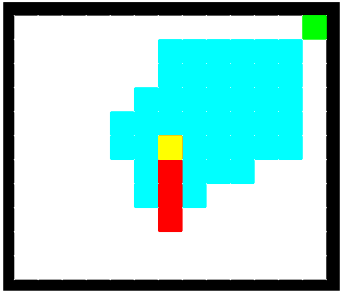

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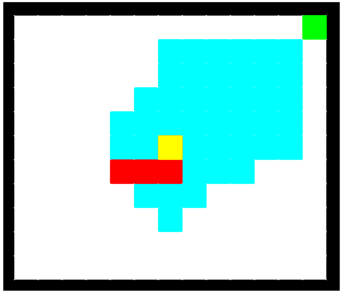

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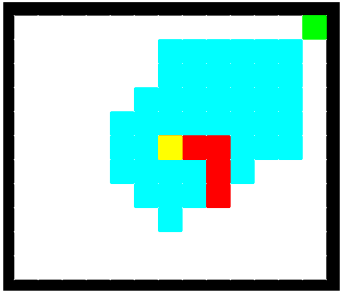

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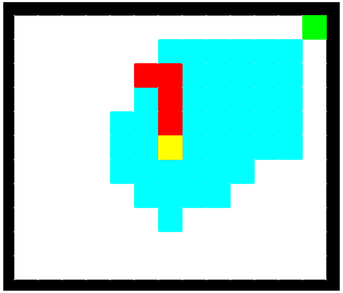

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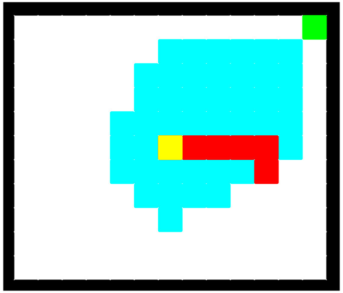

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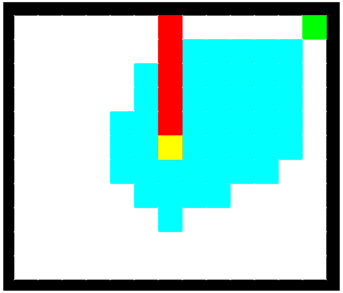

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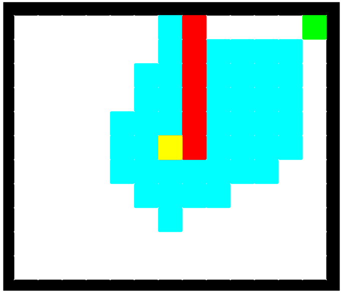

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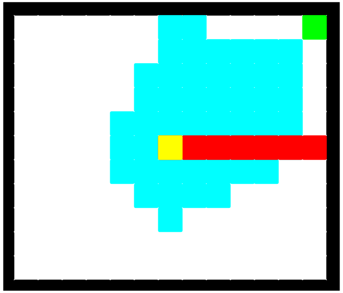

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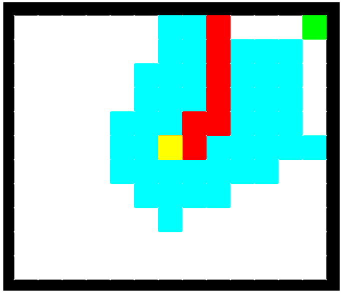

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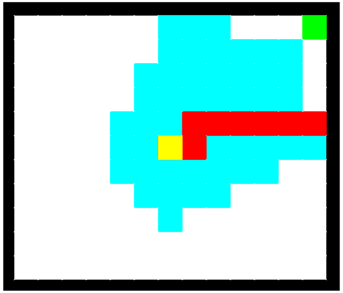

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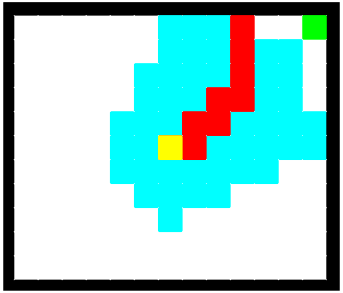

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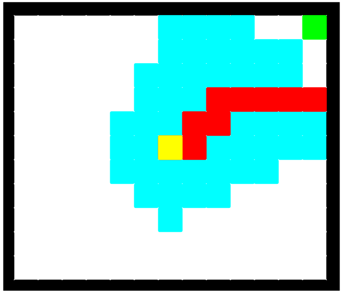

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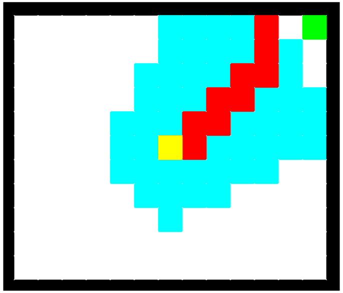

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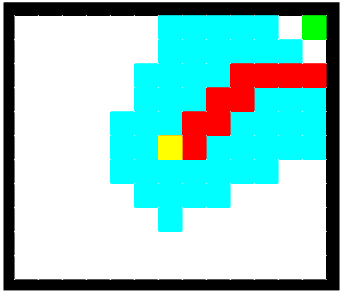

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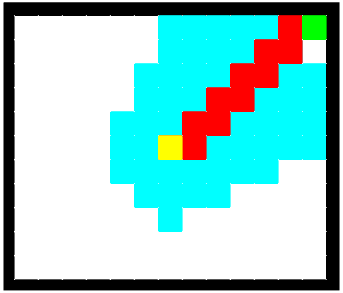

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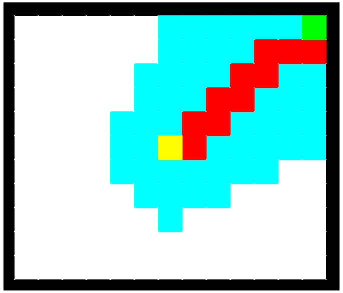

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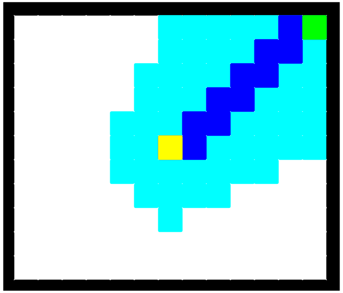

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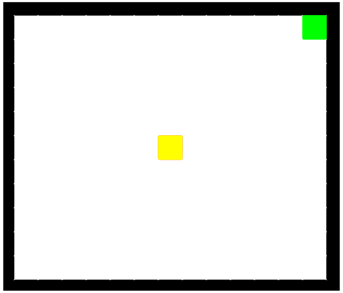

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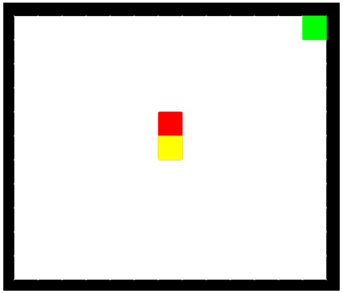

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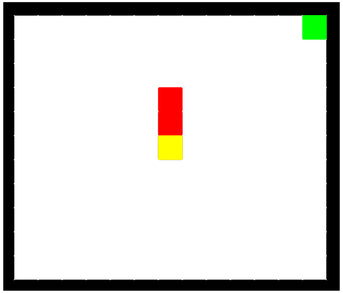

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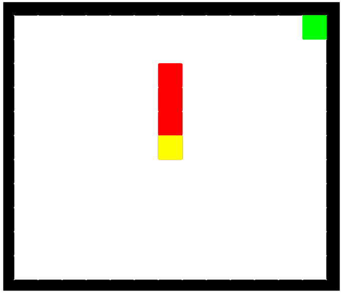

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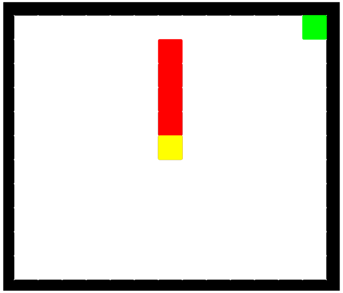

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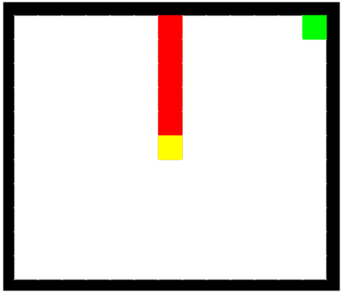

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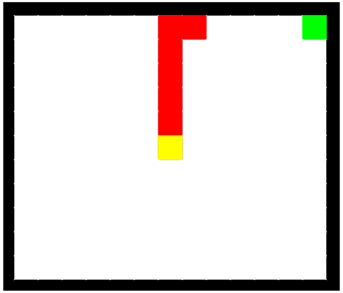

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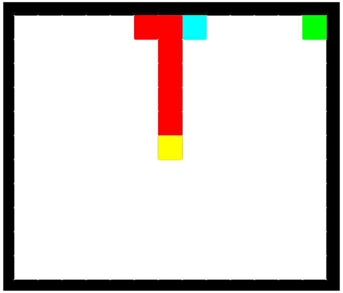

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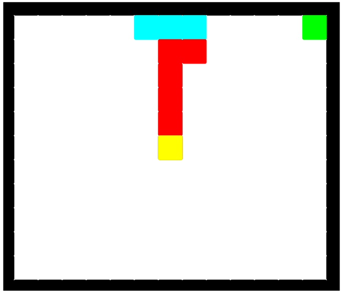

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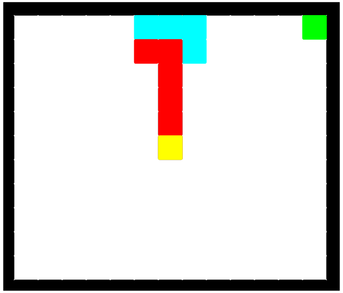

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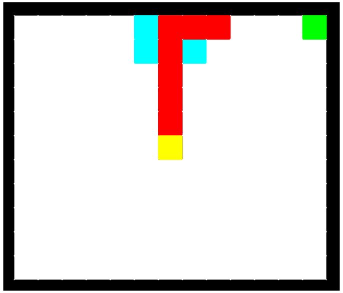

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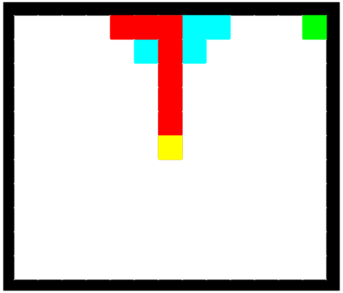

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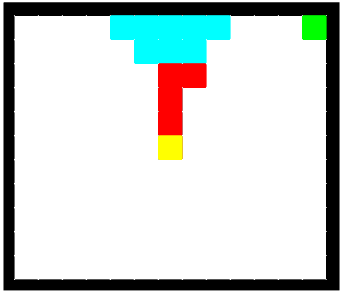

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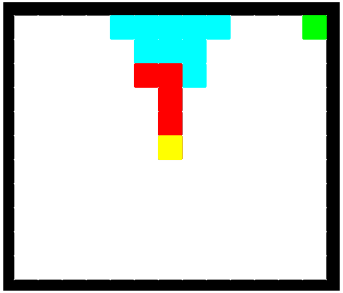

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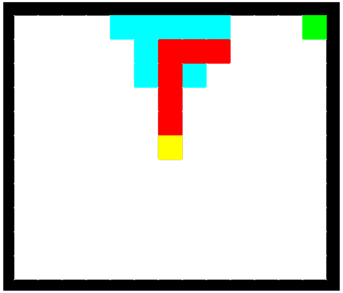

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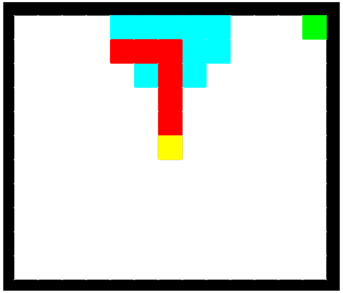

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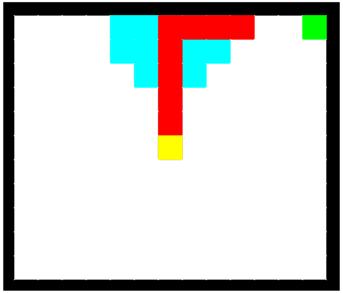

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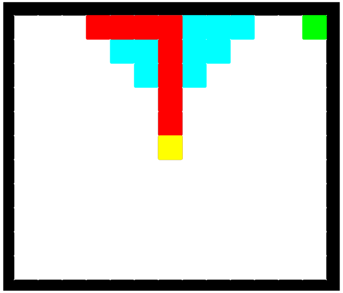

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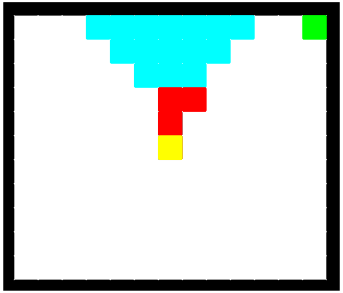

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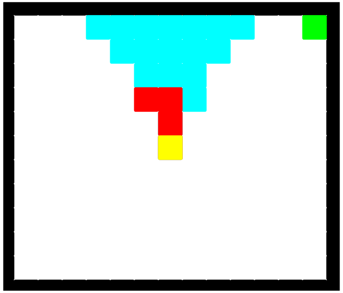

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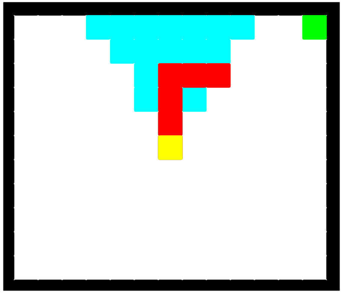

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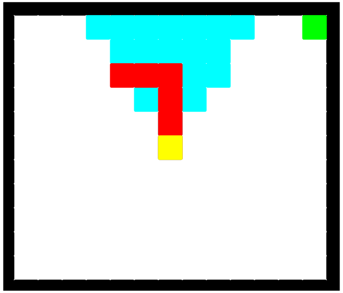
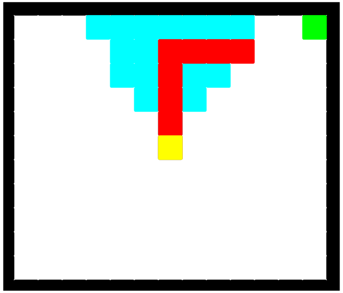
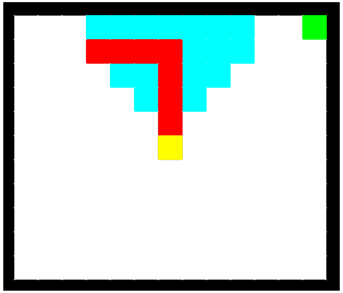


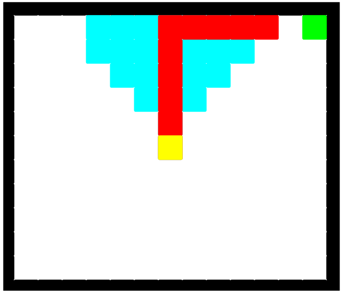
`2*min(abs(r0-r1), abs(c0-c1))`



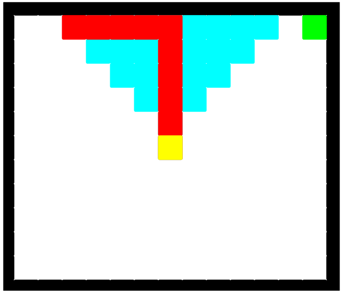
`2*min(abs(r0-r1), abs(c0-c1))`



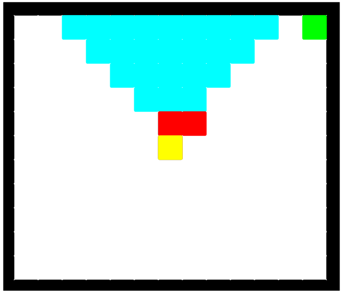
2*min(abs(r0-r1), abs(c0-c1))



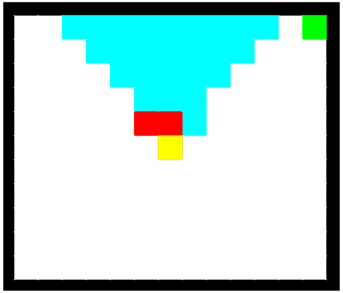
2*min(abs(r0-r1), abs(c0-c1))



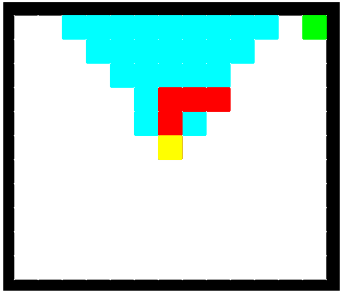
2*min(abs(r0-r1), abs(c0-c1))



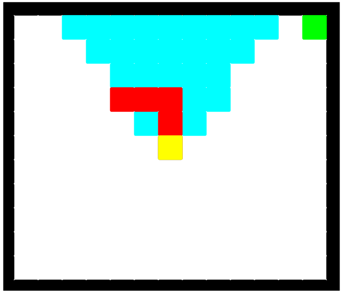
`2*min(abs(r0-r1), abs(c0-c1))`



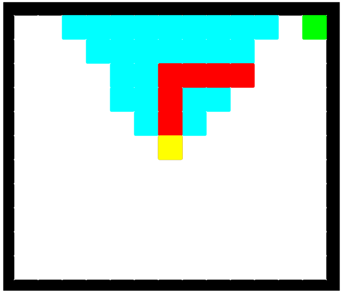
2*min(abs(r0-r1), abs(c0-c1))



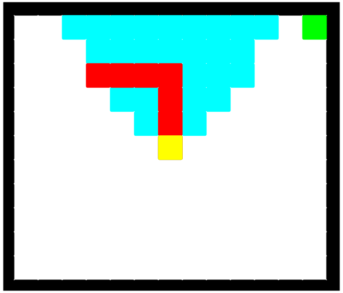
`2*min(abs(r0-r1), abs(c0-c1))`



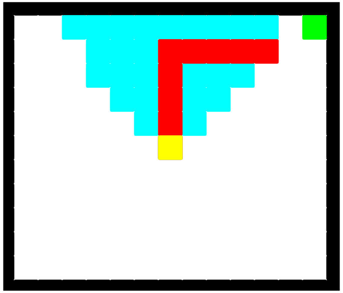
2*min(abs(r0-r1), abs(c0-c1))



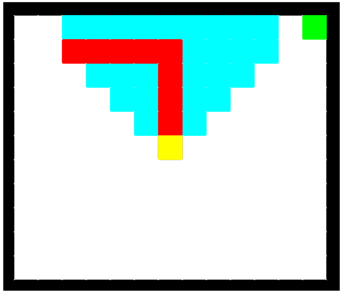
2*min(abs(r0-r1), abs(c0-c1))



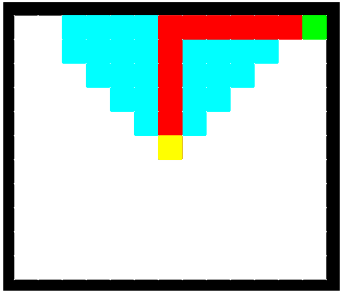
`2*min(abs(r0-r1), abs(c0-c1))`



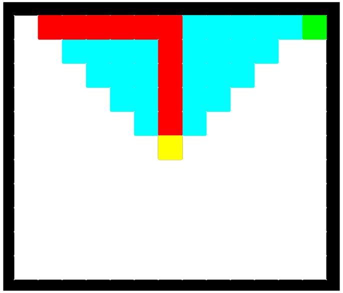
2*min(abs(r0-r1), abs(c0-c1))



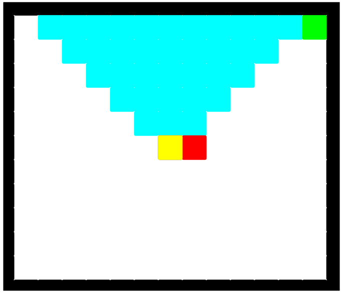
2*min(abs(r0-r1), abs(c0-c1))



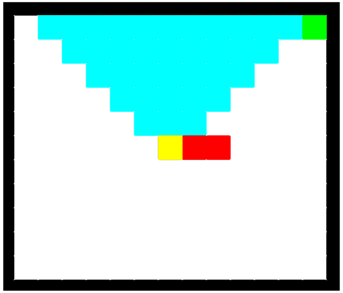
`2*min(abs(r0-r1), abs(c0-c1))`



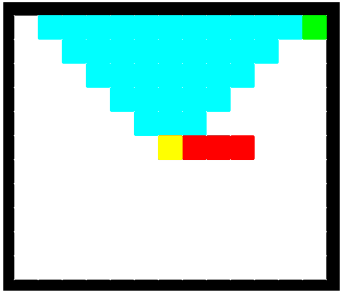
2*min(abs(r0-r1), abs(c0-c1))



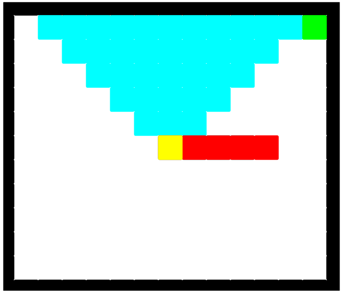
`2*min(abs(r0-r1), abs(c0-c1))`



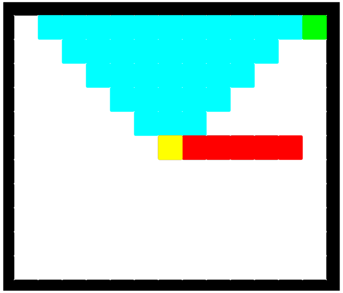
`2*min(abs(r0-r1), abs(c0-c1))`



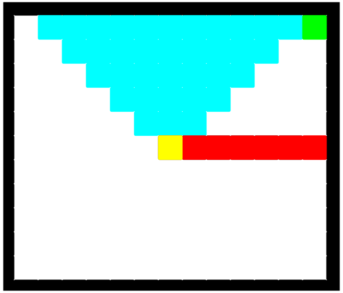
2*min(abs(r0-r1), abs(c0-c1))



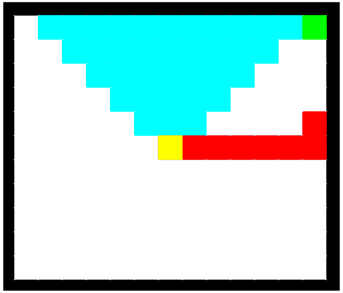
`2*min(abs(r0-r1), abs(c0-c1))`



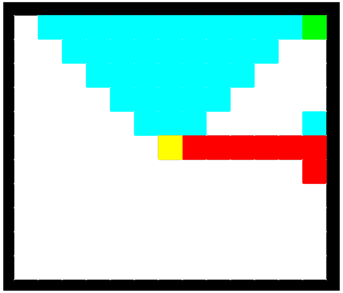
2*min(abs(r0-r1), abs(c0-c1))



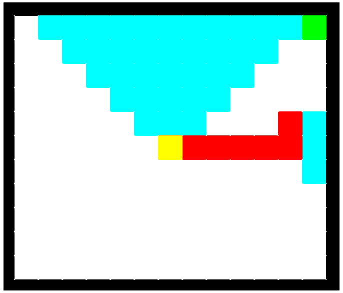
`2*min(abs(r0-r1), abs(c0-c1))`



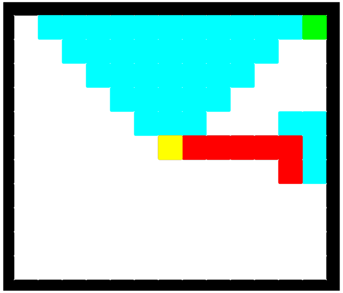
`2*min(abs(r0-r1), abs(c0-c1))`



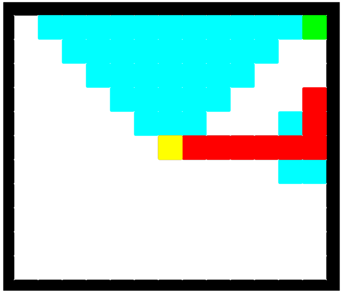
`2*min(abs(r0-r1), abs(c0-c1))`



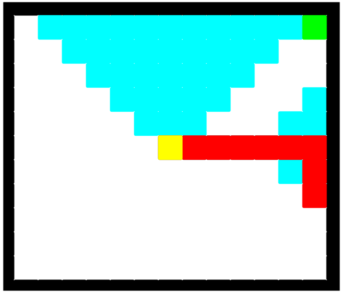
`2*min(abs(r0-r1), abs(c0-c1))`



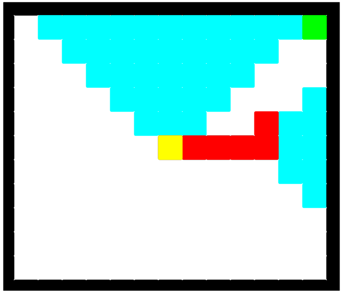
`2*min(abs(r0-r1), abs(c0-c1))`



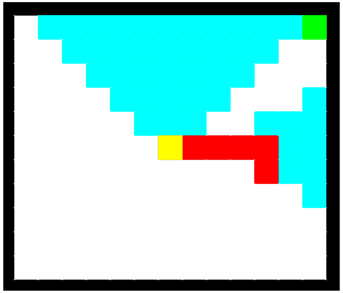
2*min(abs(r0-r1), abs(c0-c1))



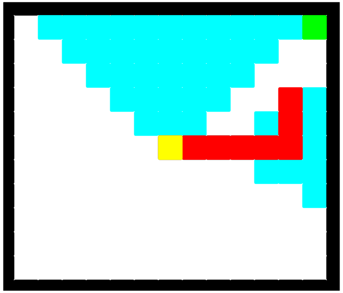
`2*min(abs(r0-r1), abs(c0-c1))`



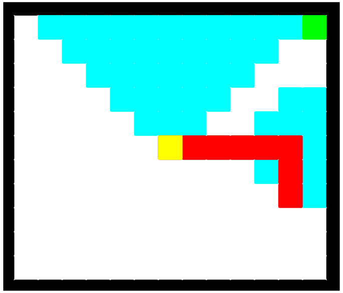
2*min(abs(r0-r1), abs(c0-c1))



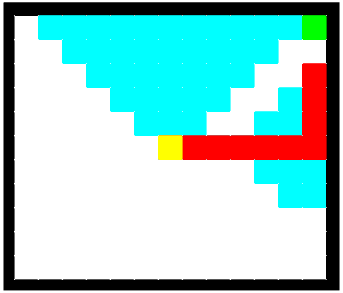
`2*min(abs(r0-r1), abs(c0-c1))`



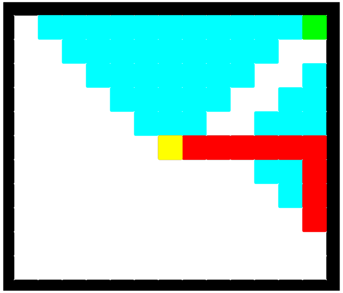
`2*min(abs(r0-r1), abs(c0-c1))`



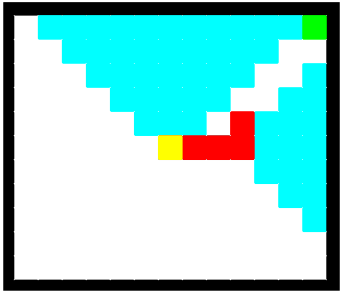
`2*min(abs(r0-r1), abs(c0-c1))`



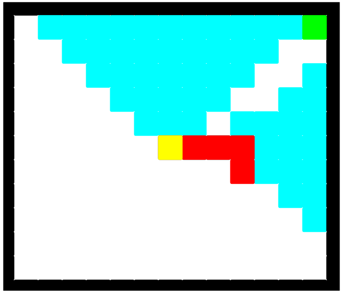
`2*min(abs(r0-r1), abs(c0-c1))`



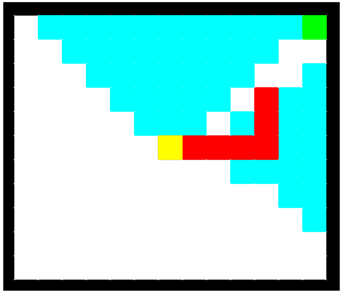
2*min(abs(r0-r1), abs(c0-c1))



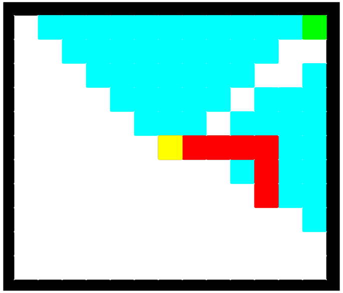
`2*min(abs(r0-r1), abs(c0-c1))`



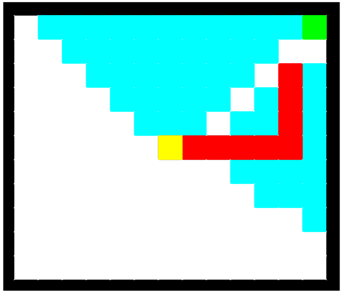
2*min(abs(r0-r1), abs(c0-c1))



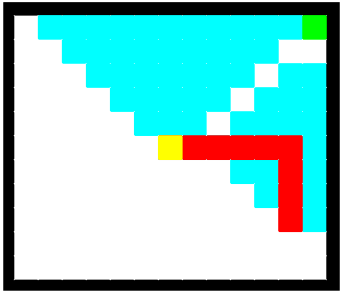
`2*min(abs(r0-r1), abs(c0-c1))`



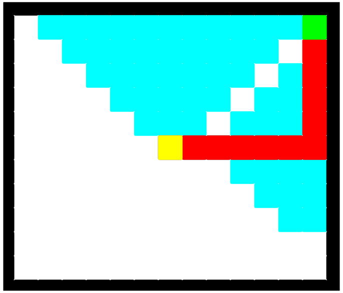
2*min(abs(r0-r1), abs(c0-c1))



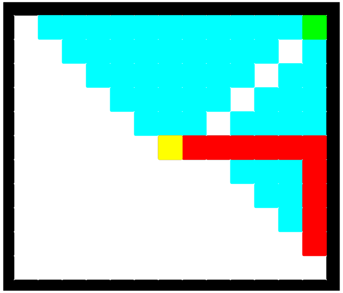
2*min(abs(r0-r1), abs(c0-c1))



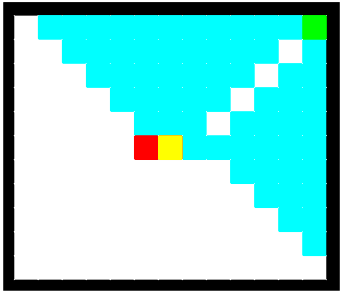
`2*min(abs(r0-r1), abs(c0-c1))`



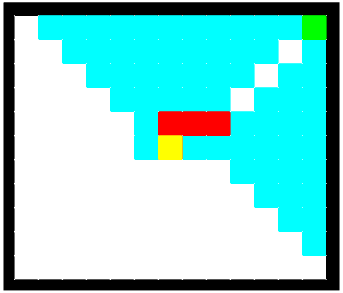
`2*min(abs(r0-r1), abs(c0-c1))`



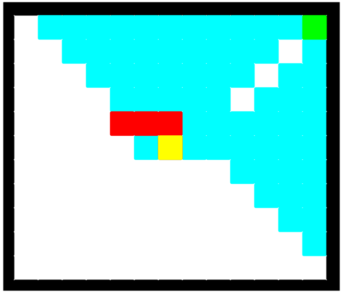
`2*min(abs(r0-r1), abs(c0-c1))`



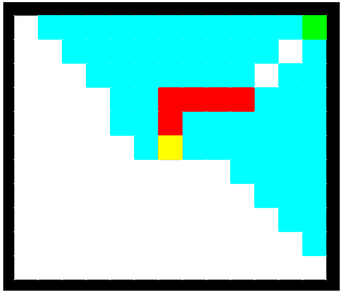
2*min(abs(r0-r1), abs(c0-c1))



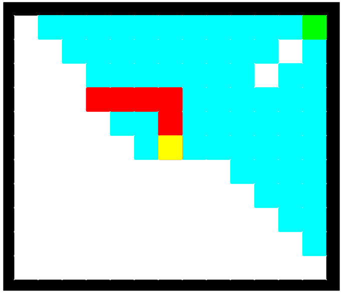
2*min(abs(r0-r1), abs(c0-c1))



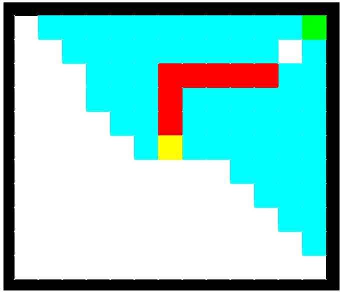
`2*min(abs(r0-r1), abs(c0-c1))`



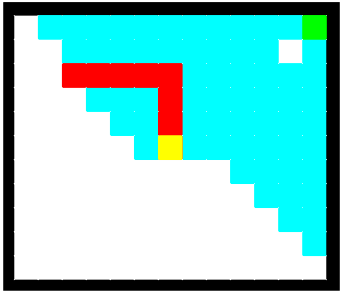
`2*min(abs(r0-r1), abs(c0-c1))`



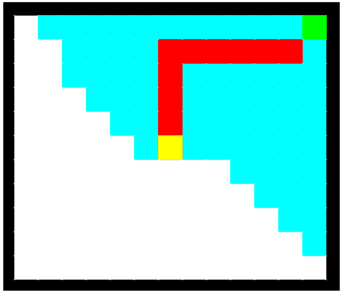
`2*min(abs(r0-r1), abs(c0-c1))`



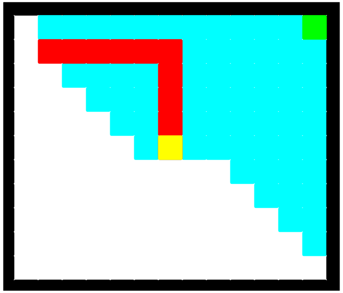
`2*min(abs(r0-r1), abs(c0-c1))`



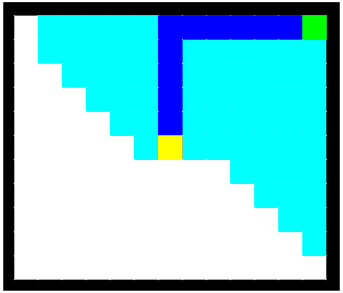
2*min(abs(r0-r1), abs(c0-c1))



`2*min(abs(r0-r1), abs(c0-c1))`



2*min(abs(r0-r1), abs(c0-c1))



Compare

Heuristic	States Expanded
`abs(r0-r1) + abs(c0-c1)`	42
`min(abs(r0-r1), abs(c0-c1))`	114
`max(abs(r0-r1), abs(c0-c1))`	60
`2*min(abs(r0-r1), abs(c0-c1))`	72

Heuristics in Other Domains

Consider the "word ladder" problem from EX12 (and last lecture).
Searching from "quiz" to "best".

Words: UC

`quiz`

Words: UC

```
quid
```

Words: UC

```
quip
```

Words: UC

`quit`

Words: UC

`quad`

Words: UC

`suit`

Words: UC

`quay`

Words: UC

`skit`

Words: UC

`slit`

Words: UC

`snit`

Words: UC

```
spit
```

Words: UC

`suet`

Words: UC

`skid`

Words: UC

`skim`

Words: UC

`skin`

Words: UC

`skip`

Words: UC

`skis`

Words: UC

`alit`

Words: UC

`flit`

Words: UC

`slat`

Words: UC

`slot`

Words: UC

```
slid
```

Words: UC

`slim`

Words: UC

`slip`

Words: UC

`knit`

Words: UC

`unit`

Words: UC

`snip`

Words: UC

`spat`

Words: UC

`spot`

Words: UC

```
spin
```

Words: UC

```
duet
```

Words: UC

`stet`

Words: UC

`sued`

Words: UC

`sues`

Words: UC

`said`

Words: UC

`shim`

Words: UC

`swim`

Words: UC

`akin`

Words: UC

`shin`

Words: UC

```
ship
```

Words: UC

`alia`

Words: UC

```
flat
```

Words: UC

`flip`

Words: UC

`plat`

Words: UC

`scat`

Words: UC

`seat`

Words: UC

`swat`

Words: UC

```
slab
```

Words: UC

`slag`

Words: UC

```
slam
```

Words: UC

```
slap
```

Words: UC

`slav`

Words: UC

`slaw`

Words: UC

`slay`

Words: UC

`blot`

Words: UC

```
clot
```

Words: UC

`plot`

Words: UC

`scot`

Words: UC

`shot`

Words: UC

`soot`

Words: UC

sloe

Words: UC

`slog`

Words: UC

```
slop
```

Words: UC

`slow`

Words: UC

`sled`

Words: UC

`slum`

Words: UC

`blip`

Words: UC

```
clip
```

Words: UC

```
knot
```

Words: UC

`snap`

Words: UC

`span`

Words: UC

`spar`

Words: UC

```
spas
```

Words: UC

`spay`

Words: UC

`spun`

Words: UC

`diet`

Words: UC

```
duct
```

Words: UC

```
dust
```

Words: UC

```
duel
```

Words: UC

`dues`

Words: UC

`stem`

Words: UC

`step`

Words: UC

`stew`

Words: UC

```
cued
```

Words: UC

`hued`

Words: UC

`rued`

Words: UC

`seed`

Words: UC

`shed`

Words: UC

`sped`

Words: UC

`surd`

Words: UC

`cues`

Words: UC

`hues`

Words: UC

`rues`

Words: UC

```
sees
```

Words: UC

```
subs
```

Words: UC

suds

Words: UC

`sums`

Words: UC

`suns`

Words: UC

`sups`

Words: UC

```
laid
```

Words: UC

`maid`

Words: UC

`paid`

Words: UC

`raid`

Words: UC

`sand`

Words: UC

`sail`

Words: UC

`whim`

Words: UC

```
sham
```

Words: UC

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swam
```

Words: UC

`swum`

Words: UC

`swig`

Words: UC

`chin`

Words: UC

`thin`

Words: UC

`shun`

Words: UC

`chip`

Words: UC

`whip`

Words: UC

`shop`

Words: UC

`ilia`

Words: UC

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aria
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Words: UC

`asia`

Words: UC

`alga`

Words: UC

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alma
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Words: UC

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feat
```

Words: UC

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fiat
```

Words: UC

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flab
```

Words: UC

`flag`

Words: UC

```
flak
```

Words: UC

```
flap
```

Words: UC

```
flaw
```

Words: UC

`flax`

Words: UC

`flay`

Words: UC

```
flop
```

Words: UC

```
peat
```

Words: UC

```
plan
```

Words: UC

`play`

Words: UC

`scab`

Words: UC

```
scan
```

Words: UC

`scar`

Words: UC

```
beat
```

Words: UC

`heat`

Words: UC

`meat`

Words: UC

`neat`

Words: UC

```
teat
```

Words: UC

`sect`

Words: UC

`sent`

Words: UC

seal

Words: UC

`seam`

Words: UC

`sear`

Words: UC

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seas
```

Words: UC

`swab`

Words: UC

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swag
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Words: UC

`swan`

Words: UC

`swap`

Words: UC

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sway
```

Words: UC

`blab`

Words: UC

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stab
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Words: UC

`shag`

Words: UC

`snag`

Words: UC

`stag`

Words: UC

`slug`

Words: UC

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clam
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Words: UC

`siam`

Words: UC

`clap`

Words: UC

`soap`

Words: UC

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claw
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Words: UC

`slew`

Words: UC

`clay`

Words: UC

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shay
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Words: UC

`stay`

Words: UC

`boot`

Words: UC

`blob`

Words: UC

`bloc`

Words: UC

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blow
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Words: UC

`coot`

Words: UC

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clod
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Words: UC

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clog
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Words: UC

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clop
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Words: UC

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cloy
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Words: UC

`plod`

Words: UC

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plop
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Words: UC

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plow
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Words: UC

`ploy`

Words: UC

SCOW

Words: UC

`shut`

Words: UC

`shod`

Words: UC

`shoe`

Words: UC

`shoo`

Words: UC

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show
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Words: UC

`foot`

Words: UC

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hoot
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Words: UC

`loot`

Words: UC

`moot`

Words: UC

`root`

Words: UC

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toot
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Words: UC

`soft`

Words: UC

`sort`

Words: UC

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soon
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Words: UC

`aloe`

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`floe`

Words: UC

`slue`

Words: UC

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flog
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Words: UC

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smog
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Words: UC

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stop
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Words: UC

`flow`

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`glow`

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`snow`

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`stow`

Words: UC

`bled`

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`alum`

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`glum`

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`plum`

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scum
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Words: UC

`slur`

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knob
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Words: UC

`know`

Words: UC

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soar
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Words: UC

`star`

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`spur`

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`spry`

Words: UC

`stun`

Words: UC

`spud`

Words: UC

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dint
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Words: UC

`dirt`

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`died`

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diem
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Words: UC

`dies`

Words: UC

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duck
```

Words: UC

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bust
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Words: UC

`gust`

Words: UC

`just`

Words: UC

`lust`

Words: UC

`must`

Words: UC

`oust`

Words: UC

`rust`

Words: UC

`dusk`

Words: UC

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fuel
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Words: UC

`dual`

Words: UC

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dull
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Words: UC

`does`

Words: UC

dyes

Words: UC

`dubs`

Words: UC

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duds
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Words: UC

`duns`

Words: UC

duos

Words: UC

`item`

Words: UC

`seem`

Words: UC

`seep`

Words: UC

`skew`

Words: UC

`spew`

Words: UC

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curd
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Words: UC

`heed`

Words: UC

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hied
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Words: UC

`hoed`

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`reed`

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`deed`

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`feed`

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meed
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Words: UC

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need
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teed
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Words: UC

`weed`

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`send`

Words: UC

`seek`

Words: UC

`seen`

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`seer`

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`shad`

Words: UC

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aped
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Words: UC

`sure`

Words: UC

surf

Words: UC

`cubs`

Words: UC

cuds

Words: UC

`cups`

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`curs`

Words: UC

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cuss
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Words: UC

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cuts
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Words: UC

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hies
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Words: UC

hoes

Words: UC

`hubs`

Words: UC

`hugs`

Words: UC

`hums`

Words: UC

`huns`

Words: UC

`huts`

Words: UC

`ryes`

Words: UC

`rubs`

Words: UC

`rugs`

Words: UC

`rums`

Words: UC

`runs`

Words: UC

`ruts`

Words: UC

`bees`

Words: UC

`fees`

Words: UC

`lees`

Words: UC

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tees
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Words: UC

`sets`

Words: UC

`sews`

Words: UC

`nubs`

Words: UC

`pubs`

Words: UC

`tubs`

Words: UC

`sibs`

Words: UC

`sobs`

Words: UC

```
buds
```

Words: UC

`muds`

Words: UC

`sods`

Words: UC

`bums`

Words: UC

gums

Words: UC

`mums`

Words: UC

`sump`

Words: UC

```
buns
```

Words: UC

```
funs
```

Words: UC

```
guns
```

Words: UC

`nuns`

Words: UC

`puns`

Words: UC

`tuns`

Words: UC

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sins
```

Words: UC

`sons`

Words: UC

sung

Words: UC

`sunk`

Words: UC

`pups`

Words: UC

`saps`

Words: UC

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sips
```

Words: UC

`sops`

Words: UC

`land`

Words: UC

`lard`

Words: UC

`laud`

Words: UC

`laic`

Words: UC

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lain
```

Words: UC

`lair`

Words: UC

`mail`

Words: UC

`maim`

Words: UC

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main
```

Words: UC

```
pard
```

Words: UC

`pail`

Words: UC

```
pain
```

Words: UC

`pair`

Words: UC

```
rail
```

Words: UC

`rain`

Words: UC

```
band
```

Words: UC

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hand
```

Words: UC

`wand`

Words: UC

```
sane
```

Words: UC

`sang`

Words: UC

`sank`

Words: UC

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bail
```

Words: UC

`fail`

Words: UC

`hail`

Words: UC

```
jail
```

Words: UC

```
nail
```

Words: UC

`tail`

Words: UC

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wail
```

Words: UC

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soil
```

Words: UC

`wham`

Words: UC

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whom
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Words: UC

`whig`

Words: UC

`whir`

Words: UC

`whit`

Words: UC

`whiz`

Words: UC

`shah`

Words: UC

`twig`

Words: UC

`cain`

Words: UC

coin

Words: UC

`chic`

Words: UC

`twin`

Words: UC

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than
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Words: UC

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then
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Words: UC

`this`

Words: UC

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chap
```

Words: UC

`chop`

Words: UC

`ilea`

Words: UC

`area`

Words: UC

`arid`

Words: UC

`aril`

Words: UC

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alms
```

Words: UC

`feet`

Words: UC

`felt`

Words: UC

`fear`

Words: UC

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fist
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Words: UC

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flew
```

Words: UC

`flex`

Words: UC

`flux`

Words: UC

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fray
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Words: UC

`pelt`

Words: UC

`pert`

Words: UC

`pest`

Words: UC

`peak`

Words: UC

peal

Words: UC

`pear`

Words: UC

`peas`

Words: UC

`clan`

Words: UC

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elan
```

Words: UC

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klan
```

Words: UC

```
pray
```

Words: UC

`boat`

Words: UC

`brat`

Words: UC

`beet`

Words: UC

```
belt
```

Words: UC

`bent`

Words: UC

`best`

Words: A*

```
quiz
```

Words: A*

```
quit
```

Words: A*

`quid`

Words: A*

`quip`

Words: A*

```
suit
```

Words: A*

```
quad
```

Words: A*

`skit`

Words: A*

```
slit
```

Words: A*

```
snit
```

Words: A*

`spit`

Words: A*

```
suet
```

Words: A*

`quay`

Words: A*

```
alit
```

Words: A*

```
flit
```

Words: A*

```
slat
```

Words: A*

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slot
```

Words: A*

`knit`

Words: A*

`unit`

Words: A*

```
spat
```

Words: A*

spot

Words: A*

```
duet
```

Words: A*

`stet`

Words: A*

`seat`

Words: A*

`blot`

Words: A*

```
dust
```

Words: A*

```
beat
```

Words: A*

```
bust
```

Words: A*

`best`

Heuristics in other domains

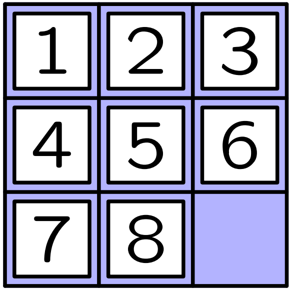
Consider the "word ladder" problem from EX12 (and last lecture).
Searching from "quiz" to "best".

UC
States expanded: 396
['quiz', 'quit', 'suit', 'slit', 'slat', 'seat', 'beat', 'best']

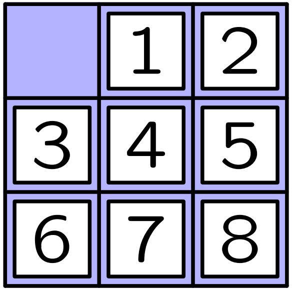
A*
States expanded: 28
['quiz', 'quit', 'suit', 'slit', 'slat', 'seat', 'beat', 'best']

Example: 8-Puzzle

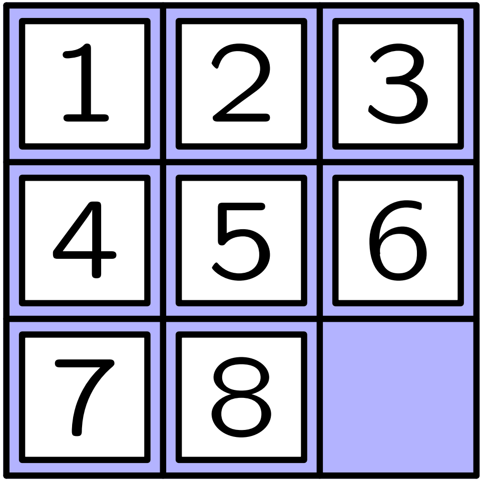
Start



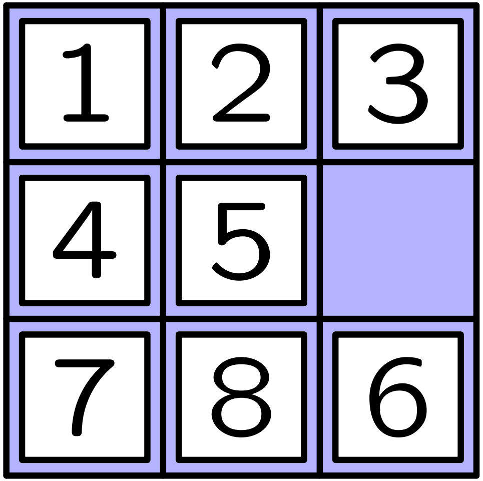
Goal



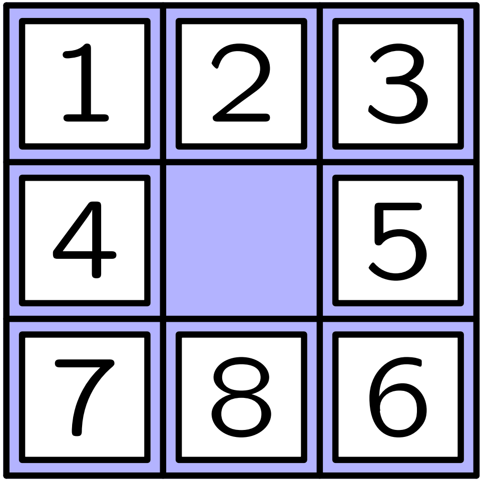
Example: 8-Puzzle



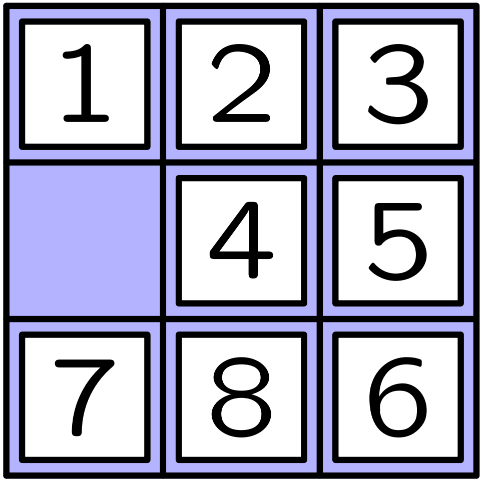
Example: 8-Puzzle



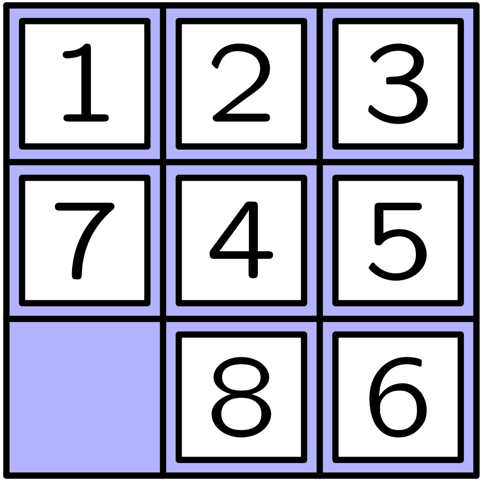
Example: 8-Puzzle



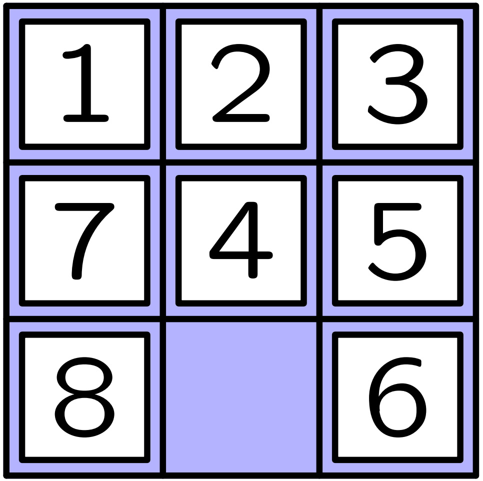
Example: 8-Puzzle



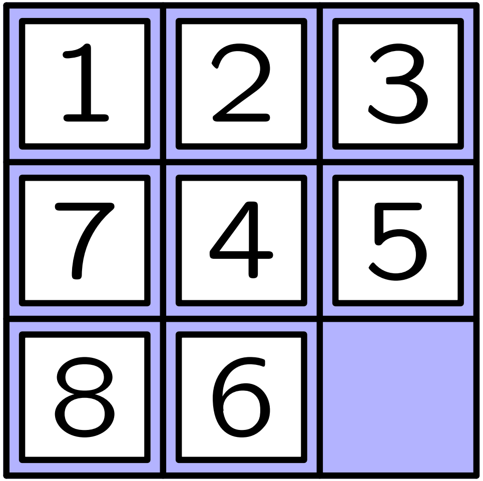
Example: 8-Puzzle



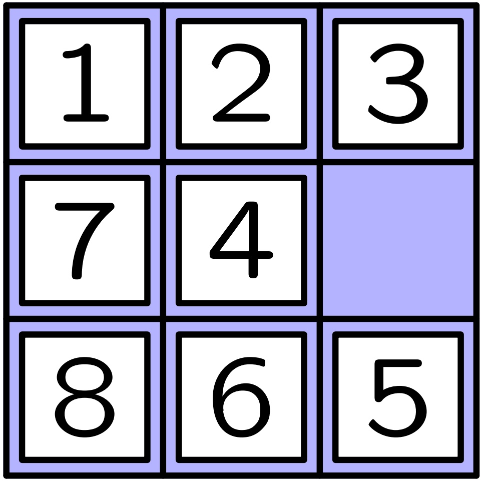
Example: 8-Puzzle



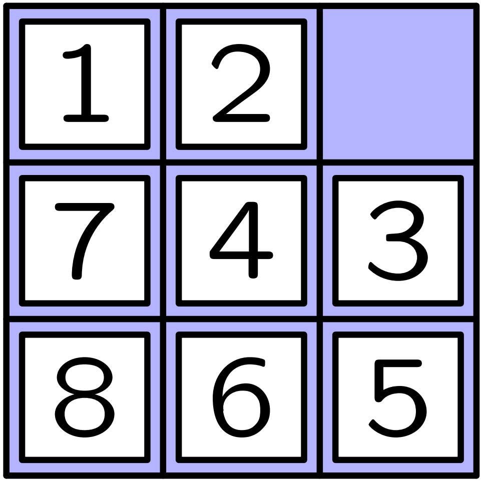
Example: 8-Puzzle



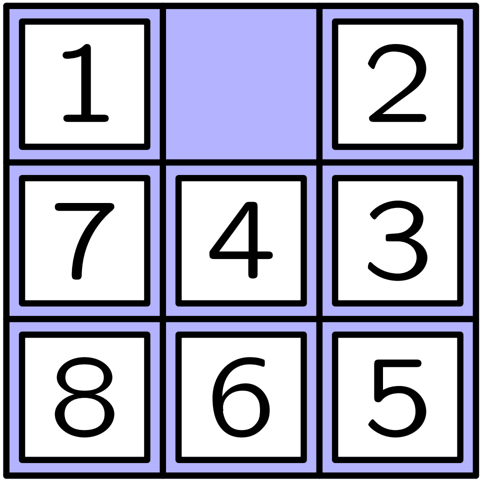
Example: 8-Puzzle



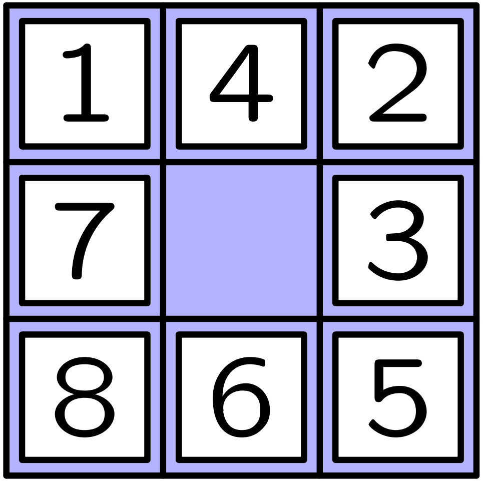
Example: 8-Puzzle



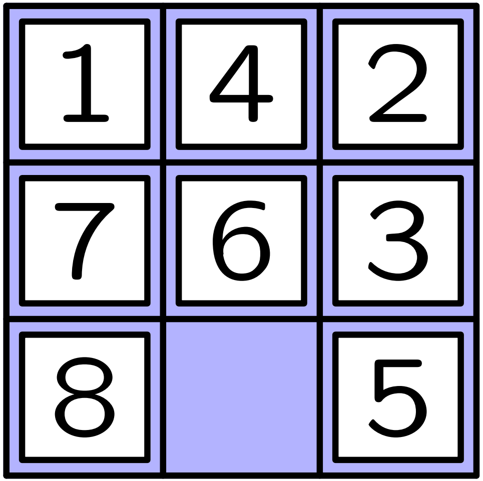
Example: 8-Puzzle



Example: 8-Puzzle



Example: 8-Puzzle



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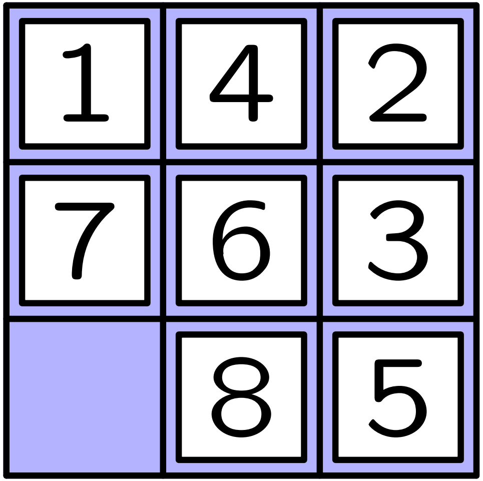
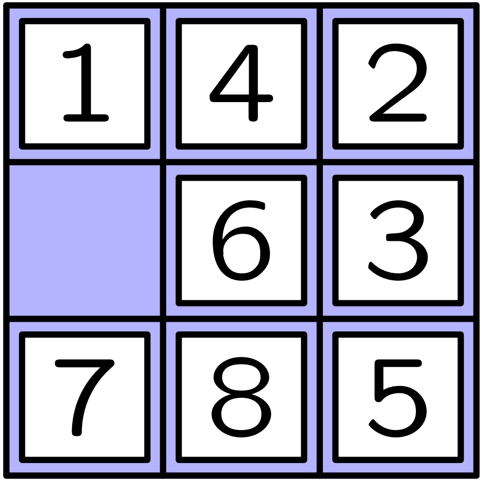
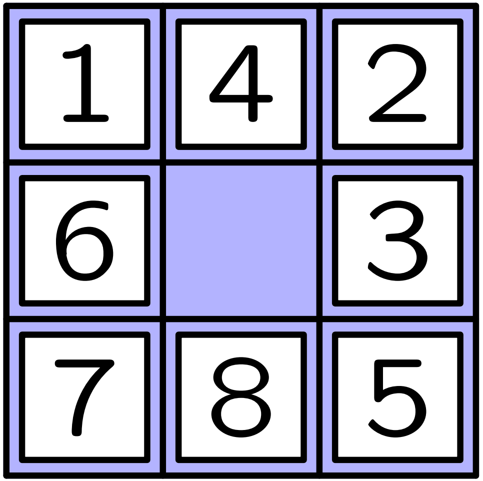


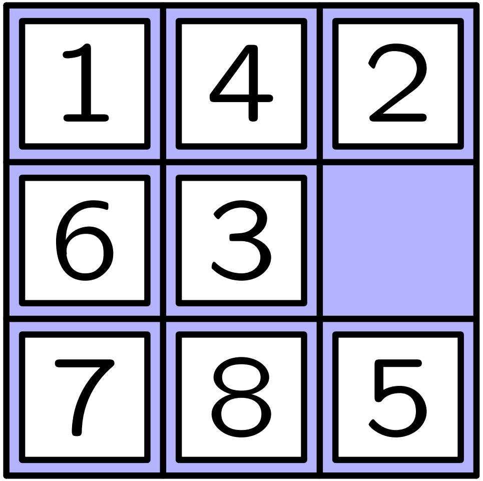
Example: 8-Puzzle



Example: 8-Puzzle



Example: 8-Puzzle



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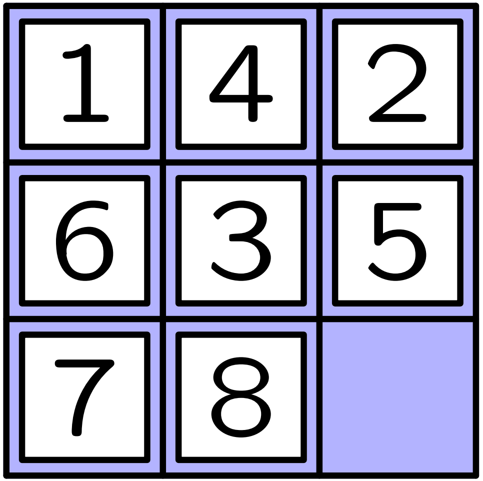
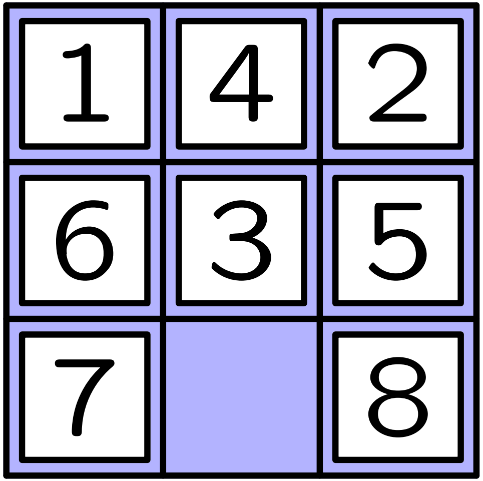
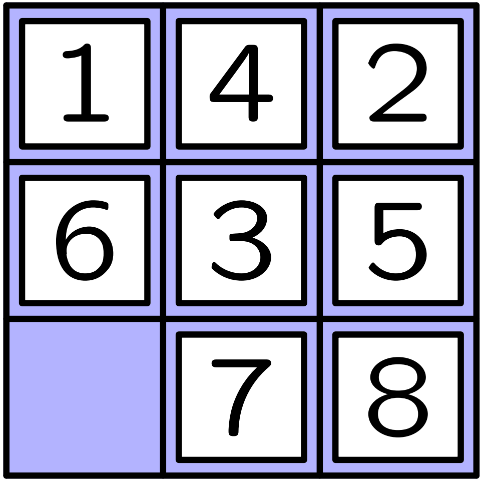


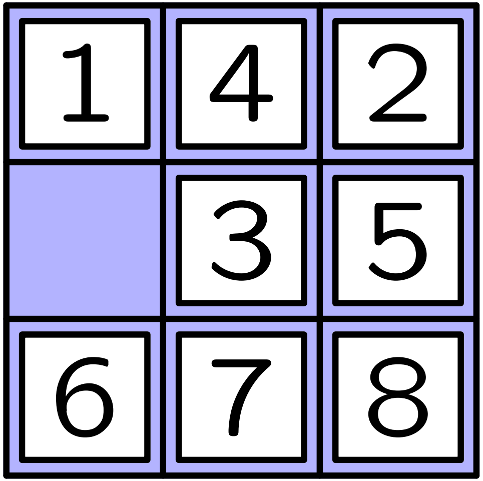
Example: 8-Puzzle



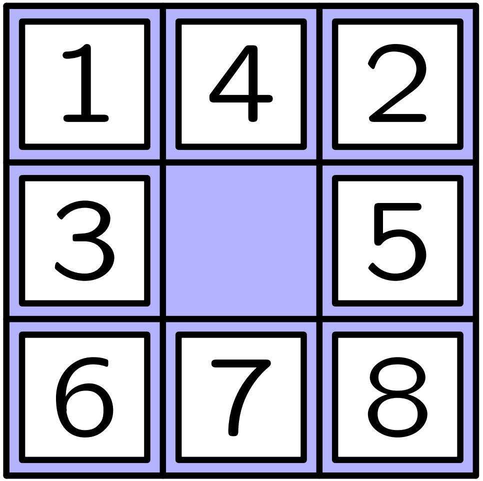
Example: 8-Puzzle



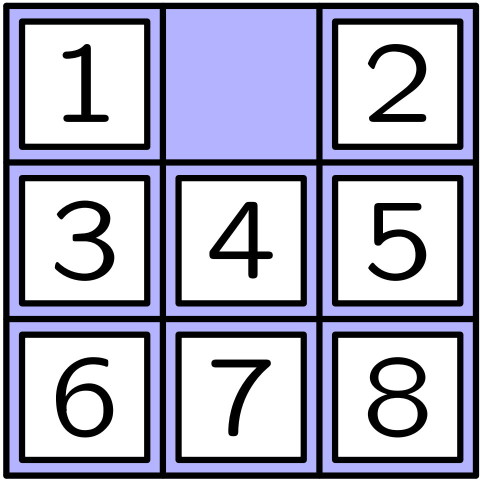
Example: 8-Puzzle



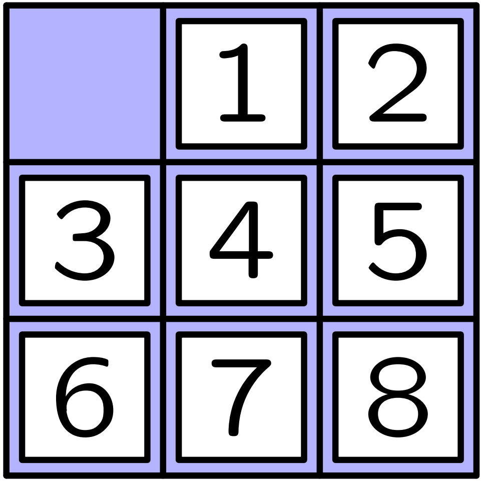
Example: 8-Puzzle



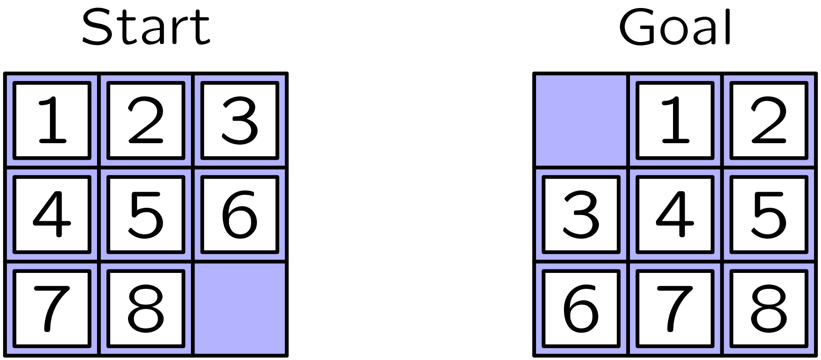
Example: 8-Puzzle



Example: 8-Puzzle



Example: 8-Puzzle



Large number of board configurations (states):

- 9! = 362,880 (if you count all)
- 9!/2 = 181,440 accessible from start state

Almost half of accessible states (84,516) are expanded by UC.

Check Yourself!

Consider three heuristics for the "eight puzzle":

1. 0
2. number of tiles out of place
3. sum over tiles of Manhattan distances to their goals

Let M_i = num. moves in the best solution using heuristic i.
Let E_i = num. states expanded using heuristic i.

Which of the following are true?

1. $M_1 = M_2 = M_3$
2. $M_1 > M_2 > M_3$
3. $E_1 = E_2 = E_3$
4. $E_1 \geq E_2 \geq E_3$
5. the same "best" solution results from all three heuristics

Check Yourself!

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Check Yourself!

Results.

Heuristics:

1. 0
2. number of tiles out of place
3. sum over tiles of Manhattan distances to their goals

Heuristic	**Expanded**	**Moves**
1	84,516	22
2	8,329	22
3	1,348	22

Recap

Developed a new class of search algorithms: uniform cost
Developed a new class of optimizations: heuristics

Summary of Search Algorithms:

Algorithm	**Agenda**	**Goal Test**	**DP**	**Guarantees**[†]
DFS				
BFS				
UC				

[†] Provided a path exists

Recap

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Developed a new class of optimizations: heuristics

Summary of Search Algorithms:

Algorithm	**Agenda**	**Goal Test**	**DP**	**Guarantees**[†]
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BFS				
UC				

[†] Provided a path exists
[*] In a finite search domain

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BFS	Queue (FIFO)	Visit	Visited Set	Shortest Path
UC				

† Provided a path exists
* In a finite search domain

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BFS	Queue (FIFO)	Visit	Visited Set	Shortest Path
UC	Priority Queue	Expand	Expanded Set	Least-cost Path

† Provided a path exists
* In a finite search domain