

# 6.01

Lecture 12: Graph Search

## 6.01: Introduction to EECS I

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The **intellectual themes** in 6.01 are recurring themes in engineering:

- design of complex systems
- modeling and controlling physical systems
- augmenting physical systems with computation
- building systems that are robust to uncertainty

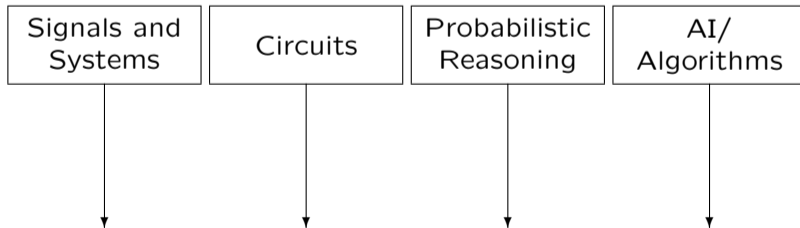
## 6.01: Introduction to EECS I

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Approach: focus on **key concepts** to pursue **in depth**



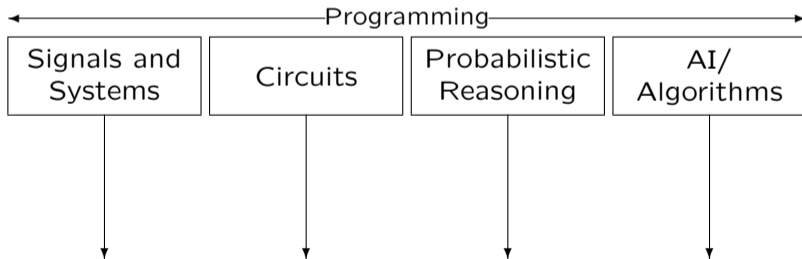
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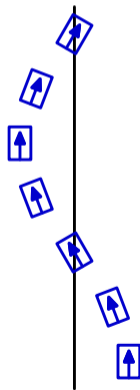
# Module 1: Signals and Systems

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Modeling and analyzing behavior of physical systems

**Topics:** Feedback Control Systems

**Lab Exercises:** Wall-finder, Wall-follower, Jousting



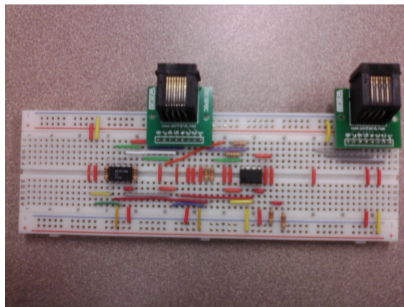
# Module 2: Circuits

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Designing, constructing, and analyzing physical systems

**Topics:** Resistive Networks, Op-Amps, Linearity and Equivalence

**Lab Exercises:** Design a new sensory modality for the robot



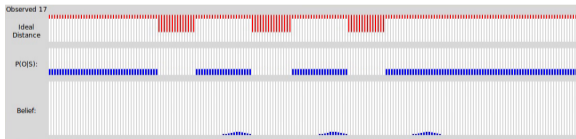
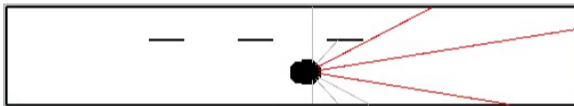
# Module 3: Bayesian Reasoning

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Modeling uncertainty and designing robust systems

**Topics:** Subjective Probability, Bayesian Inference

**Lab Exercises:** Localization and Parking



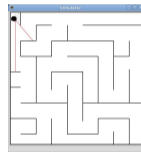
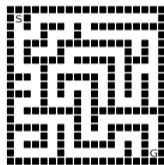
# Module 4: Planning

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Augmenting physical systems with computation.

**Topics:** Graph Search

**Lab Exercises:** Solving mazes, Path planning on maps





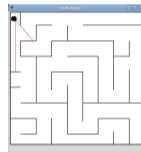
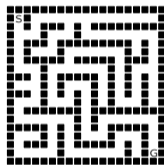
# Module 4: Planning

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Augmenting physical systems with computation.

**Topics:** Graph Search

**Lab Exercises:** Solving mazes, Path planning on maps



# Graph Search (Path Planning)

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What is a graph?

- Some set  $V$  of vertices
- Some collection  $E$  of edges connecting vertices

## Example: 8-Puzzle

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Start

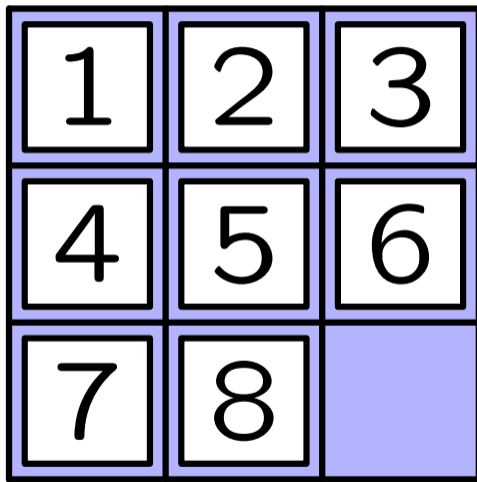
1	2	3
4	5	6
7	8	

Goal

	1	2
3	4	5
6	7	8

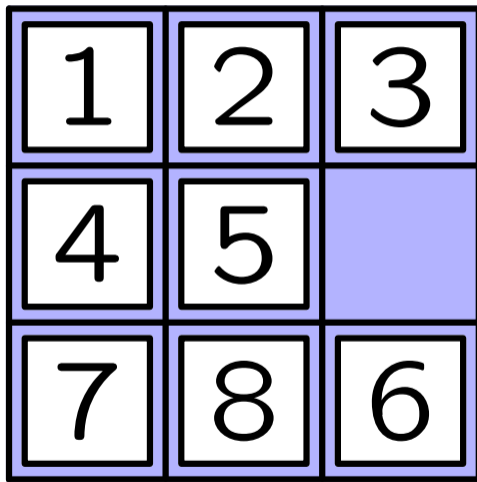
## Example: 8-Puzzle

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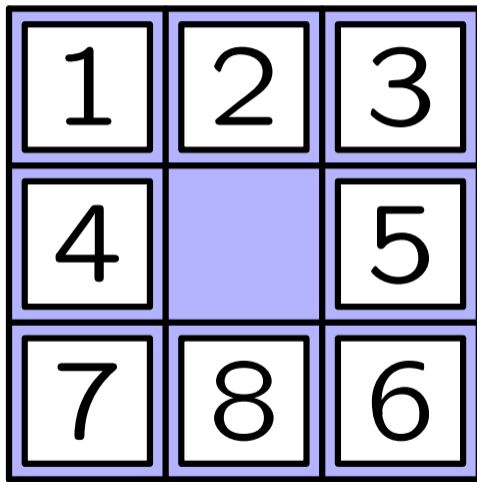
## Example: 8-Puzzle

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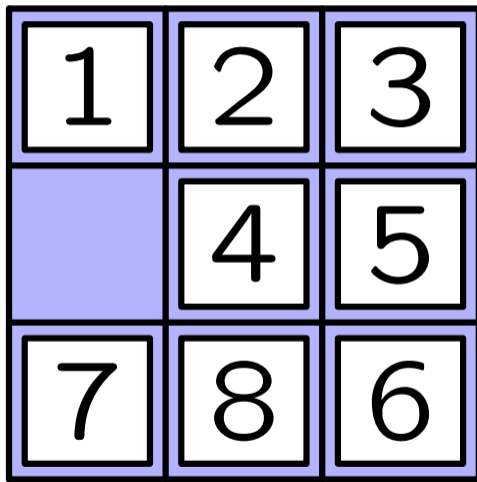
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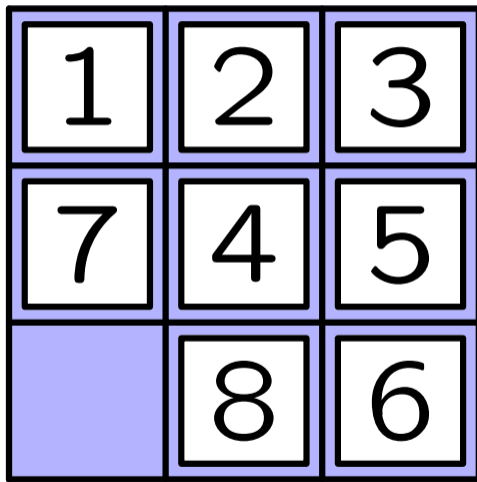
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## Example: 8-Puzzle

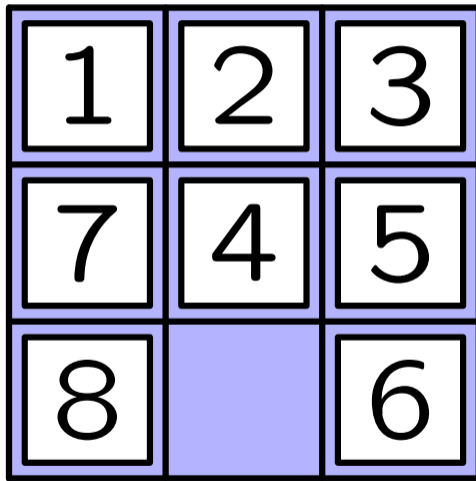
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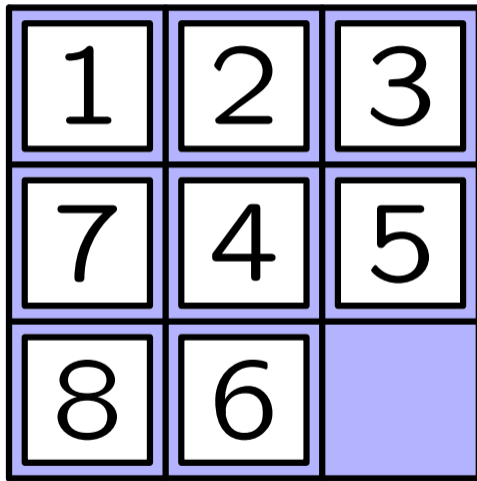
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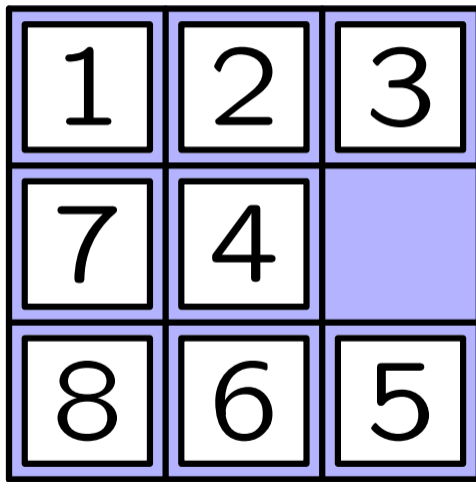
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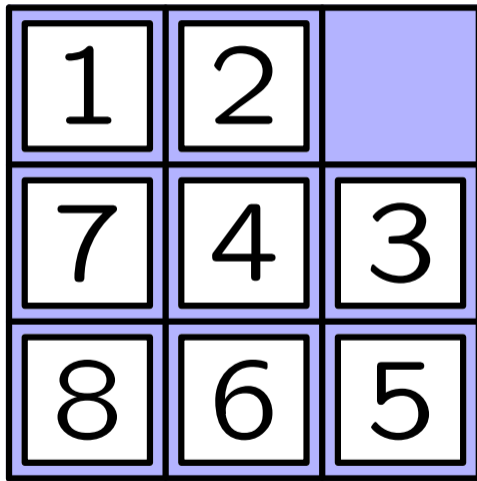
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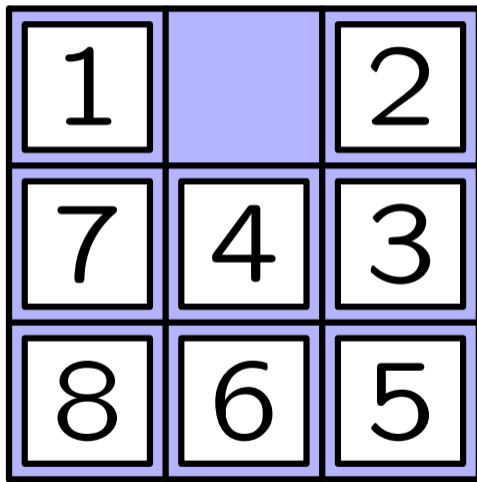
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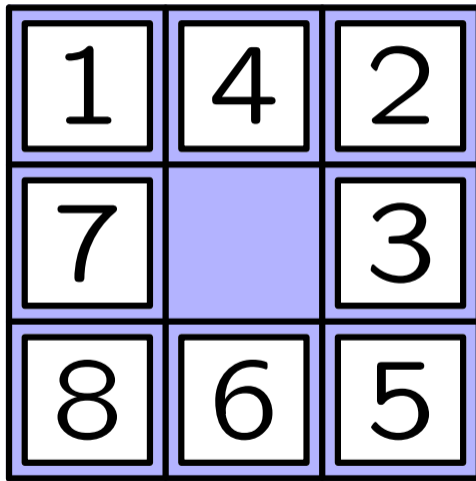
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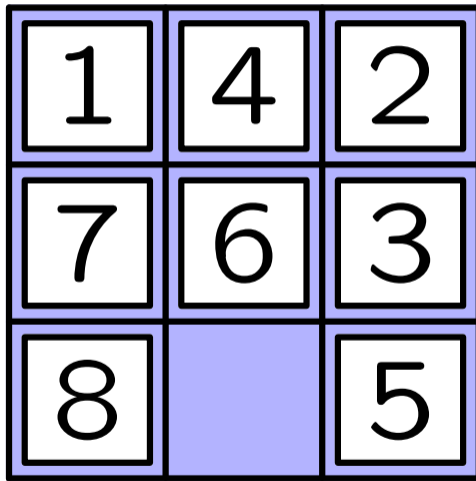
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## Example: 8-Puzzle

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1	4	2
7	6	3
	8	5



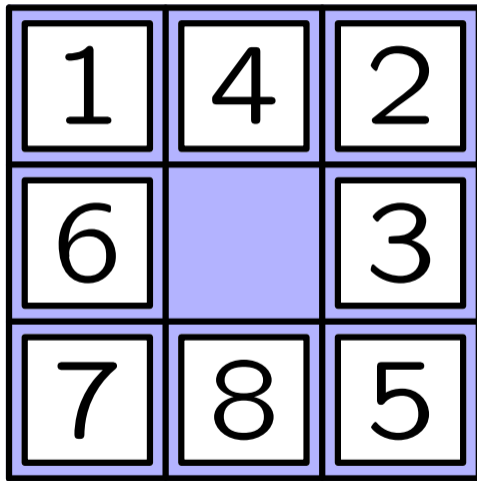
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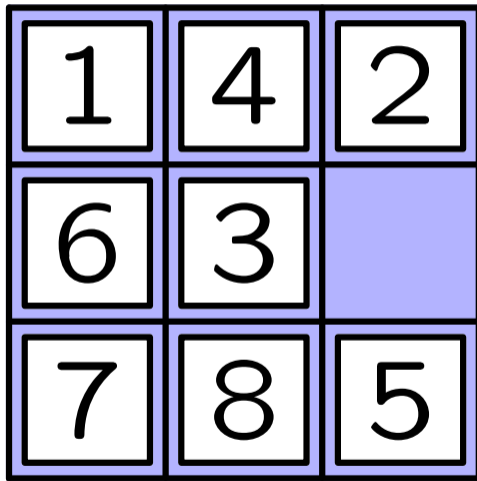
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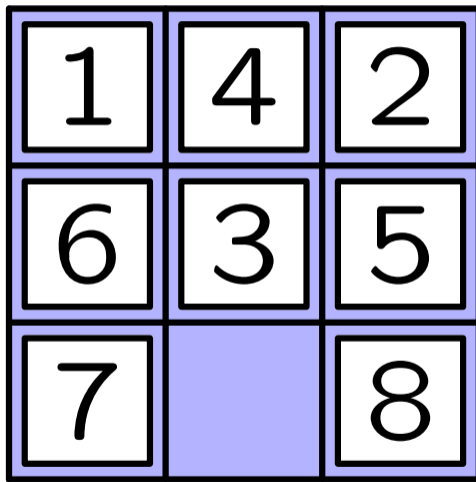
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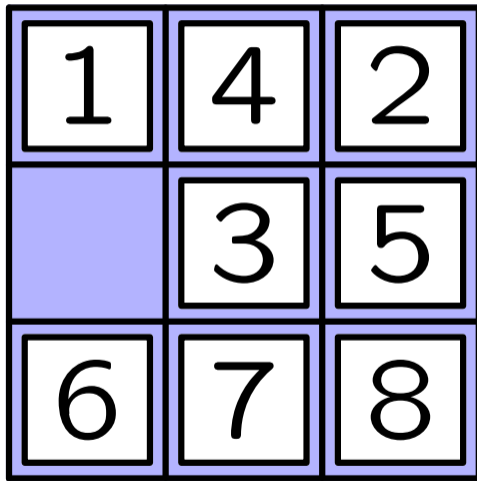
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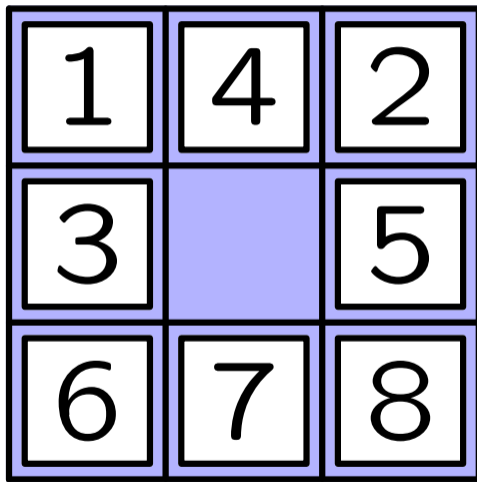
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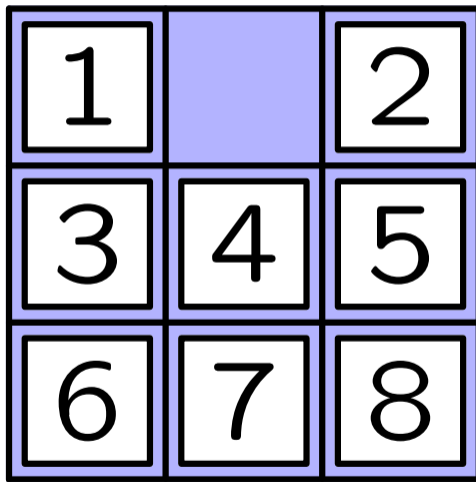
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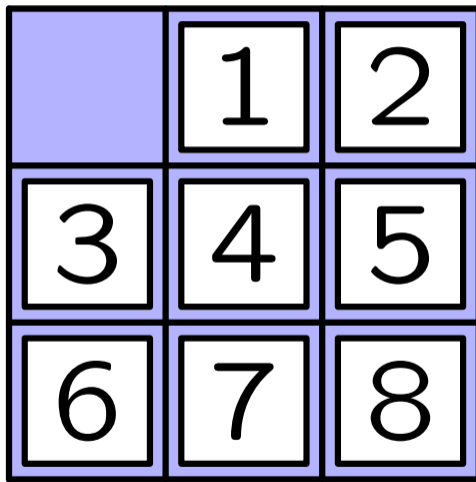
## Example: 8-Puzzle

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## Example: 8-Puzzle

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## Check Yourself

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Start

1	2	3
4	5	6
7	8	

Goal

	1	2
3	4	5
6	7	8

How many different board configurations (states) exist?

1.  $8^2 = 64$
2.  $9^2 = 81$
3.  $8! = 40,320$
4.  $9! = 362,880$
5. None of the above

## Check Yourself

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1	2	3
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We could have to look through as many as  $9! = 362,880$  states (or even more if we're not careful!)

- And we would have to figure out for each state, which of the other states can be reached through a single move.

Is the solution with 22 moves optimal? Do shorter solutions exist?

Do we have to look at all 362,880 configurations to be sure?

# Graph Search

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In this module:

- Develop algorithms to systematically “search” through a graph
- Analyze how well the algorithms perform

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- Develop algorithms to systematically “search” through a graph
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- Optimize the algorithms:
  - Find “better” paths (results)
  - Consider fewer cases (speed)

# Graph Search

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In this module:

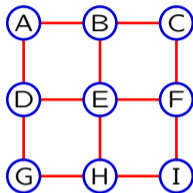
- Develop algorithms to systematically “search” through a graph
- Analyze how well the algorithms perform
- Optimize the algorithms:
  - Find “better” paths (results)
  - Consider fewer cases (speed)
- Observe the algorithms at work in multiple contexts
  - Robot path-planning
  - Route Planning in USA
  - Language
  - Biology



## Example: Grid Search

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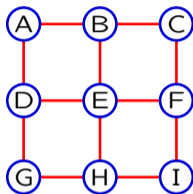
Find path between 2 points on a rectangular grid.



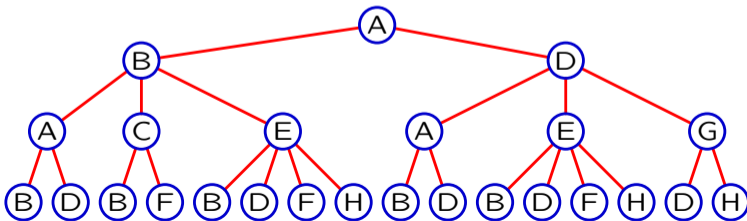
# Example: Grid Search

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Find path between 2 points on a rectangular grid.



Represent **all possible paths** from *A* with a **tree**:

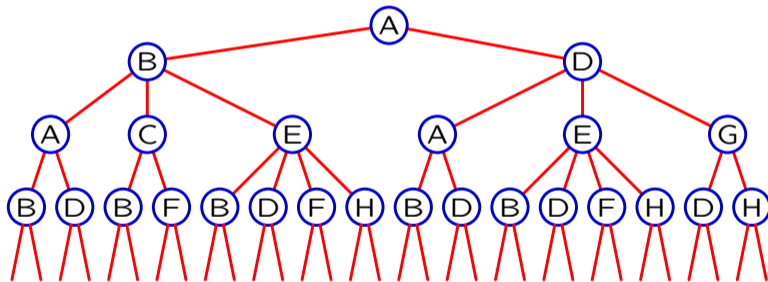


# Problem?

---

Notice that there are infinitely many paths.

The tree is infinitely large!

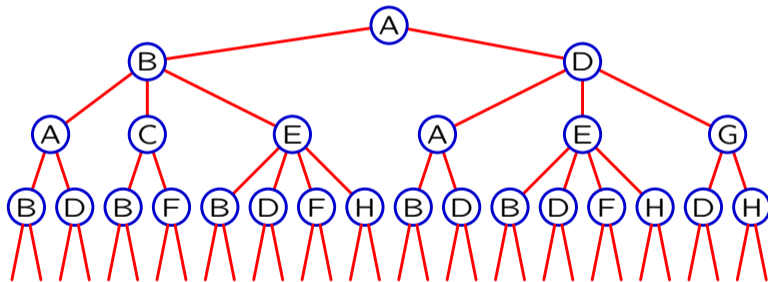


# Problem?

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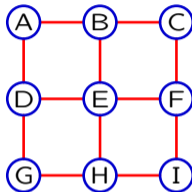
Strategy:

construct the tree **incrementally** while looking for a path

# Python Representation of Grid

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Represent the grid as instance of class Grid



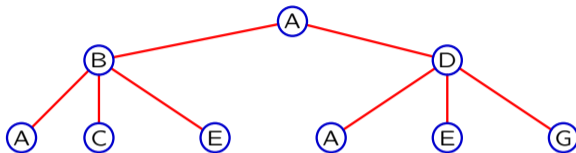
```
class Grid:
    def __init__(self, width, height, start, goal):
        self.width = width
        self.height = height
        self.start = start
        self.goal = goal
```

```
grid = Grid(3, 3, (0,0), (2,2))
```

# Search Trees in Python

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Represent each **node** in the tree as an instance of `SearchNode`



Note: no explicit representation for *entire* tree

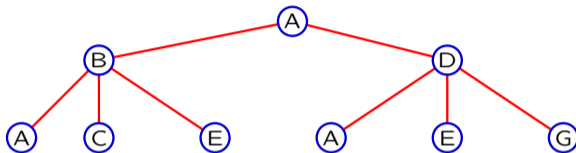
Issues:

- need to “grow” the tree as we search it
- need to reconstruct paths in tree

# Search Trees in Python

---

Represent each **node** in the tree as an instance of `SearchNode`



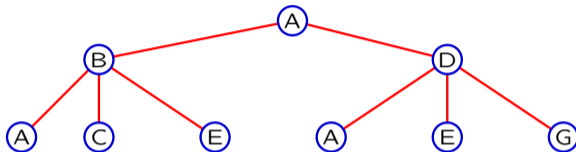
```
class SearchNode:
    def __init__(self, state, parent):
        self.state = state
        self.parent = parent

    def path(self):
        p = []
        node = self
        while node:
            p.append(node.state)
            node = node.parent
        return p[::-1]
```

# Pathfinding Algorithm

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Construct the tree and find a path to the goal.



Algorithm:

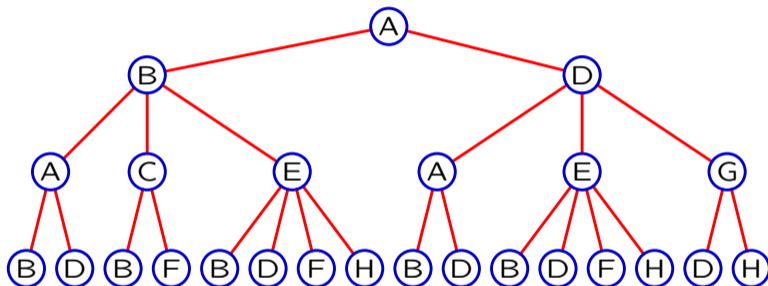
- Initialize **agenda** (list of nodes to consider)
- Repeat the following:
  - Remove one node from the agenda ("expand")
  - Add that node's successors to the agenda ("visit")until **goal is found** or **agenda is empty**
- Return resulting path



# Order Matters!

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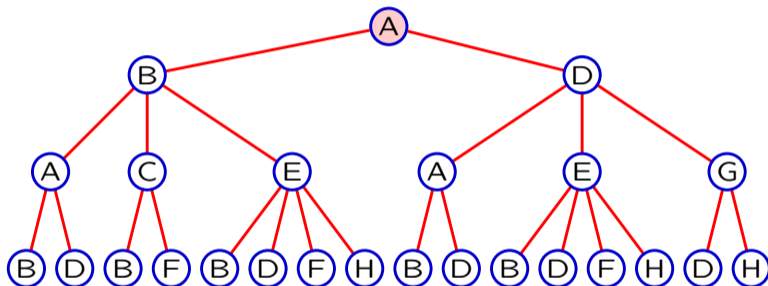
Strategy: Replace last node in agenda by its successors



# Order Matters!

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Strategy: Replace last node in agenda by its successors

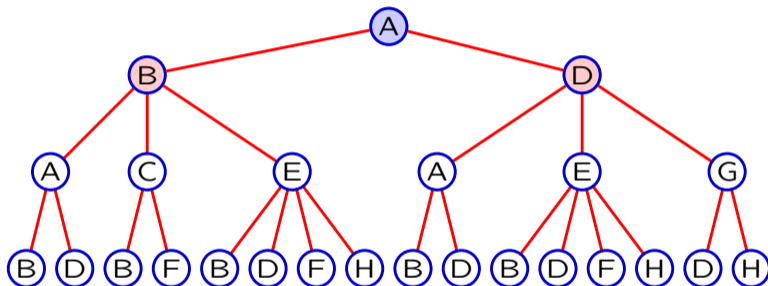


Agenda: A

# Order Matters!

---

Strategy: Replace last node in agenda by its successors

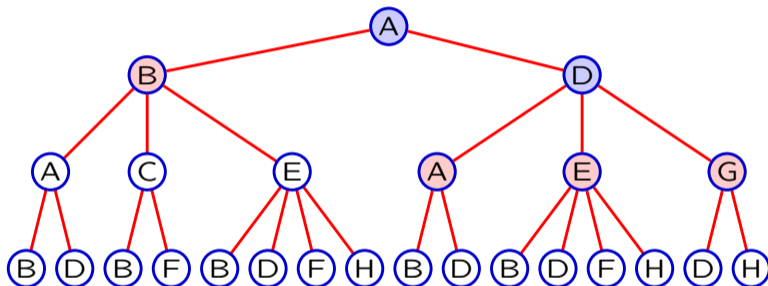


Agenda: **A** AB AD

# Order Matters!

---

Strategy: Replace last node in agenda by its successors

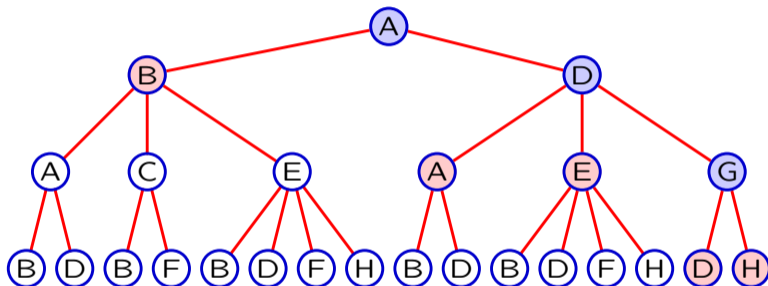


Agenda: ~~A~~ AB ~~AD~~ ADA ADE ADG

# Order Matters!

---

Strategy: Replace last node in agenda by its successors

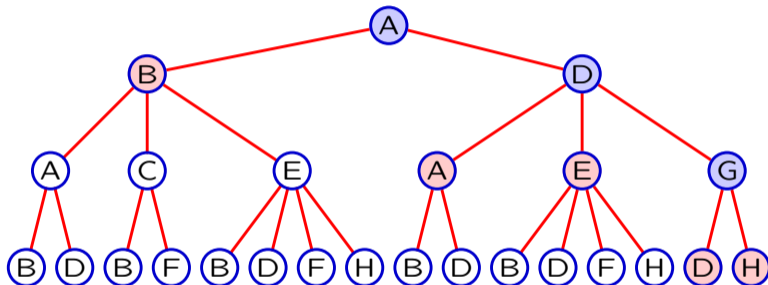


Agenda: A AB ~~AD~~ ADA ADE **ADG** **ADGD** **ADGH**

# Order Matters!

---

Strategy: Replace last node in agenda by its successors



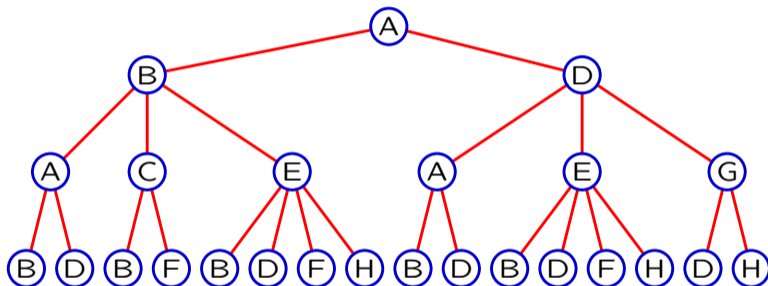
Agenda: A AB ~~AD~~ ADA ADE **ADG** **ADGD** **ADGH**

*Depth-first* Search

# Order Matters!

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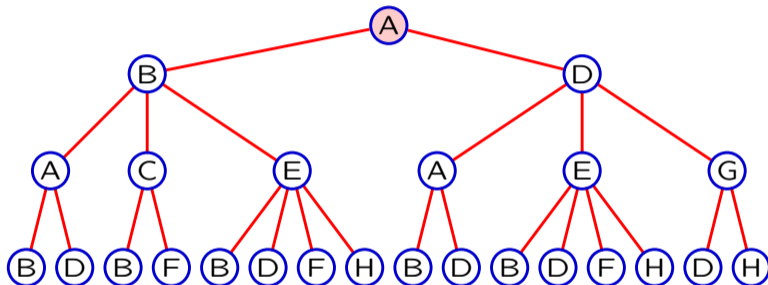
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# Order Matters!

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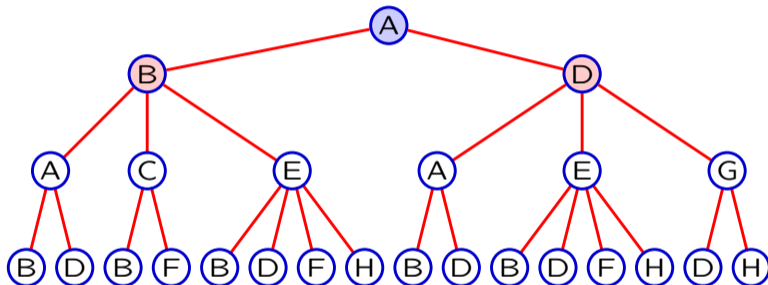
Agenda: A



# Order Matters!

---

Strategy: Replace first node in agenda by its successors

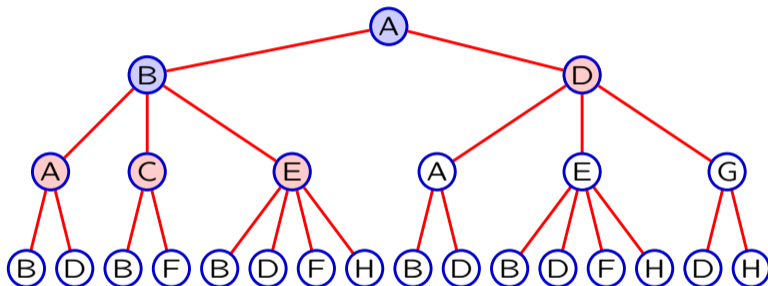


Agenda: **A** AB AD

# Order Matters!

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Strategy: Replace first node in agenda by its successors

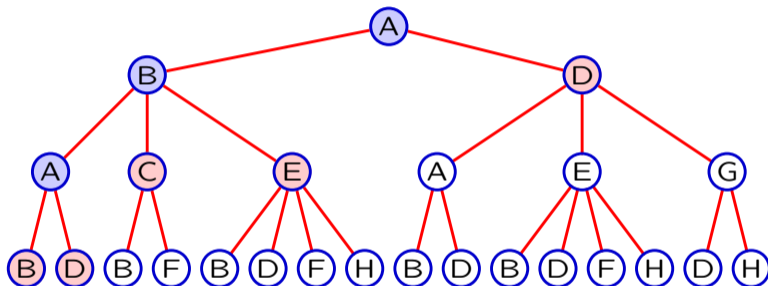


Agenda: A AB ABA ABC ABE AD

# Order Matters!

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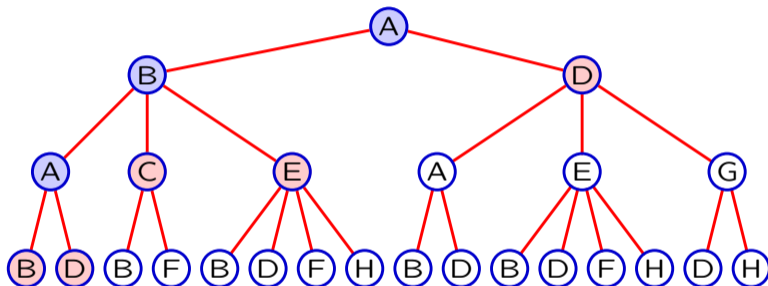
Strategy: Replace first node in agenda by its successors



Agenda: A AB **ABA** **ABAB** **ABAD** ABC ABE AD

# Order Matters!

Strategy: Replace first node in agenda by its successors



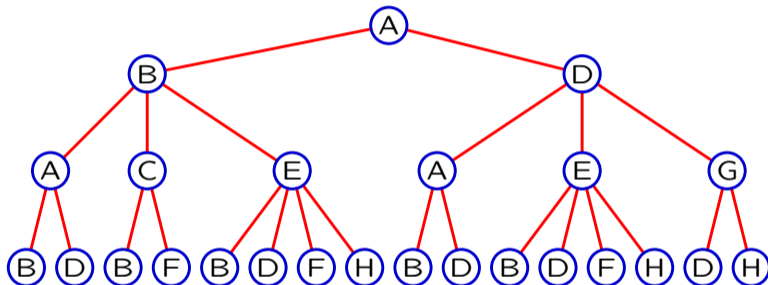
Agenda: ~~A~~ ~~AB~~ ~~ABA~~ ~~ABAB~~ ~~ABAD~~ ABC ABE AD

Still *Depth-first* Search

# Order Matters!

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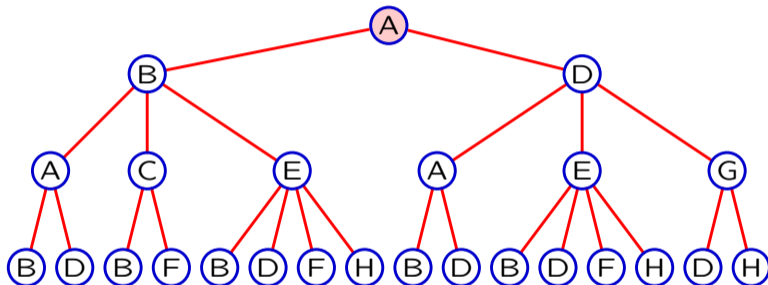
Strategy: Remove first node and add its successors **to end**



# Order Matters!

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Strategy: Remove first node and add its successors **to end**

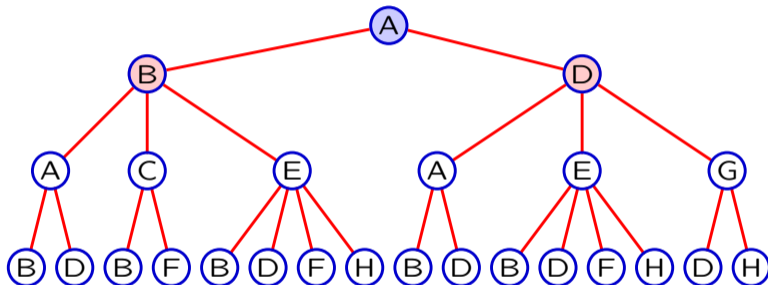


Agenda: **A**

# Order Matters!

---

Strategy: Remove first node and add its successors **to end**



Agenda: **A** AB AD

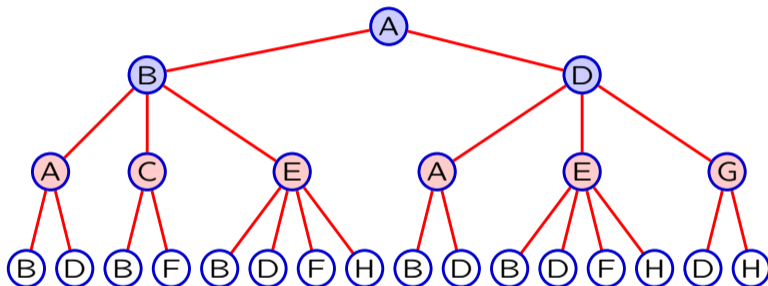




# Order Matters!

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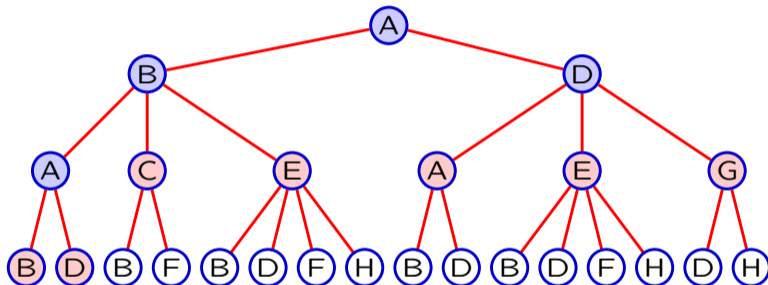


Agenda: A AB ~~AD~~ ABA ABC ABE **ADA ADE ADG**

# Order Matters!

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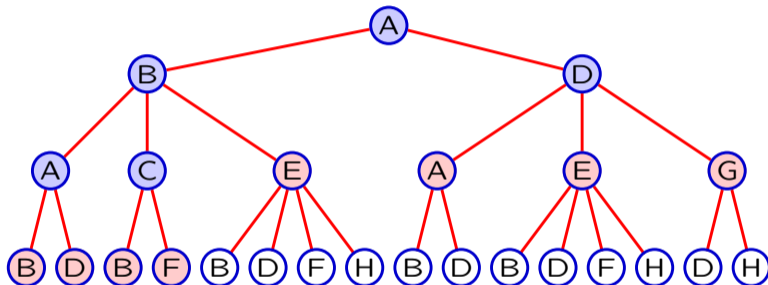
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Agenda: A AB AD ~~ABA~~ ABC ABE ADA ADE ADG **ABAB ABAD**

# Order Matters!

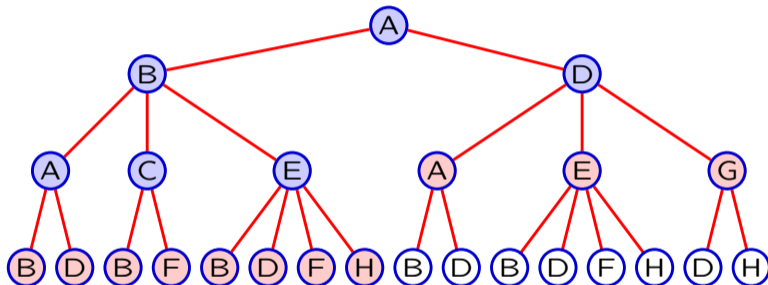
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Agenda: A AB AD ABA **ABC** ABE ADA ADE ADG ABAB ABAD **ABCB**  
**ABCF**

# Order Matters!

Strategy: Remove first node and add its successors to end

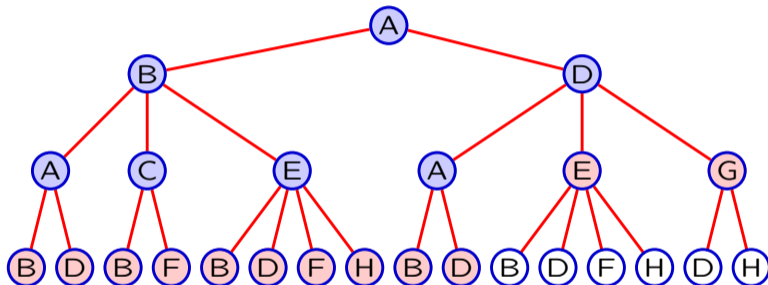


Agenda: A AB AD ~~ABA~~ ~~ABC~~ ABE ADA ADE ADG ABAB ABAD ABCB  
ABCF ABEB ABED ABEF ABEH

# Order Matters!

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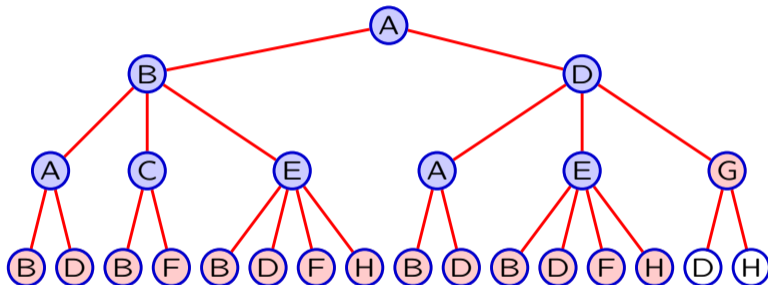
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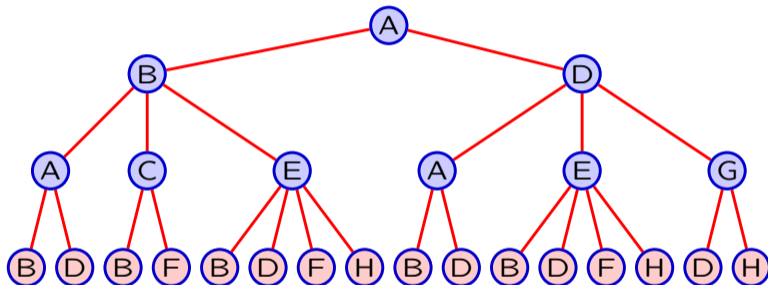


Agenda: A AB AD ABA ABC ABE ADA ADE ADG ABAB ABAD ABCB  
ABCF ABEB ABED ABEF ABEH ADAB ADAD ADEB ADED ADEF ADEH

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---

Strategy: Remove first node and add its successors **to end**

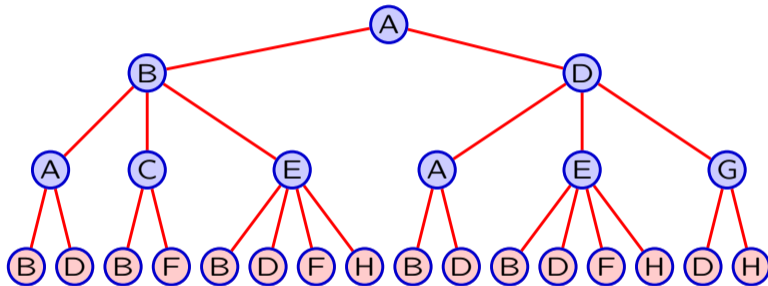


Agenda: A AB AD ~~ABA~~ ~~ABC~~ ABE ADA ADE **ADG** ABAB ABAD ABCB  
ABCF ABEB ABED ABEF ABEH ADAB ADAD ADEB ADED ADEF ADEH  
**ADGD ADGH**

# Order Matters!

---

Strategy: Remove first node and add its successors to end



Agenda: A AB AD ~~ABA ABC ABE ADA ADE~~ **ADG** ABAB ABAD ABCB  
ABCF ABEB ABED ABEF ABEH ADAB ADAD ADEB ADED ADEF ADEH  
**ADGD ADGH**

*Breadth-first* Search



# Order Matters!

---

Depth-First Search (DFS):

- Push and Pop from same side of agenda
- Works down one branch of the tree before moving on to another branch

Breadth-First Search (BFS):

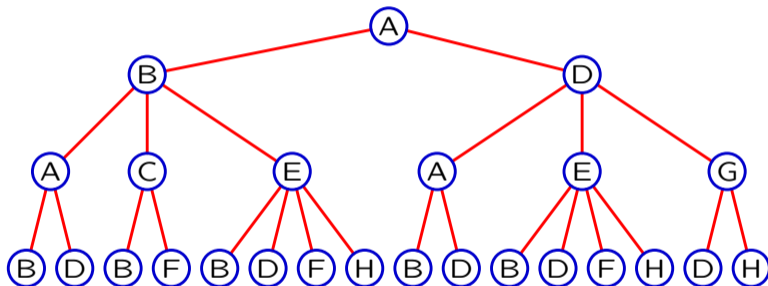
- Push and Pop from different sides of agenda
- Considers all paths of length  $n$  before considering paths of length  $n + 1$

# Too Much Searching

---

Find path between 2 points on a rectangular grid.

Represent **all possible paths** with a **tree**:



But don't need to consider all nodes!

# Pruning

---

“Prune” the tree to reduce the amount of work.

## **Pruning Strategy 1:**

Don't consider any path the visits the same state twice.

# Pruning

---

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Algorithm:

- Initialize **agenda** (list of nodes to consider)
  - Repeat the following:
    - Remove one node from the agenda
    - Add each child (of that node) to the agenda **if its state is not in the parent's path.**
- until **goal is found** or **agenda is empty**
- Return resulting path

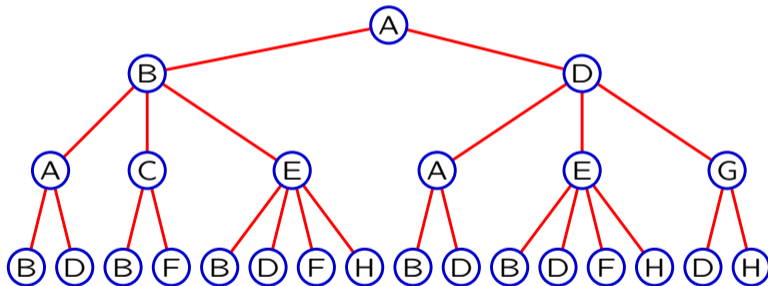
# Pruning

---

“Prune” the tree to reduce the amount of work.

## Pruning Strategy 1:

Don't consider any path that contains the same state twice.

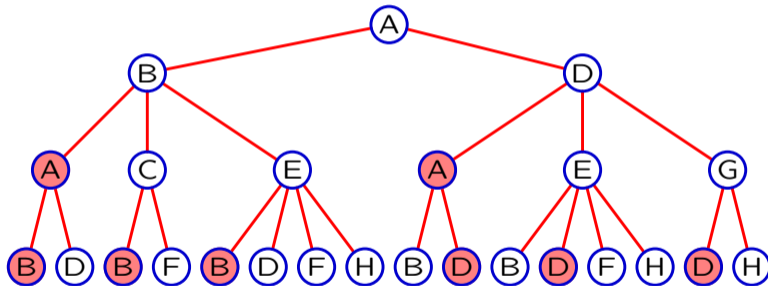


# Pruning

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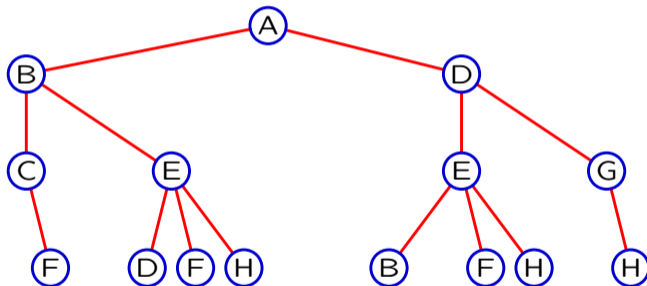
# Pruning

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# Pruning

---

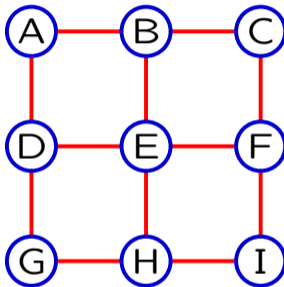
Under strategy 1, BFS in the 3x3 grid still visits 16 nodes...



# Pruning

---

Under strategy 1, BFS in the 3x3 grid still visits 16 nodes...  
but there are only 9 states!



We should be able to reduce the search even further.

# Dynamic Programming

---

Basic idea behind dynamic programming:

- Break big problem into easy ones, solve and combine.
- Remember the solutions to the easy problems for later use.

# Dynamic Programming

---

Basic idea behind dynamic programming:

- Break big problem into easy ones, solve and combine.
- Remember the solutions to the easy problems for later use.

Appropriate if problem has:

- *optimal substructure*: best solution is combination of optimal solutions to sub-problems
- *overlapping sub-problems*: same sub-problem occurs many times while solving overall problem

# Dynamic Programming

---

As applies to search:

(Depends slightly on which algorithm we're using)

# Dynamic Programming

---

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(Depends slightly on which algorithm we're using)

**BFS:** The shortest path  $S \rightarrow X \rightarrow G$  is made up of the shortest path  $S \rightarrow X$  and the shortest path  $X \rightarrow G$ .

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(Depends slightly on which algorithm we're using)

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The moral: once we have found a path  $S \rightarrow X$ , we don't need to spend time looking for other paths through  $X$ .

# Dynamic Programming

---

As applies to search:

(Depends slightly on which algorithm we're using)

**BFS:** The shortest path  $S \rightarrow X \rightarrow G$  is made up of the shortest path  $S \rightarrow X$  and the shortest path  $X \rightarrow G$ .

**DFS:** A path  $S \rightarrow X \rightarrow G$  is made up of a path  $S \rightarrow X$  and a path  $X \rightarrow G$ .

The moral: once we have found a path  $S \rightarrow X$ , we don't need to spend time looking for other paths through  $X$ .

Said another way: Many paths that include  $S \rightarrow X$ , but don't need to recompute while exploring rest of path (*memoization*); and once have a satisfactory path  $S \rightarrow X$ , don't need to keep looking for others (*dynamic programming*).



# Dynamic Programming

---

As applied to graph search: Don't consider any path that visits a state that you have already visited via some other path.

Need to remember which states we have visited to avoid visiting them again.

# Dynamic Programming

---

As applied to graph search: Don't consider any path that visits a state that you have already visited via some other path.

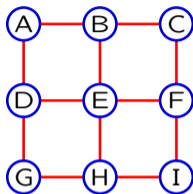
Need to remember which states we have visited to avoid visiting them again.

Algorithm:

- Initialize **visited set**
  - Initialize **agenda** (list of nodes to consider)
  - Repeat the following:
    - Remove one node from the agenda
    - Add each child (of that node) to the agenda **if its state is not already in the visited set**, and **add each of these new states to the visited set**
- until **goal is found** or **agenda is empty**
- Return resulting path

## Check Yourself!

---

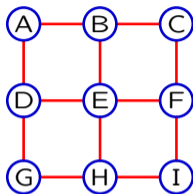


Consider a breadth-first search with dynamic programming, from  $A$  to  $I$ . How many states are visited?

1. 2
2. 4
3. 6
4. 8
5. 10

## Check Yourself!

---



Consider a breadth-first search with dynamic programming, from  $A$  to  $I$ . How many states are visited?

1. 2
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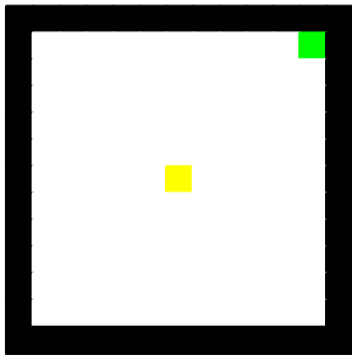
# Before we continue...

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Properties of DFS/BFS

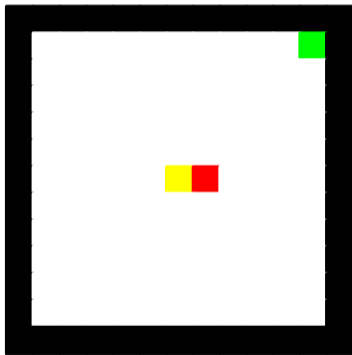
# Grid: BFS 1

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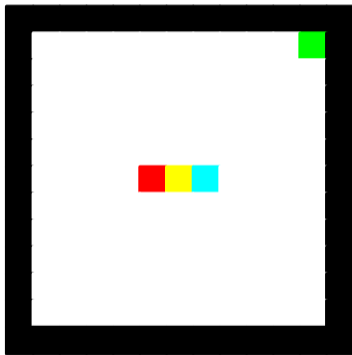
# Grid: BFS 1

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# Grid: BFS 1

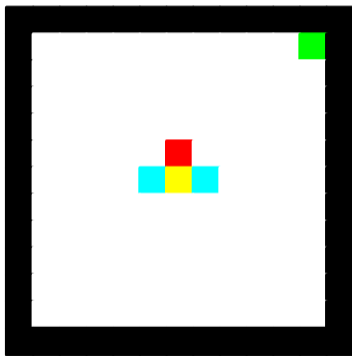
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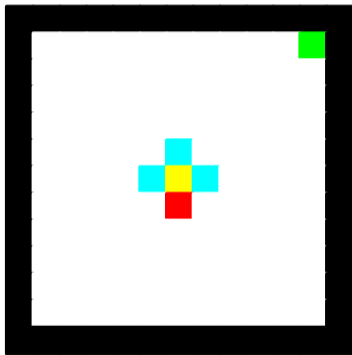
# Grid: BFS 1

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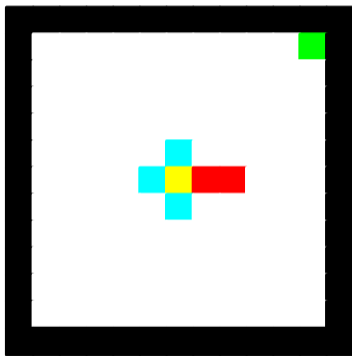
# Grid: BFS 1

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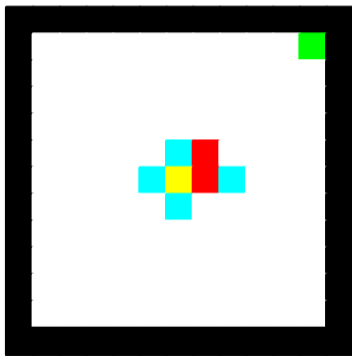
# Grid: BFS 1

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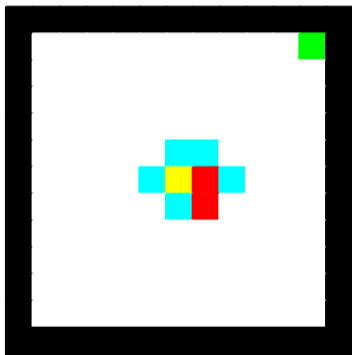
# Grid: BFS 1

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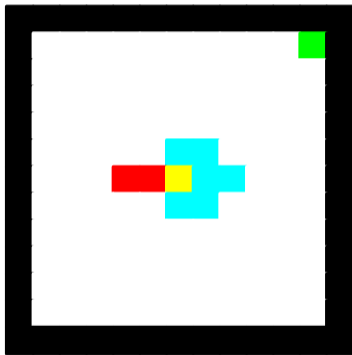
# Grid: BFS 1

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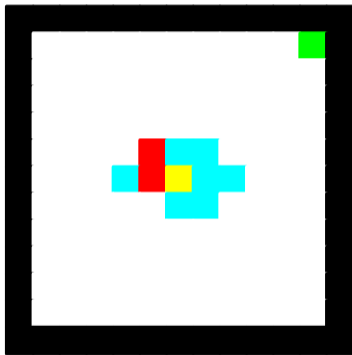
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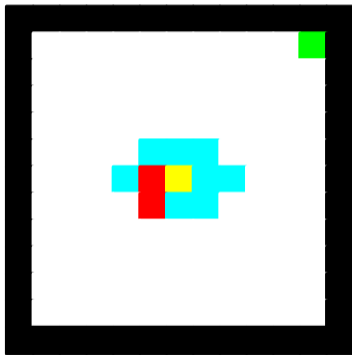
# Grid: BFS 1

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# Grid: BFS 1

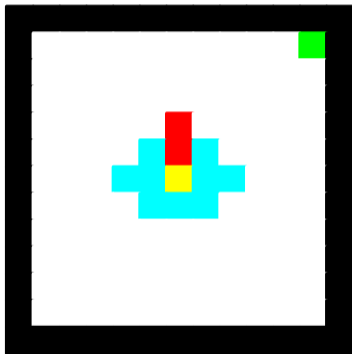
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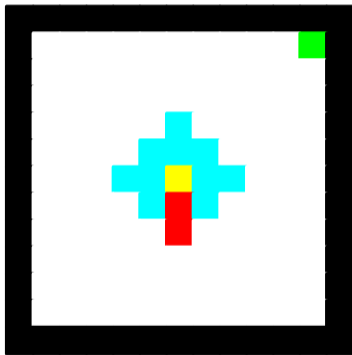
# Grid: BFS 1

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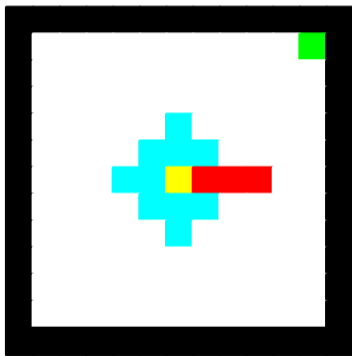
# Grid: BFS 1

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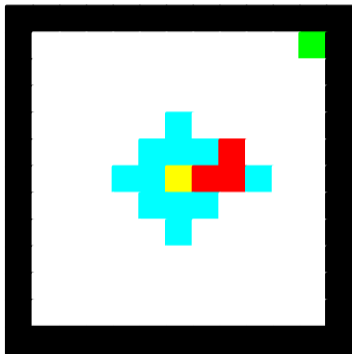
# Grid: BFS 1

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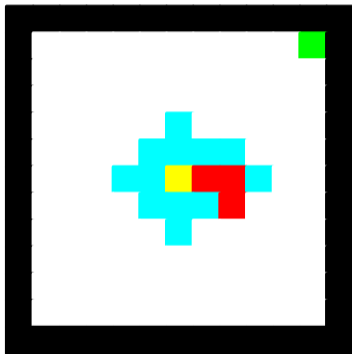
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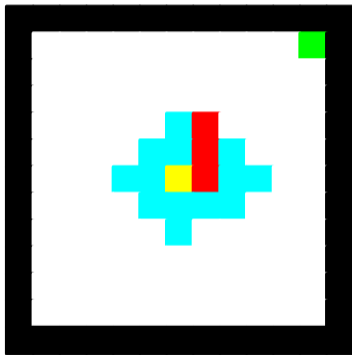
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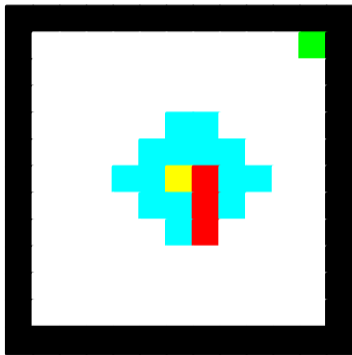
# Grid: BFS 1

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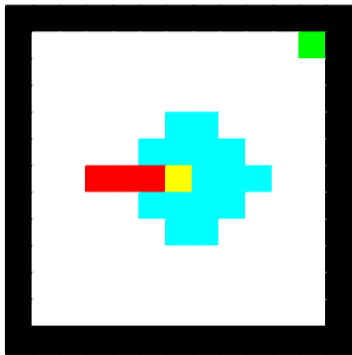
# Grid: BFS 1

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# Grid: BFS 1

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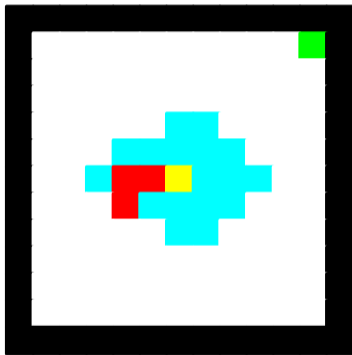






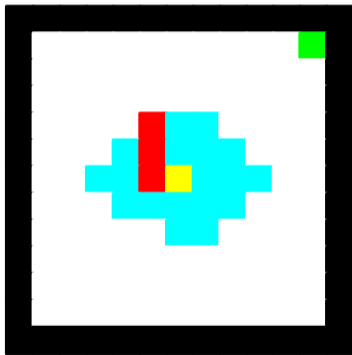
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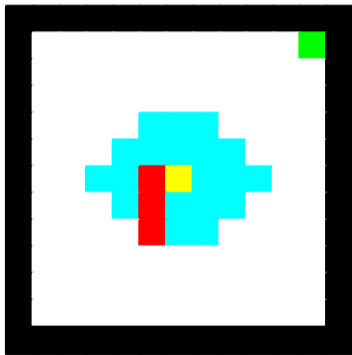
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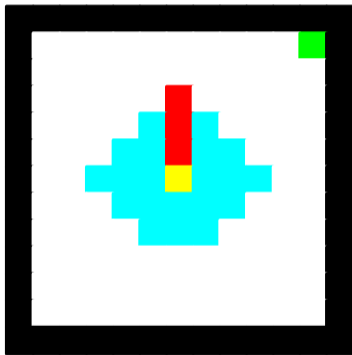
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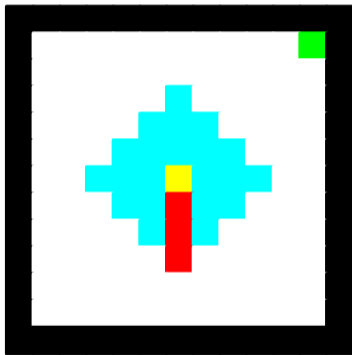
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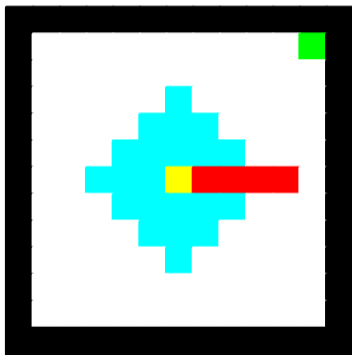
# Grid: BFS 1

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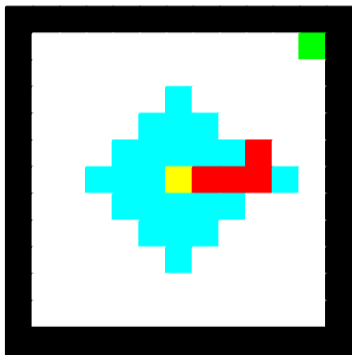
# Grid: BFS 1

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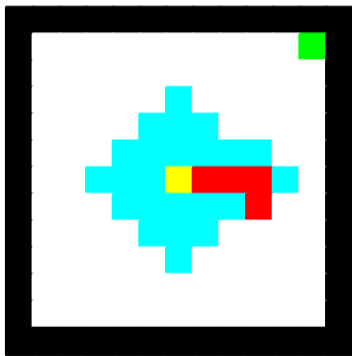
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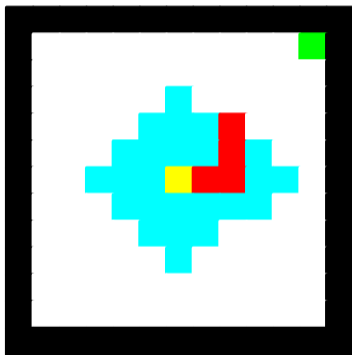
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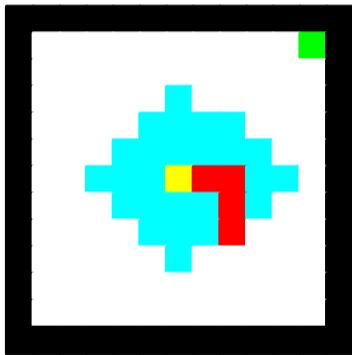
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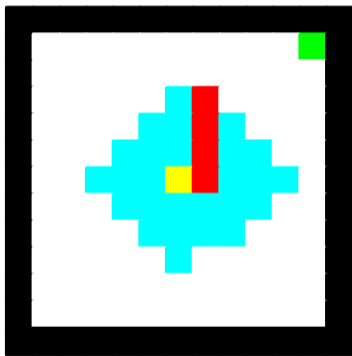
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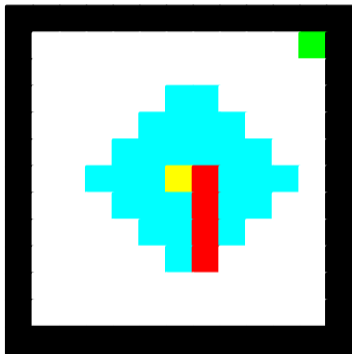
# Grid: BFS 1

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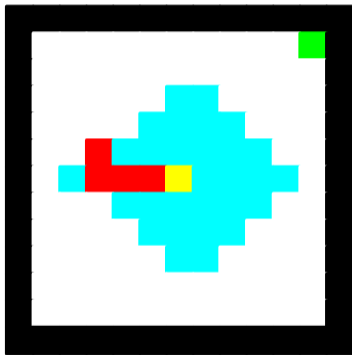
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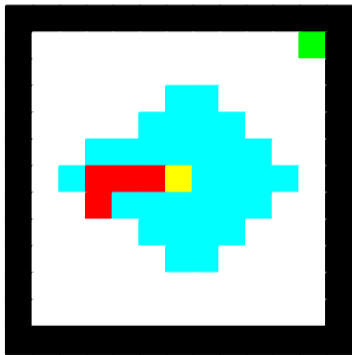
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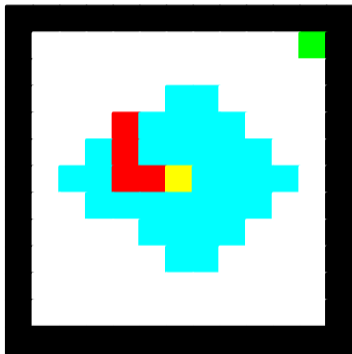
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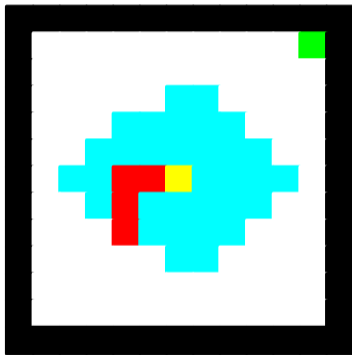
# Grid: BFS 1

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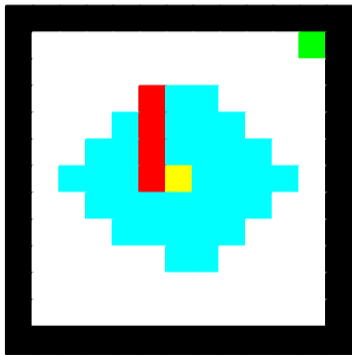
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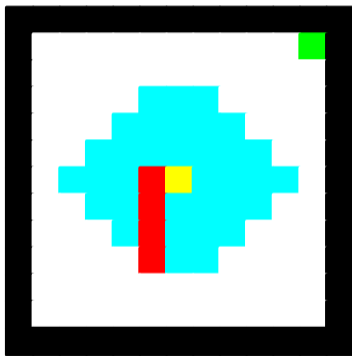
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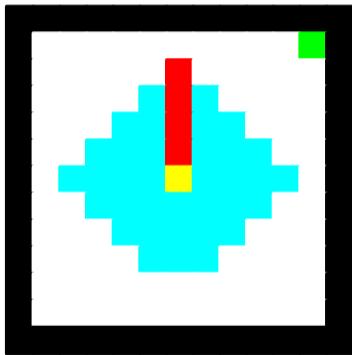
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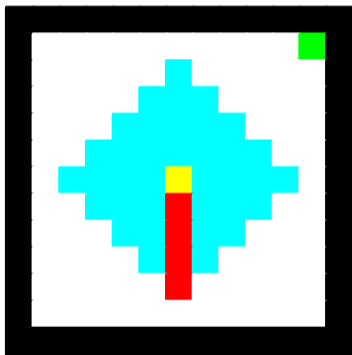
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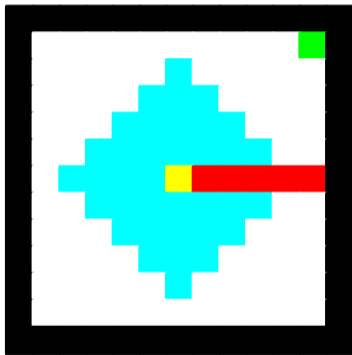
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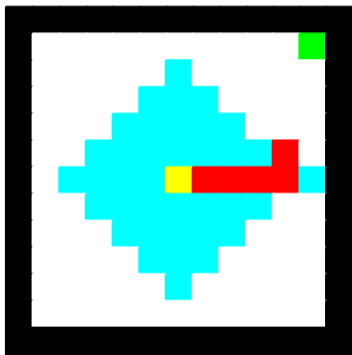
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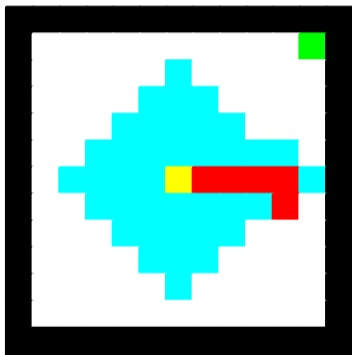
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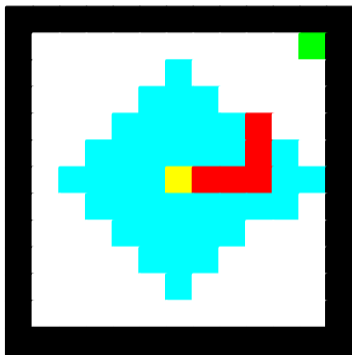
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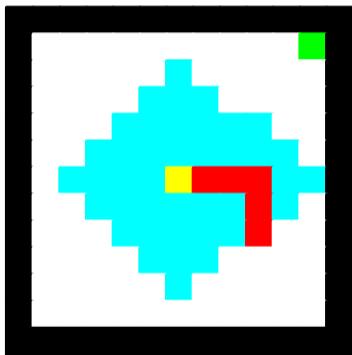
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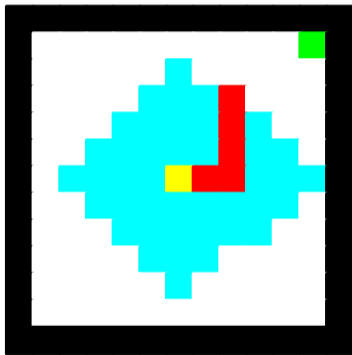
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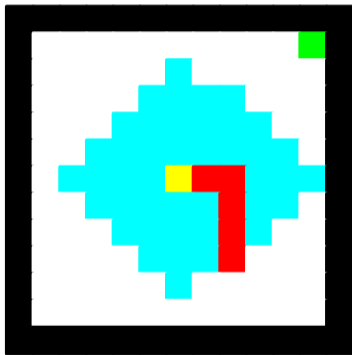
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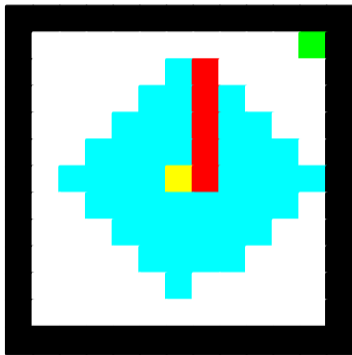
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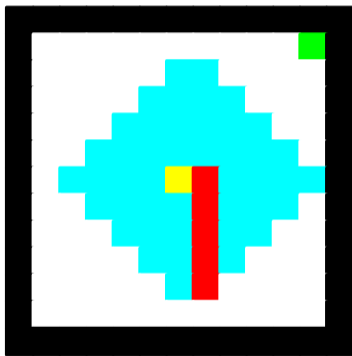
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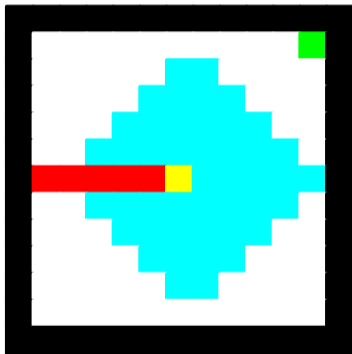
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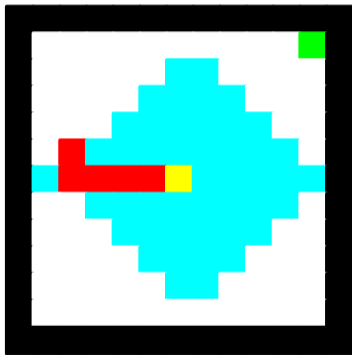
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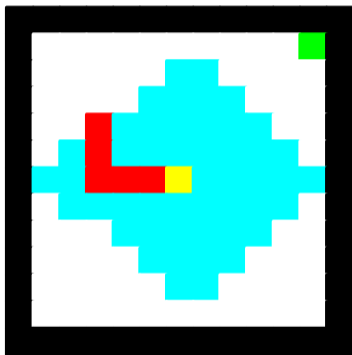
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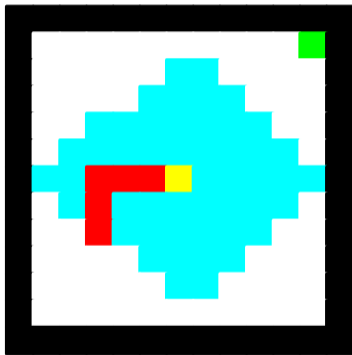
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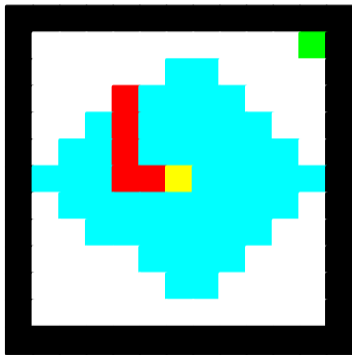
# Grid: BFS 1

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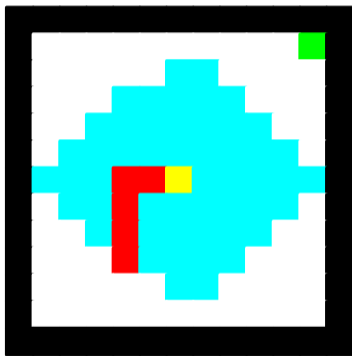
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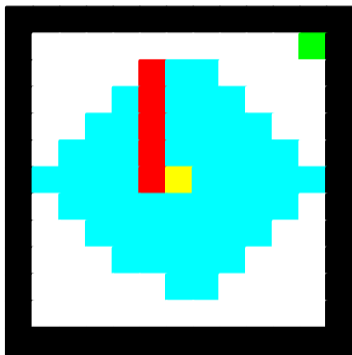
# Grid: BFS 1

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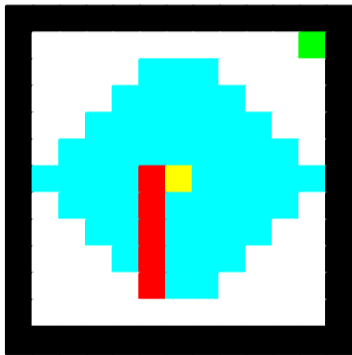
# Grid: BFS 1

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# Grid: BFS 1

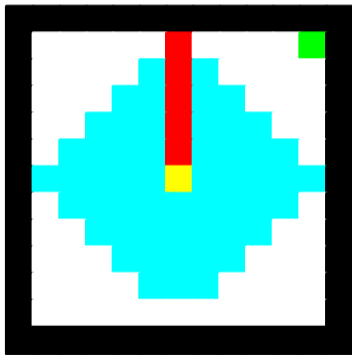
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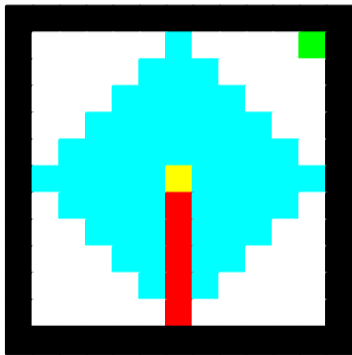
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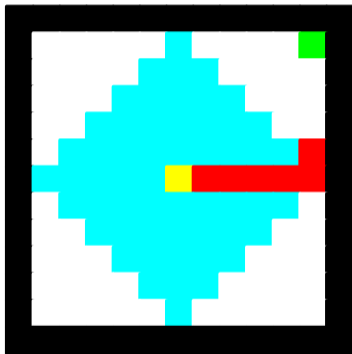
# Grid: BFS 1

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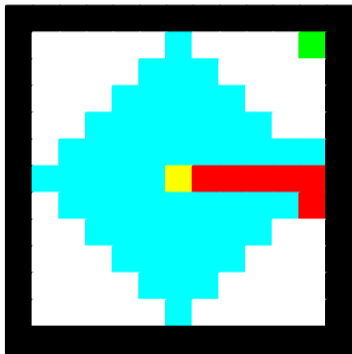
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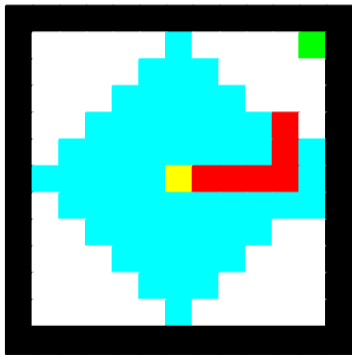
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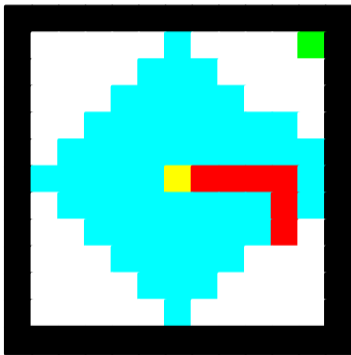
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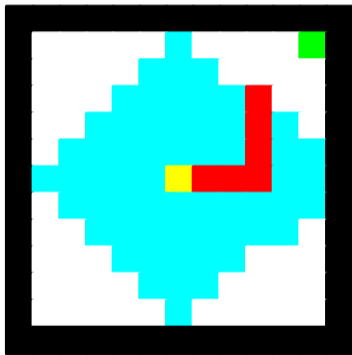
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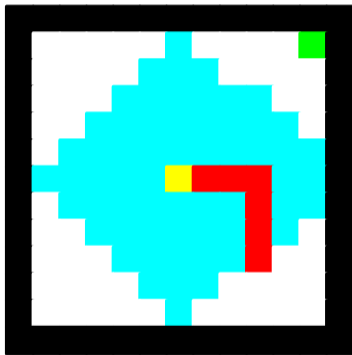
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# Grid: BFS 1

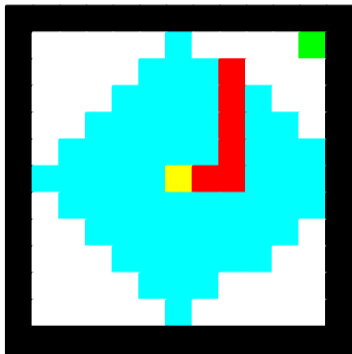
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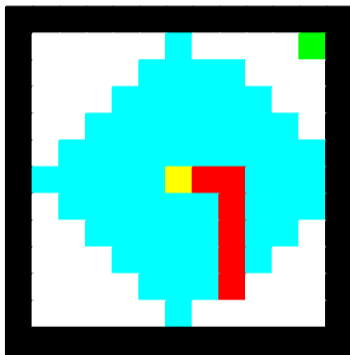
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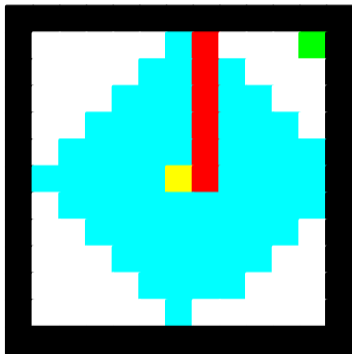
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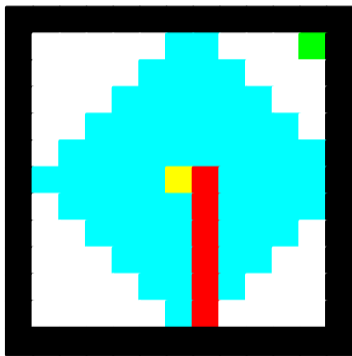
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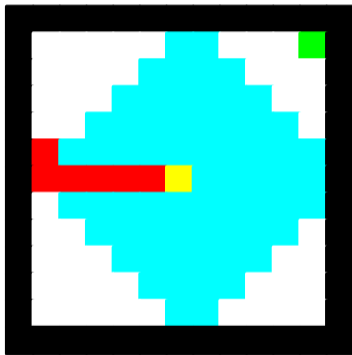
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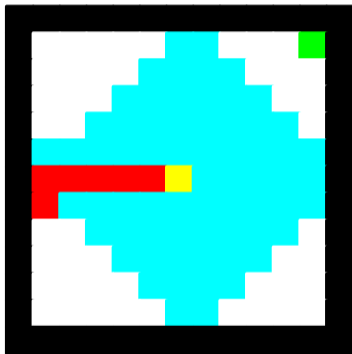
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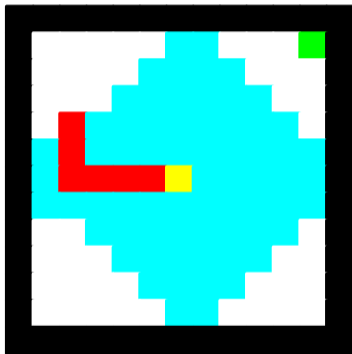
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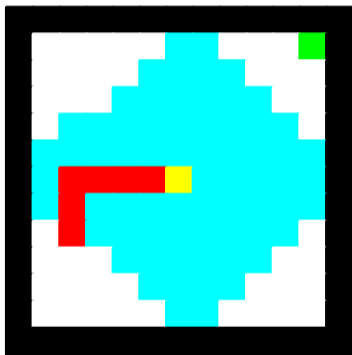
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# Grid: BFS 1

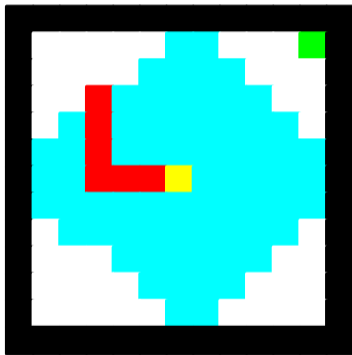
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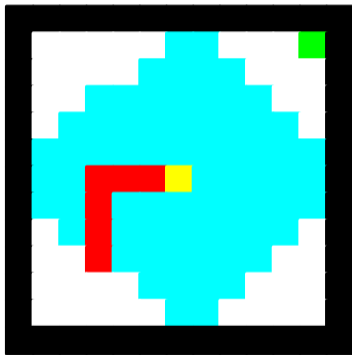
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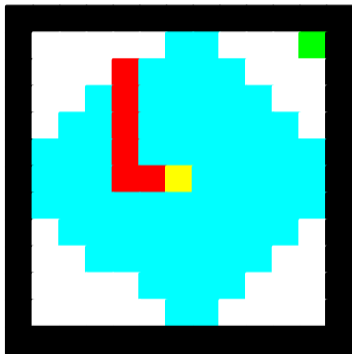
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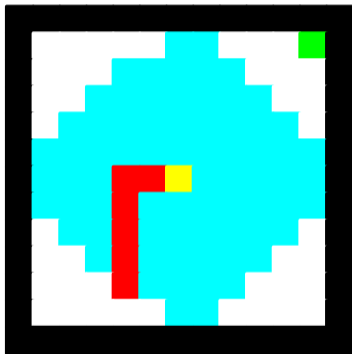
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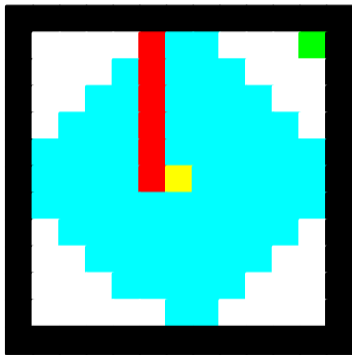
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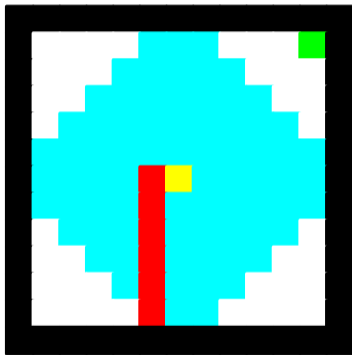
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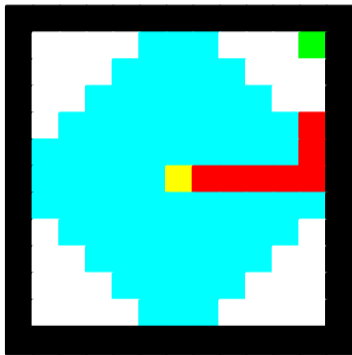
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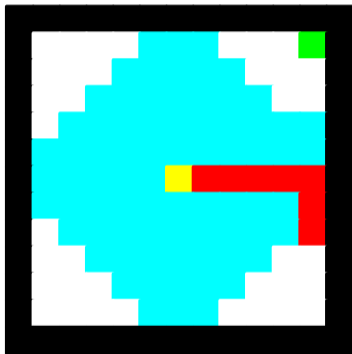
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# Grid: BFS 1

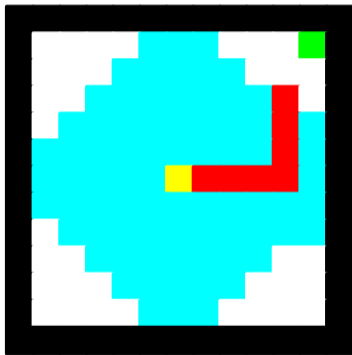
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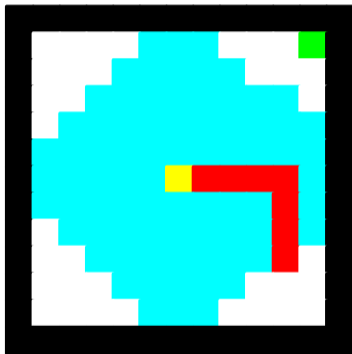
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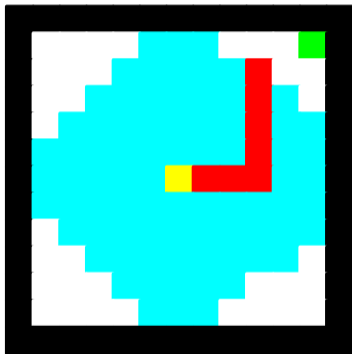
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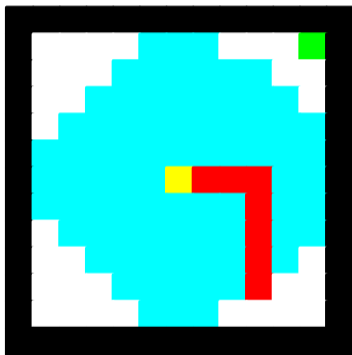
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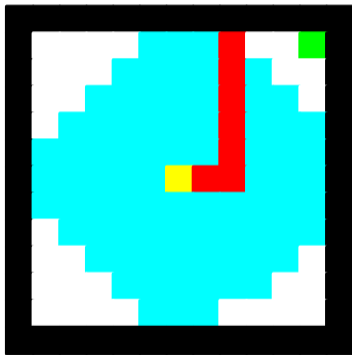
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# Grid: BFS 1

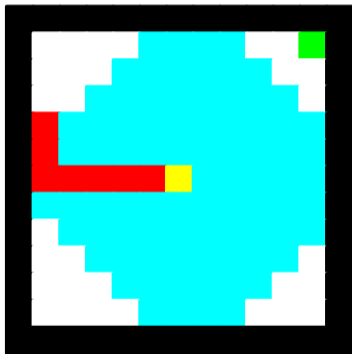
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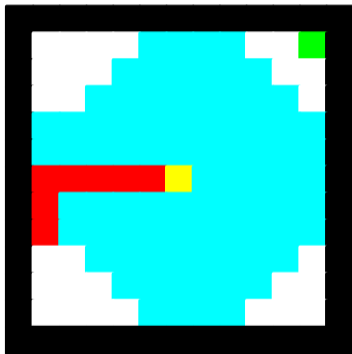
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# Grid: BFS 1

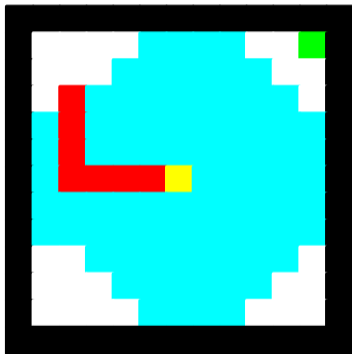
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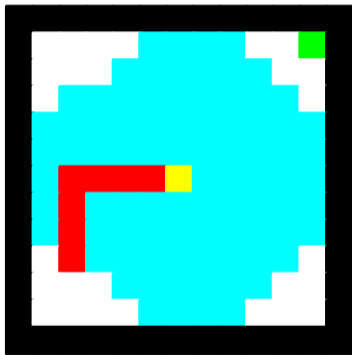
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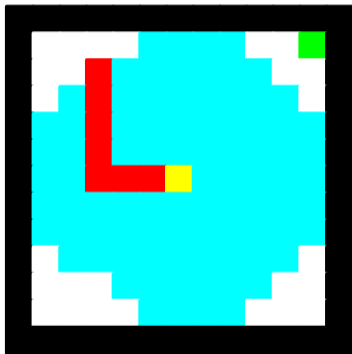
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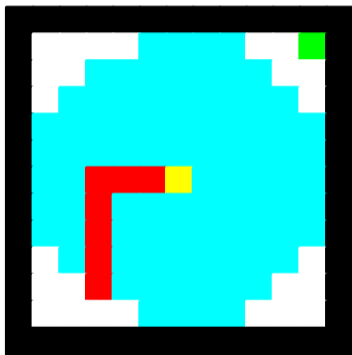
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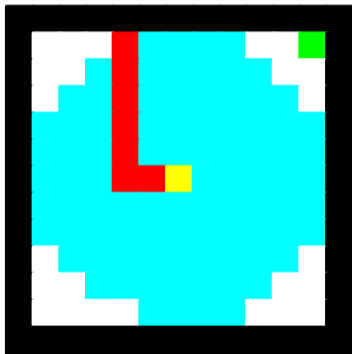
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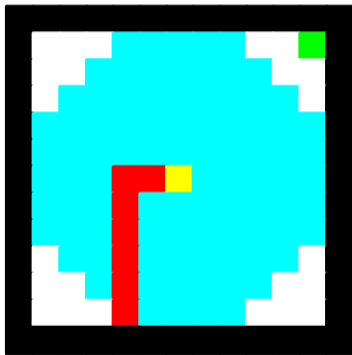
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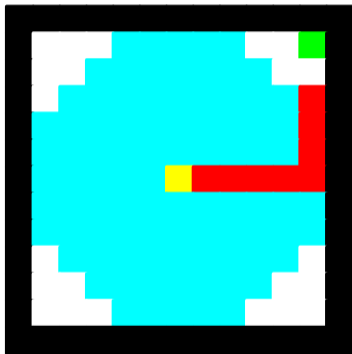
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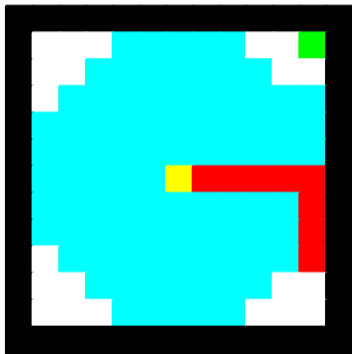
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# Grid: BFS 1

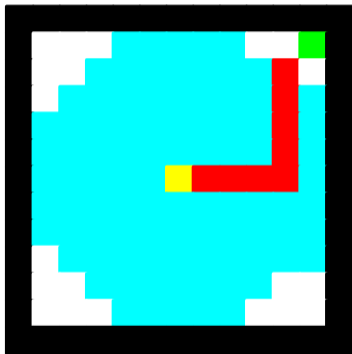
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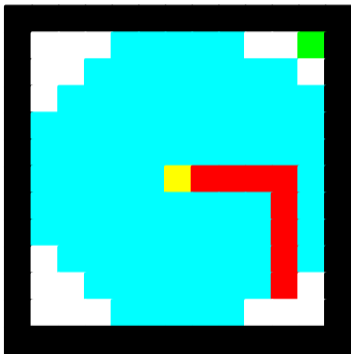
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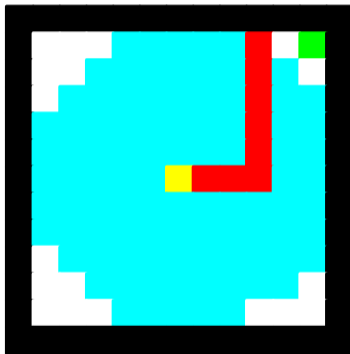
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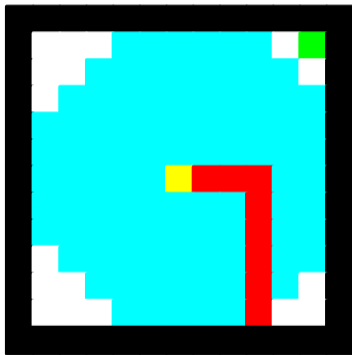
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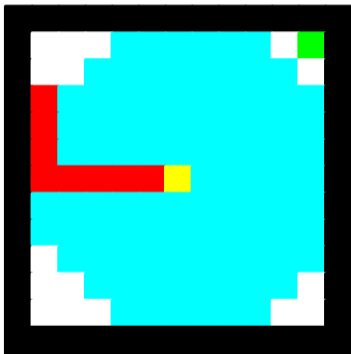
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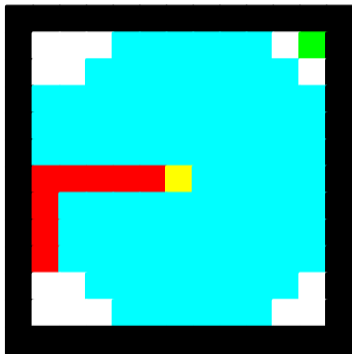
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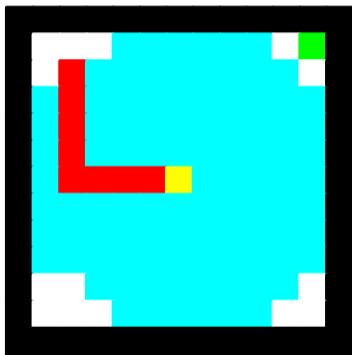
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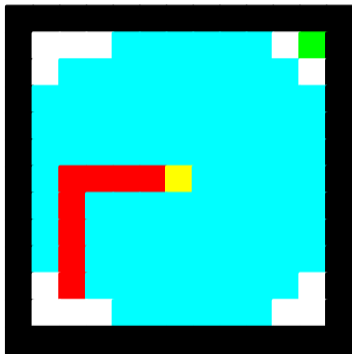
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# Grid: BFS 1

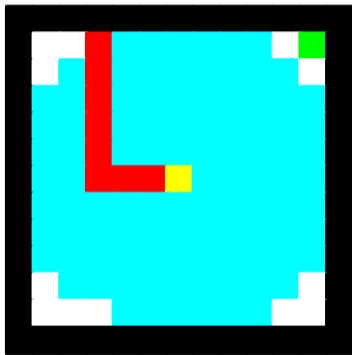
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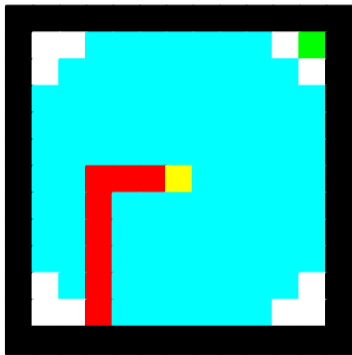
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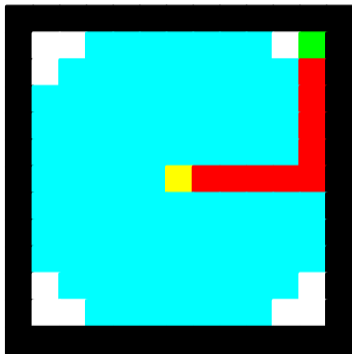
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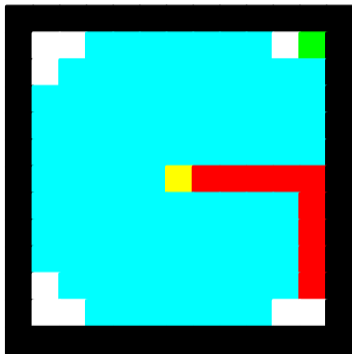
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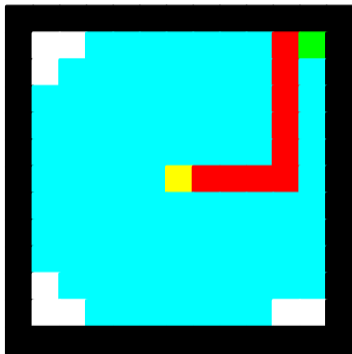
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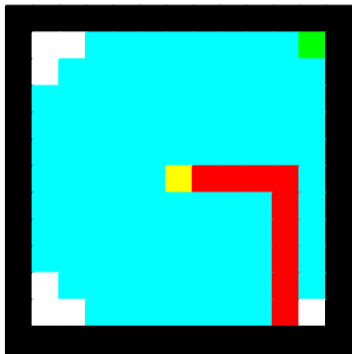
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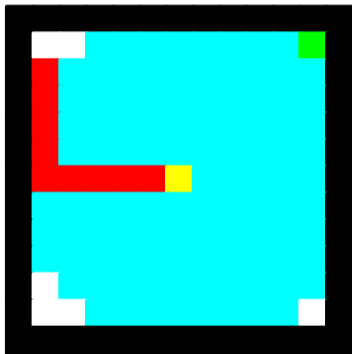
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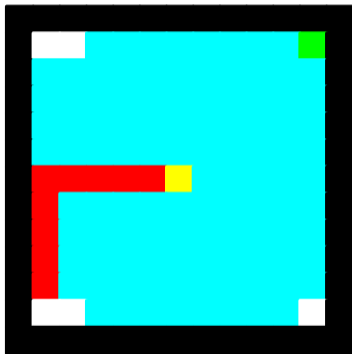
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# Grid: BFS 1

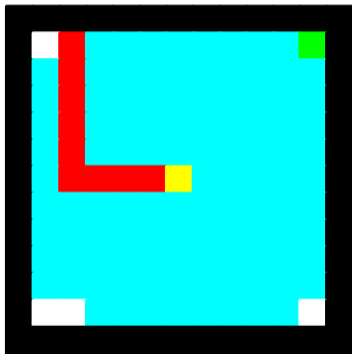
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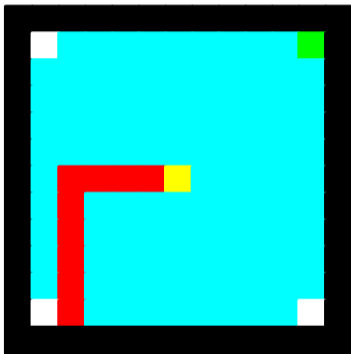
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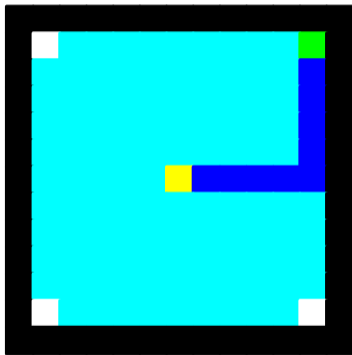
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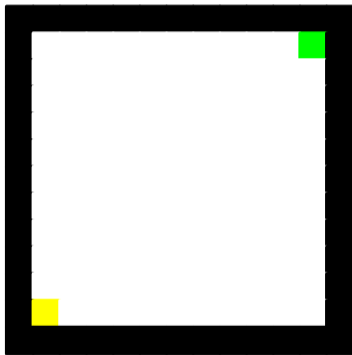
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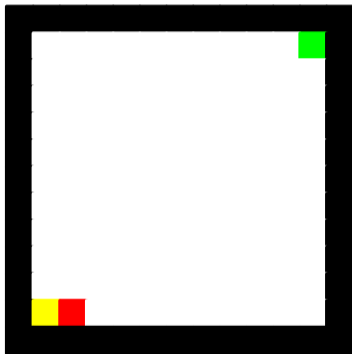
## Grid: BFS 2

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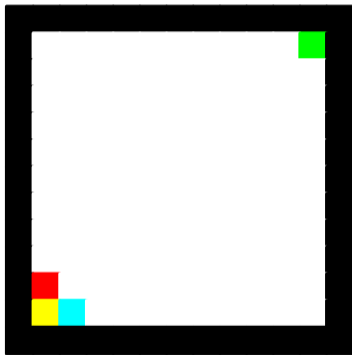
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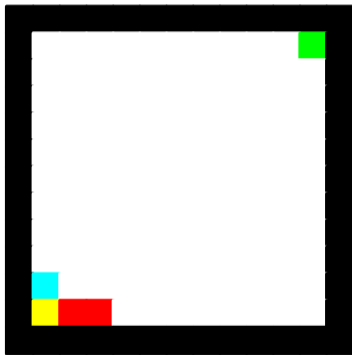
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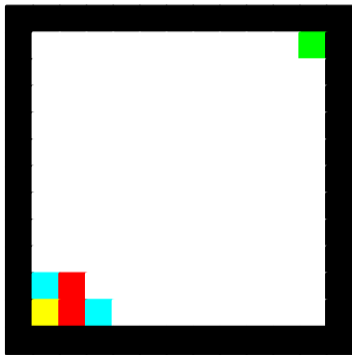
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## Grid: BFS 2

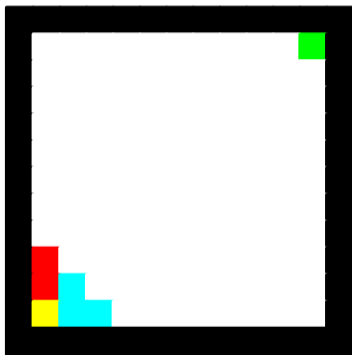
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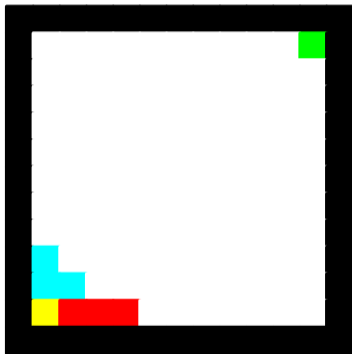
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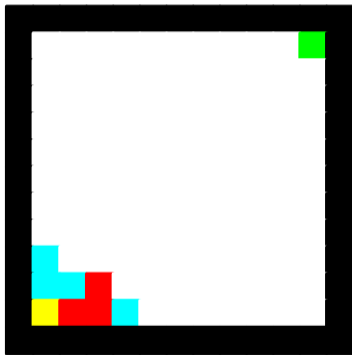
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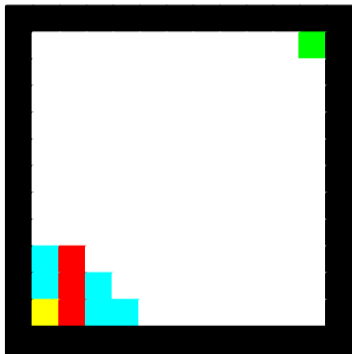
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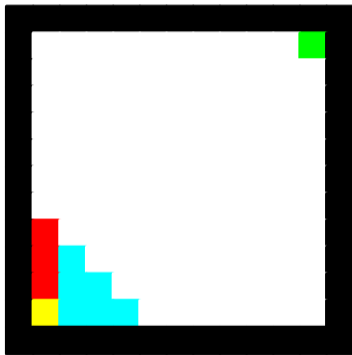
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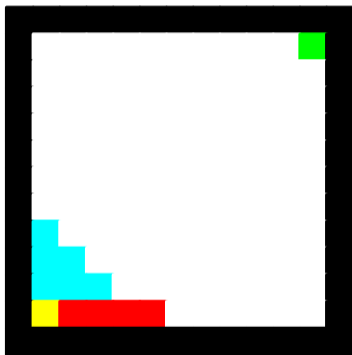
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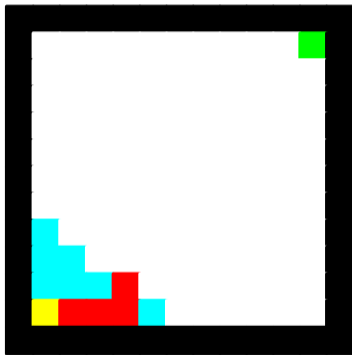
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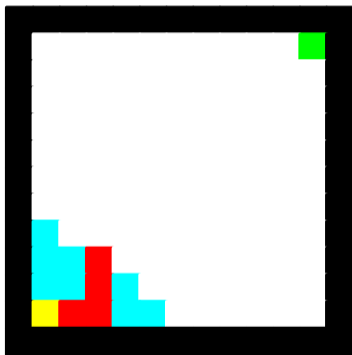
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## Grid: BFS 2

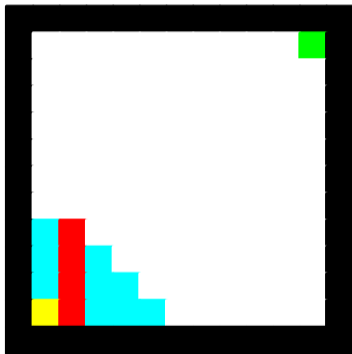
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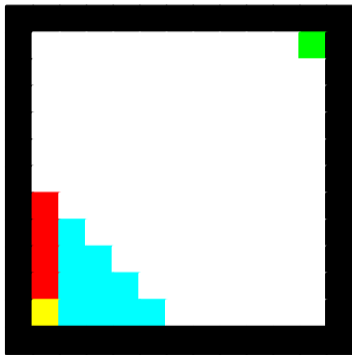
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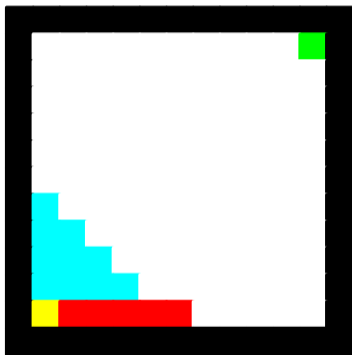
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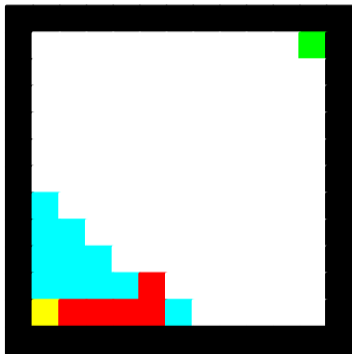
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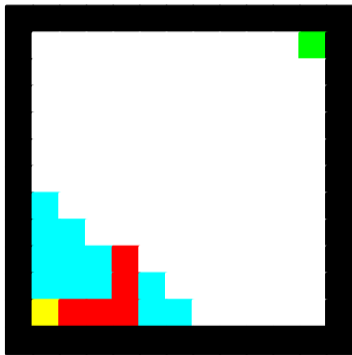
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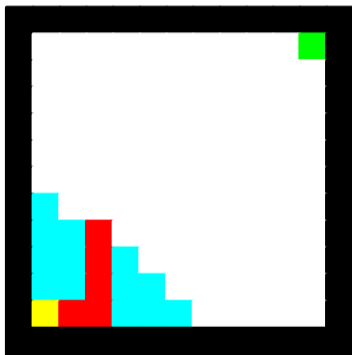
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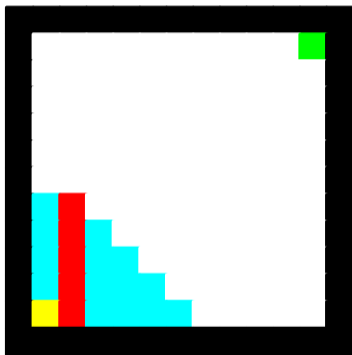
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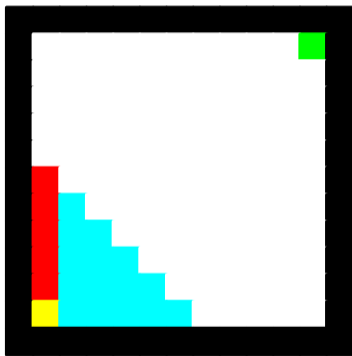
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## Grid: BFS 2

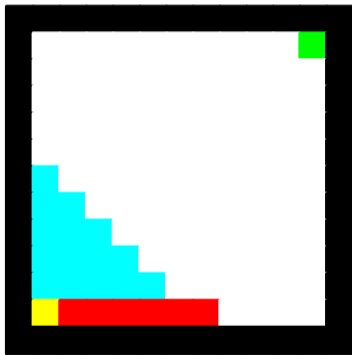
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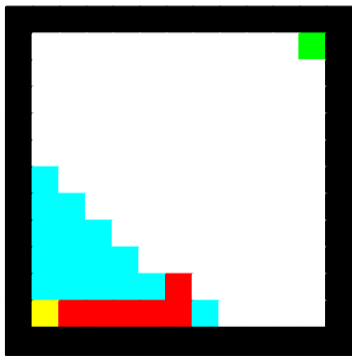
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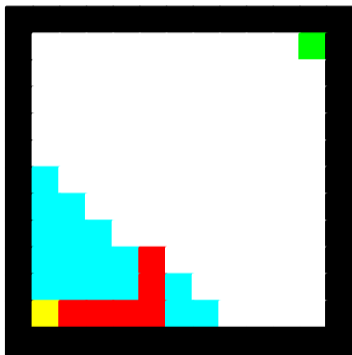
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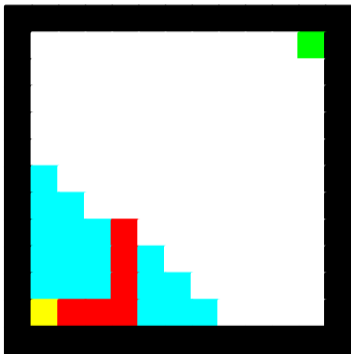
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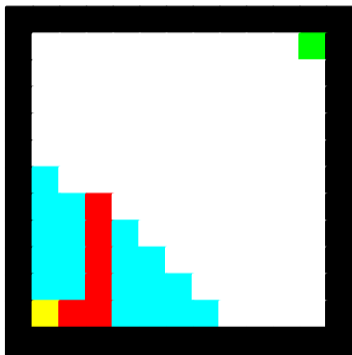
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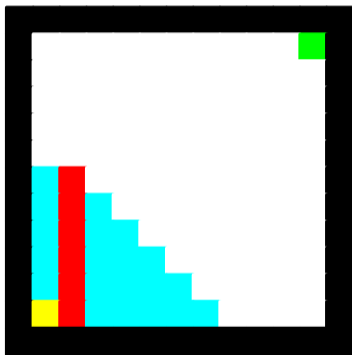
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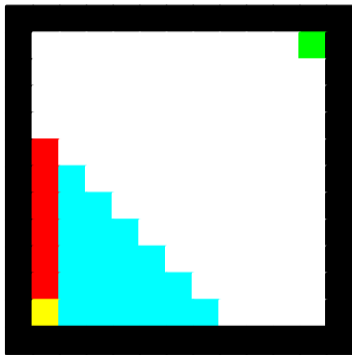
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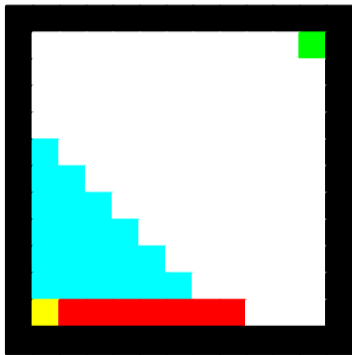
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## Grid: BFS 2

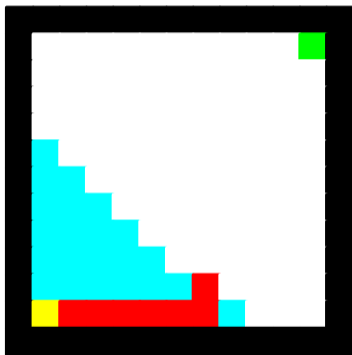
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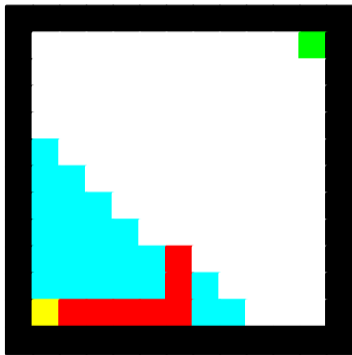
## Grid: BFS 2

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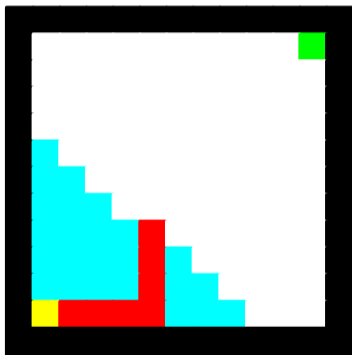
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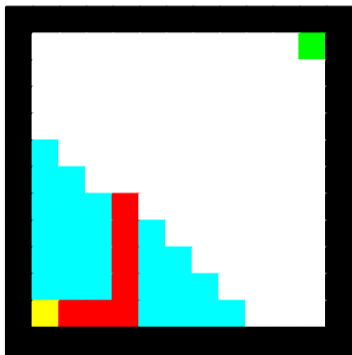
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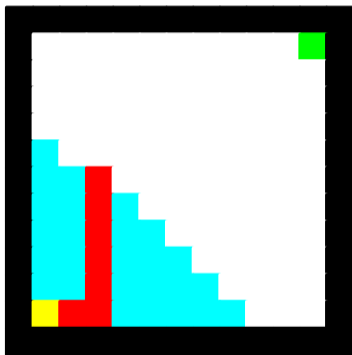
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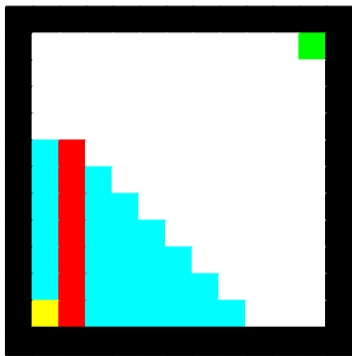
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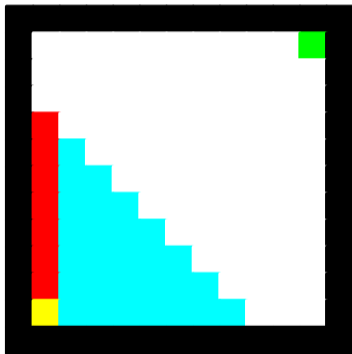
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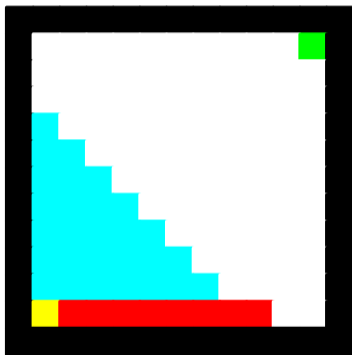
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## Grid: BFS 2

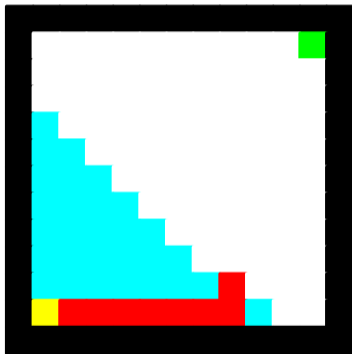
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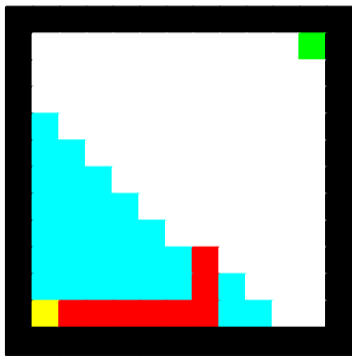
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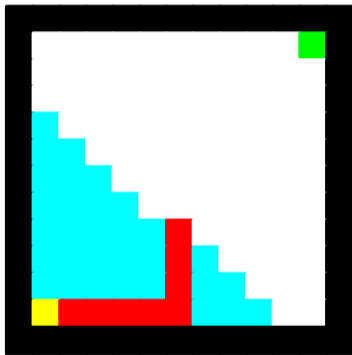
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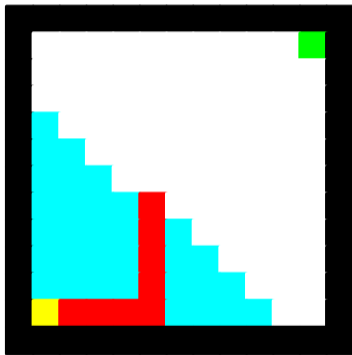
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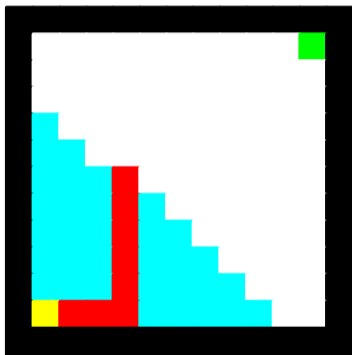
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# Grid: BFS 2

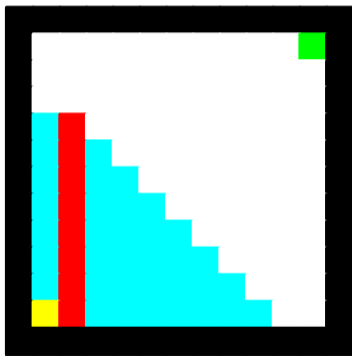
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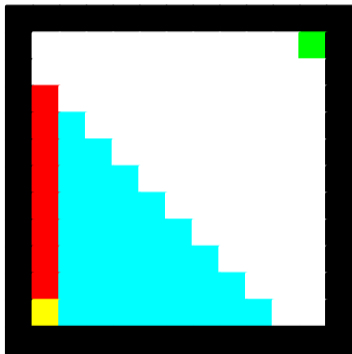
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## Grid: BFS 2

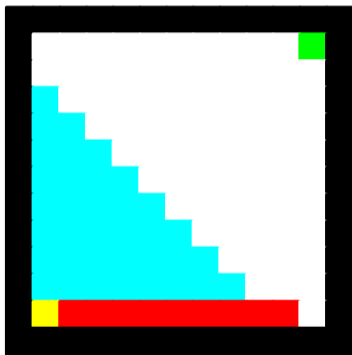
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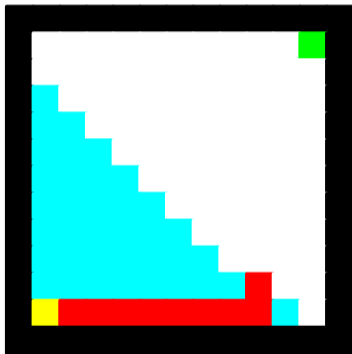
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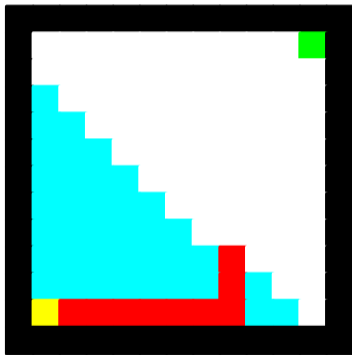
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## Grid: BFS 2

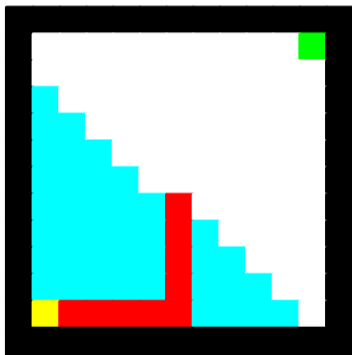
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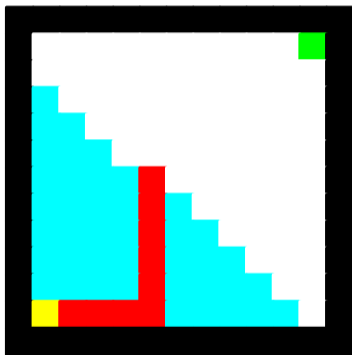
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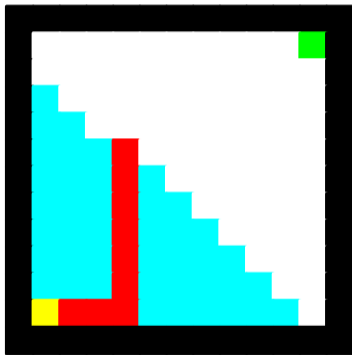
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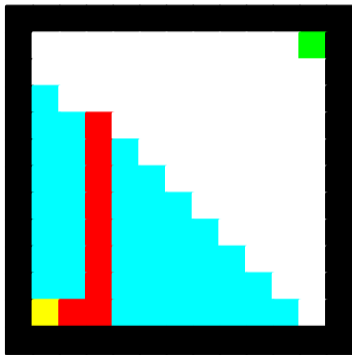
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## Grid: BFS 2

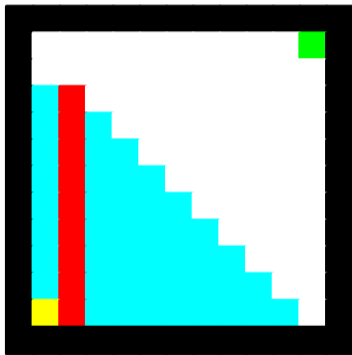
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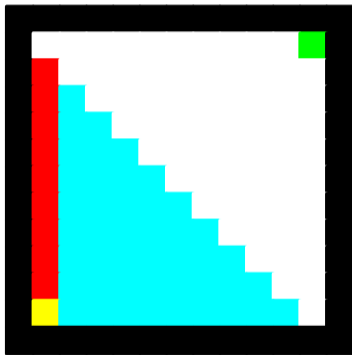
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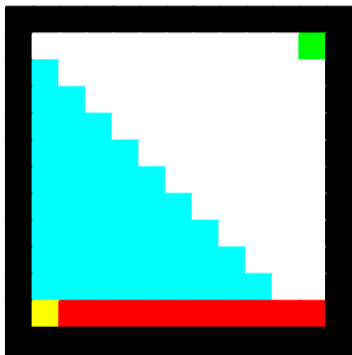
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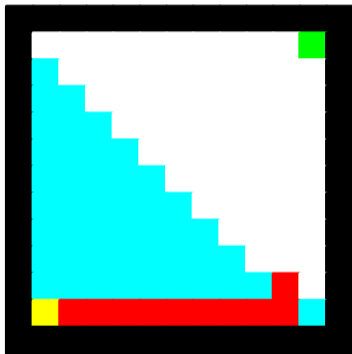
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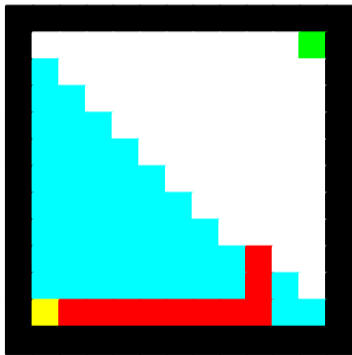
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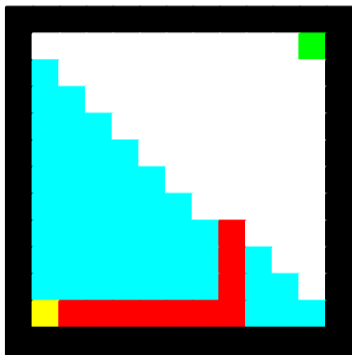
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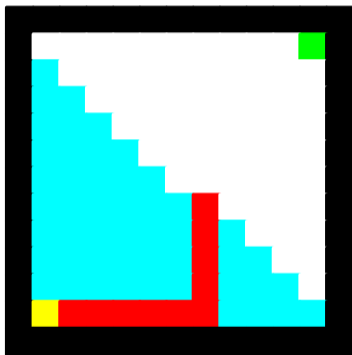
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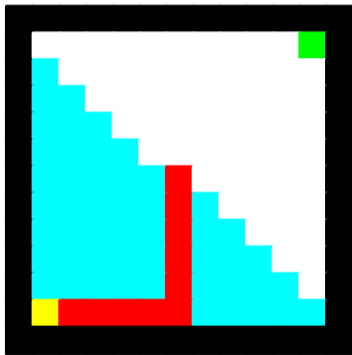
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## Grid: BFS 2

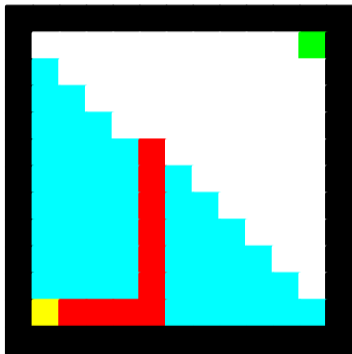
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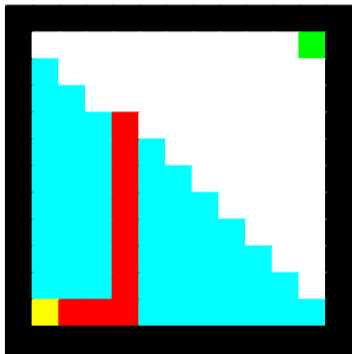
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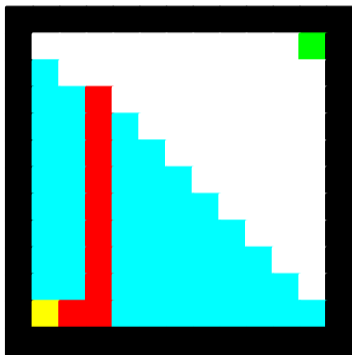
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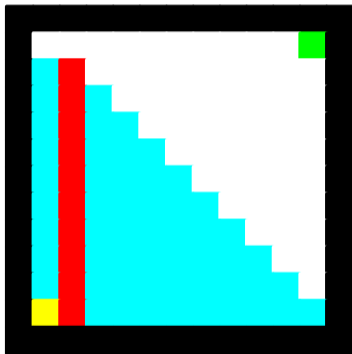
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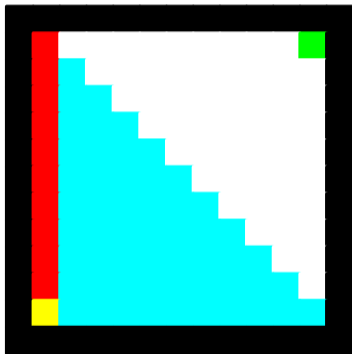
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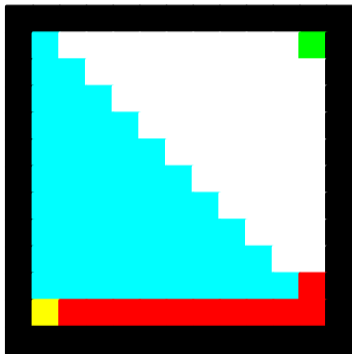
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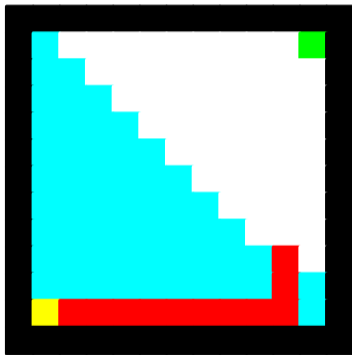
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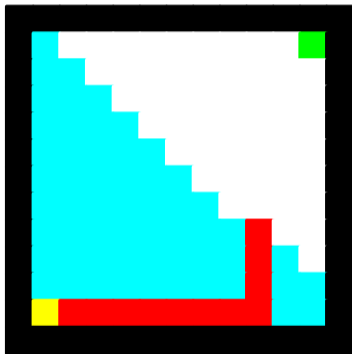
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## Grid: BFS 2

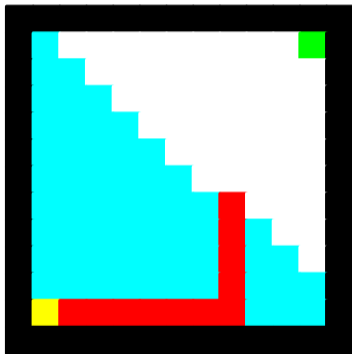
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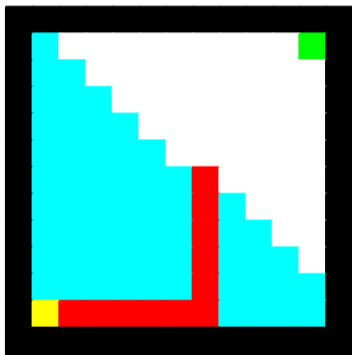
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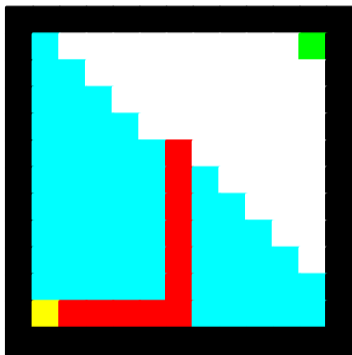
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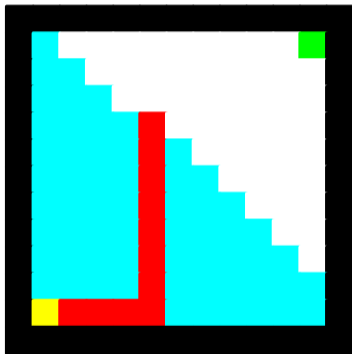
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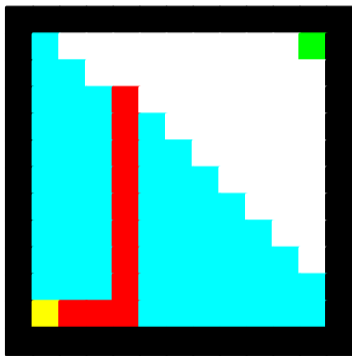
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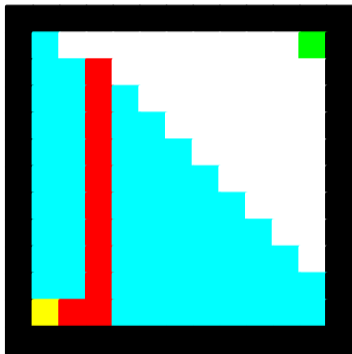
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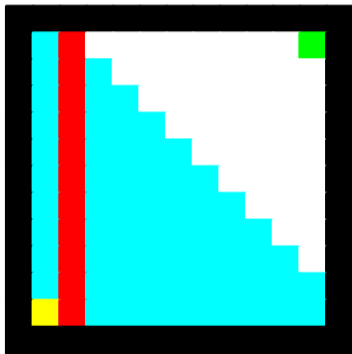
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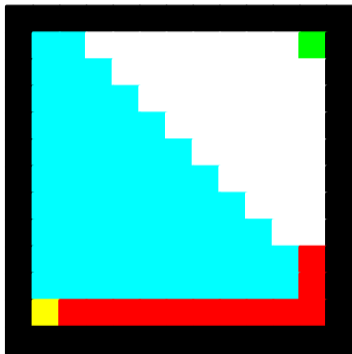
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## Grid: BFS 2

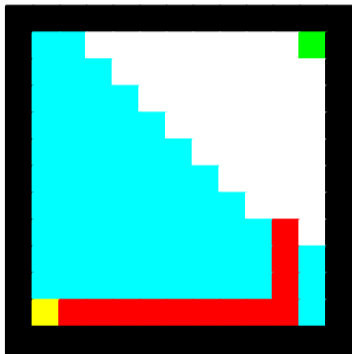
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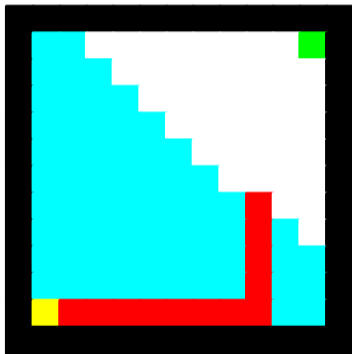
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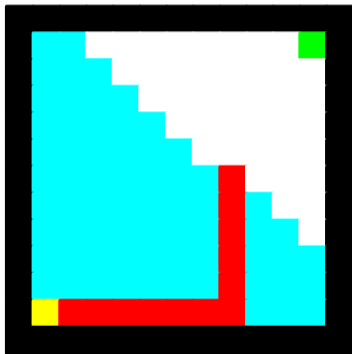
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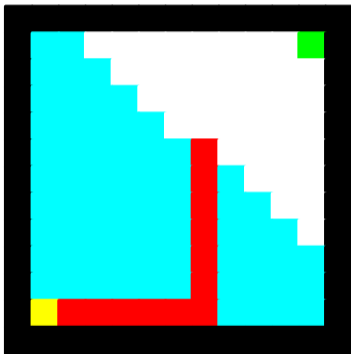
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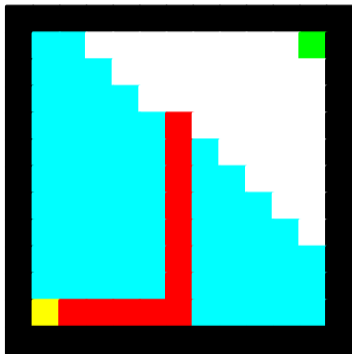
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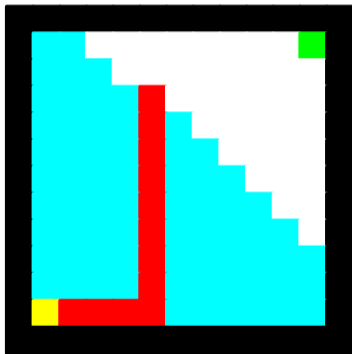
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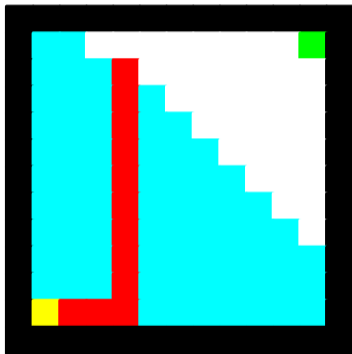
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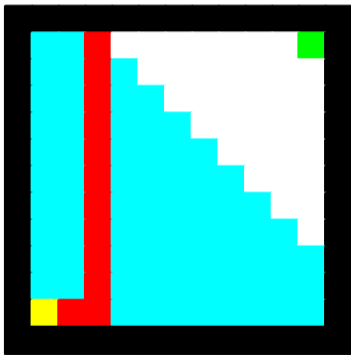
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## Grid: BFS 2

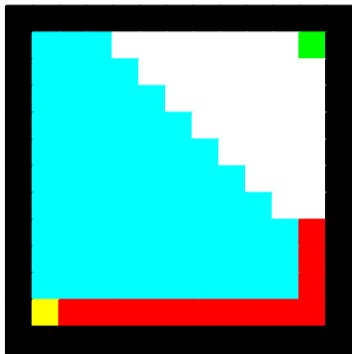
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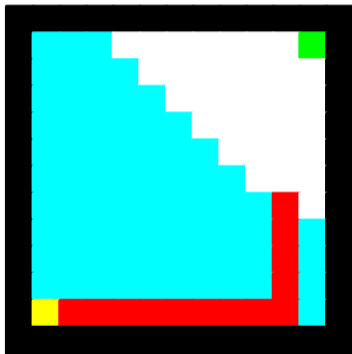
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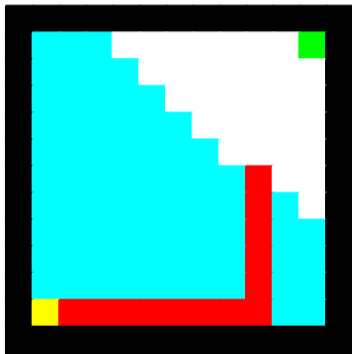
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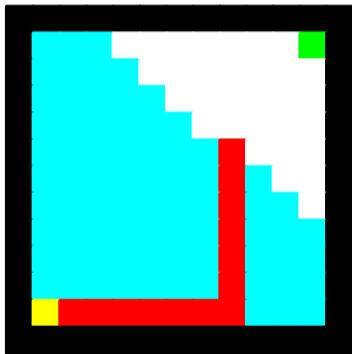
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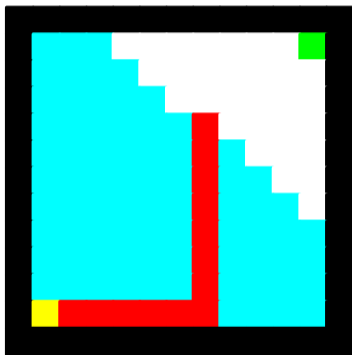
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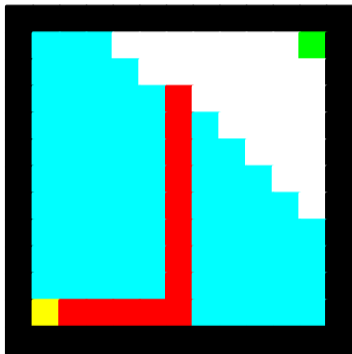
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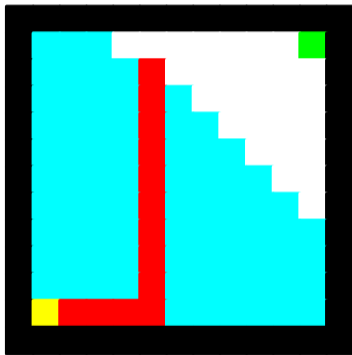
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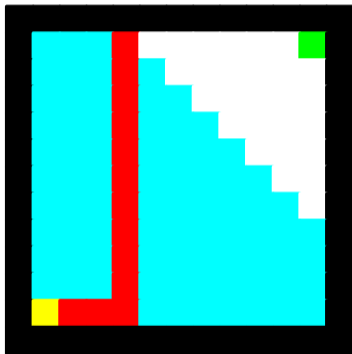
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## Grid: BFS 2

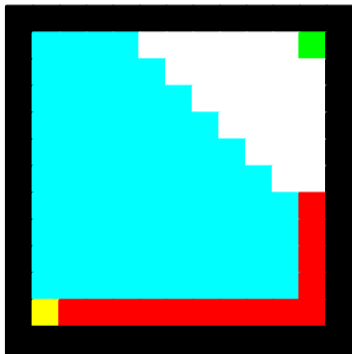
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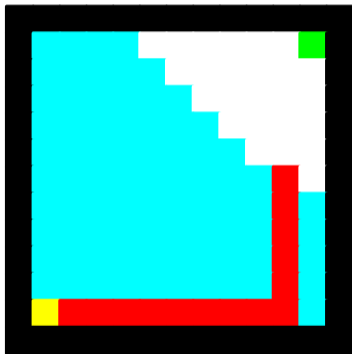
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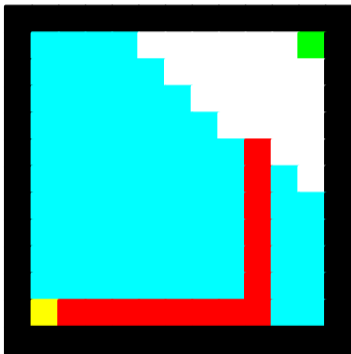
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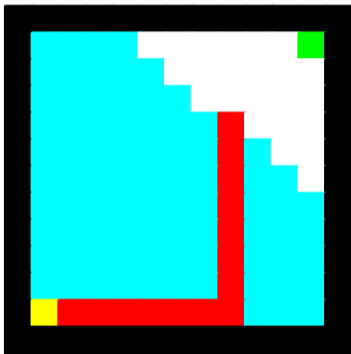
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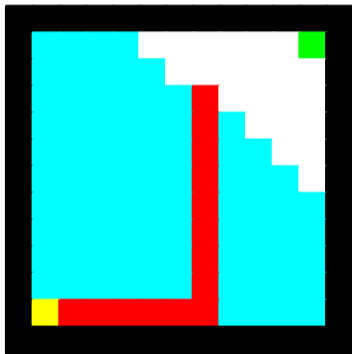
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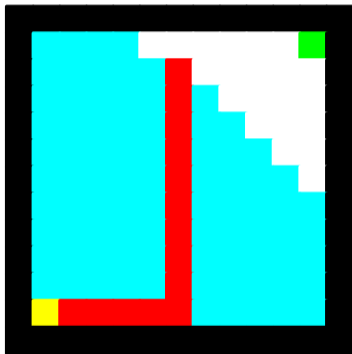
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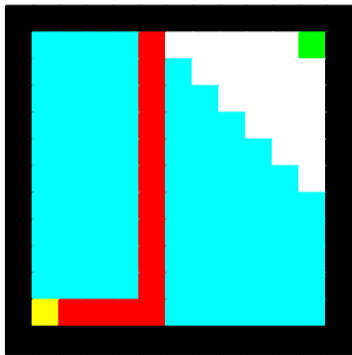
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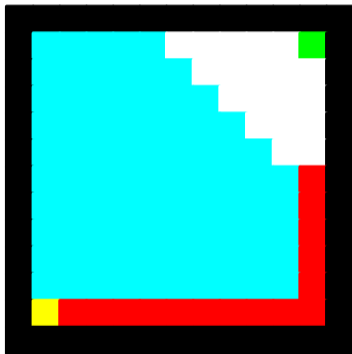
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## Grid: BFS 2

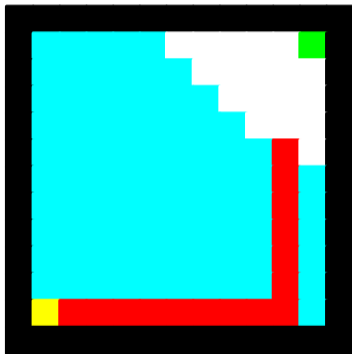
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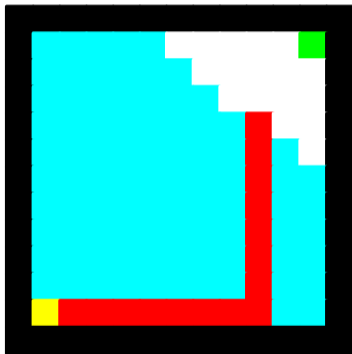
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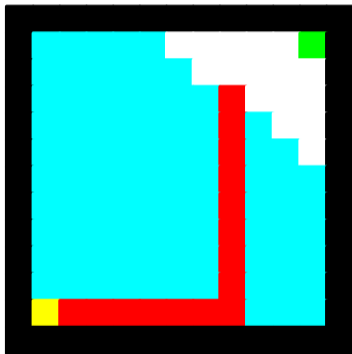
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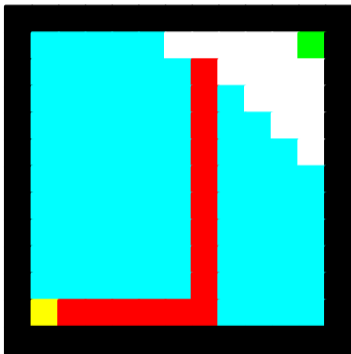
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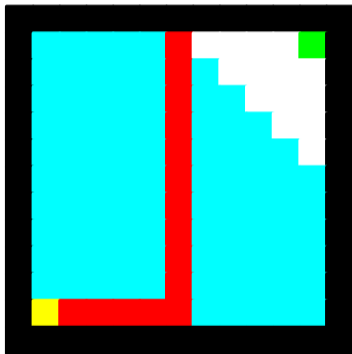
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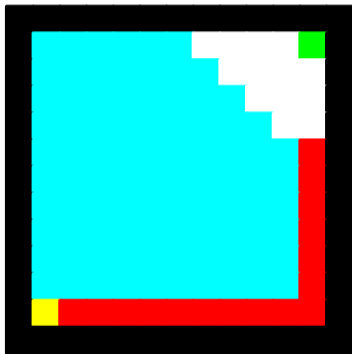
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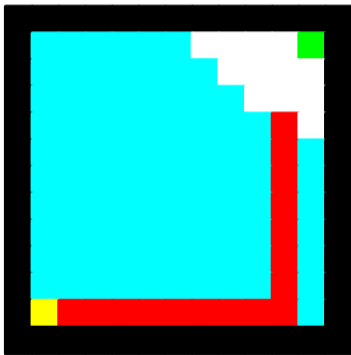
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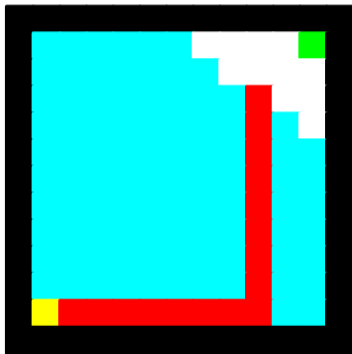
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# Grid: BFS 2

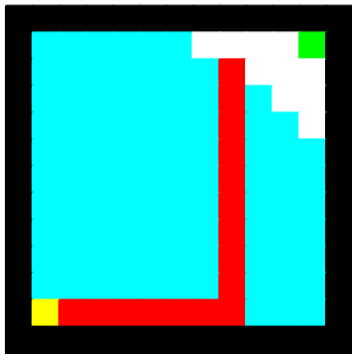
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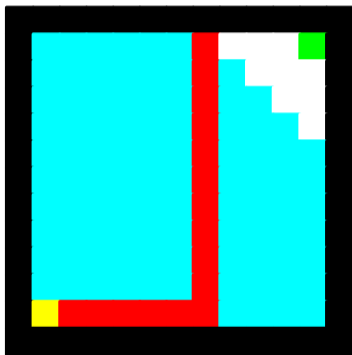
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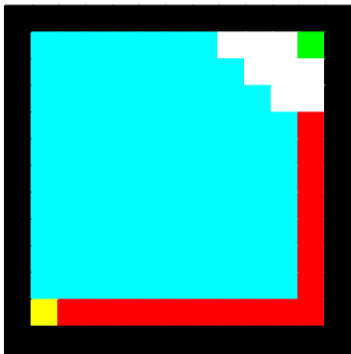
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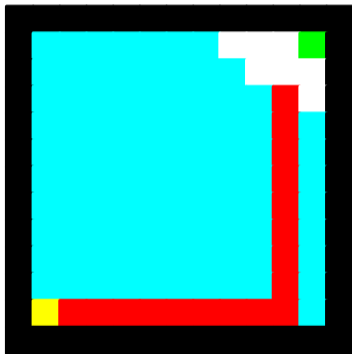
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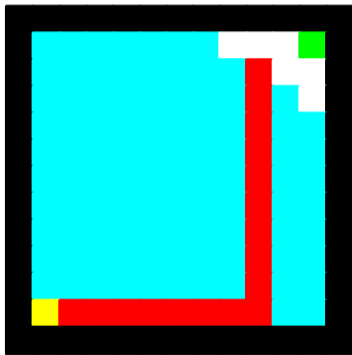
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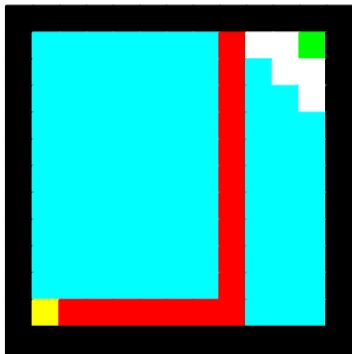
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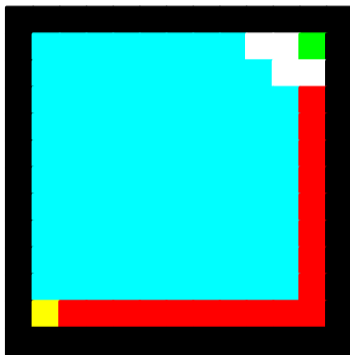
## Grid: BFS 2

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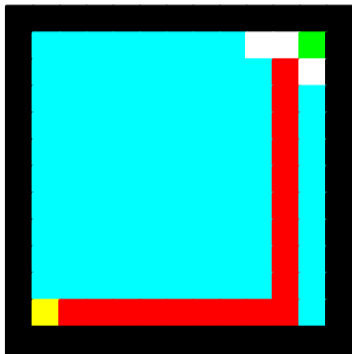
## Grid: BFS 2

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## Grid: BFS 2

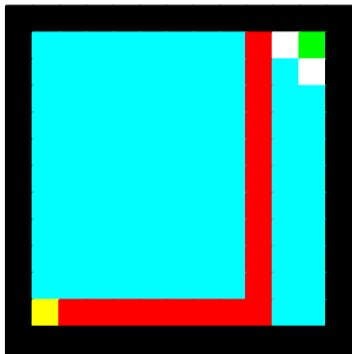
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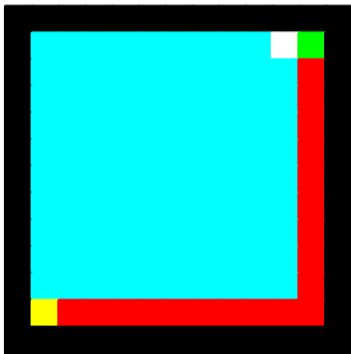
## Grid: BFS 2

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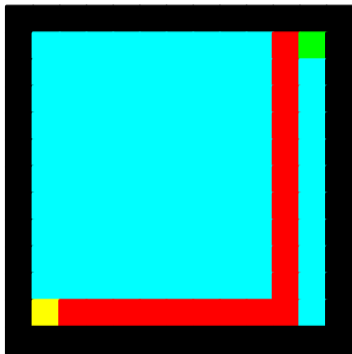
## Grid: BFS 2

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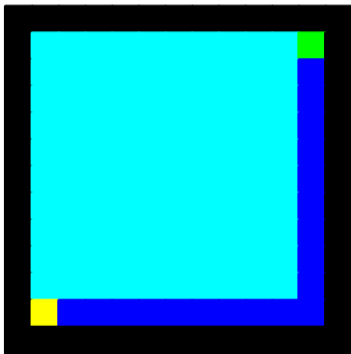
## Grid: BFS 2

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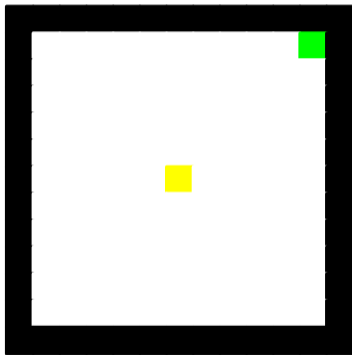
## Grid: BFS 2

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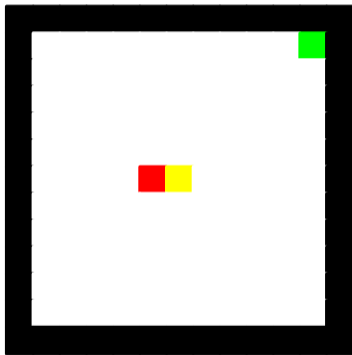
# Grid: DFS 1

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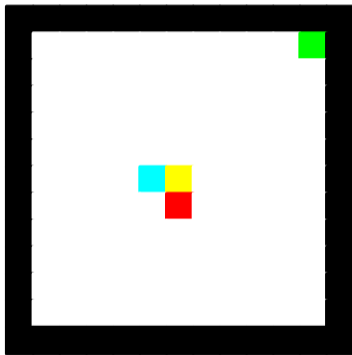
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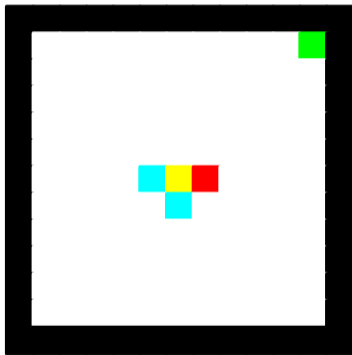
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# Grid: DFS 1

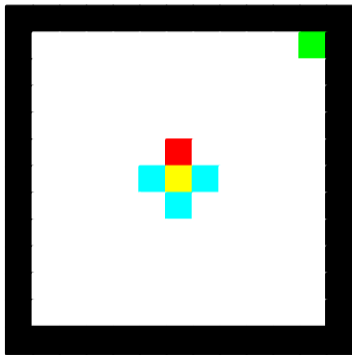
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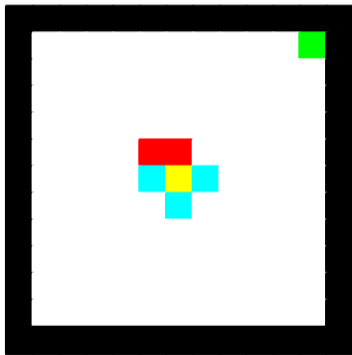
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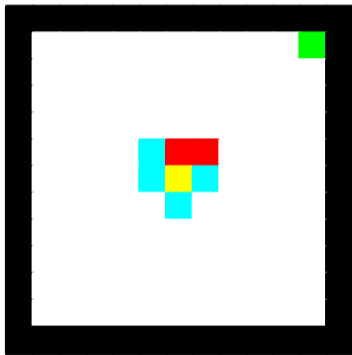
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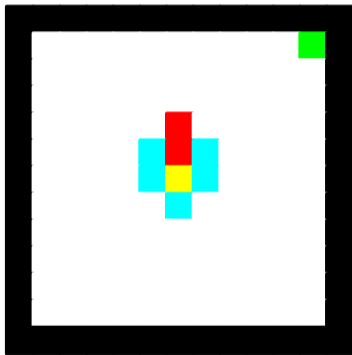
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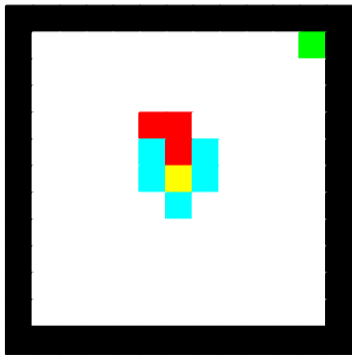
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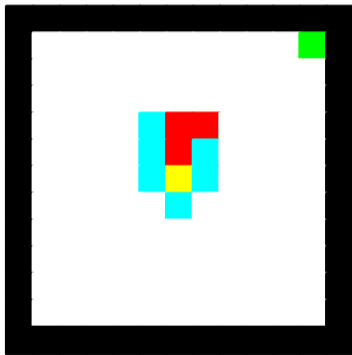
# Grid: DFS 1

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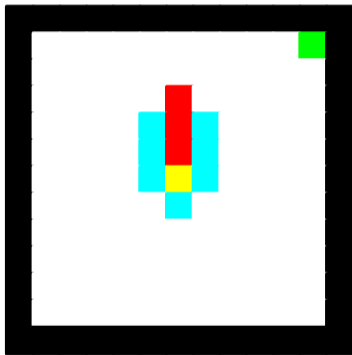
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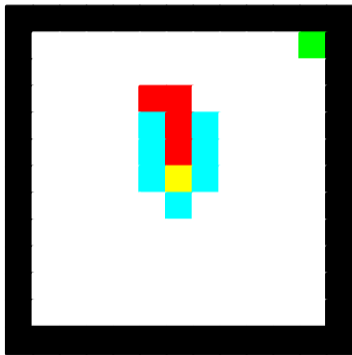
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# Grid: DFS 1

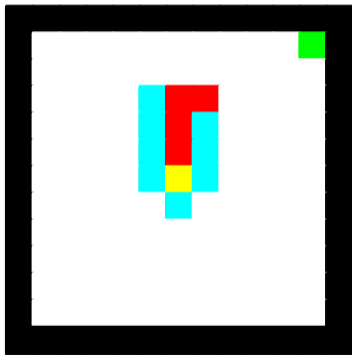
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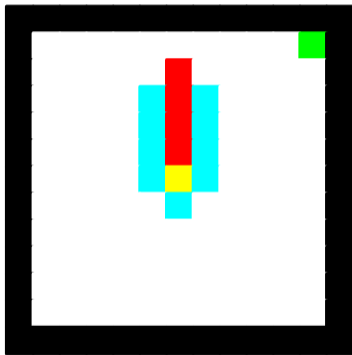
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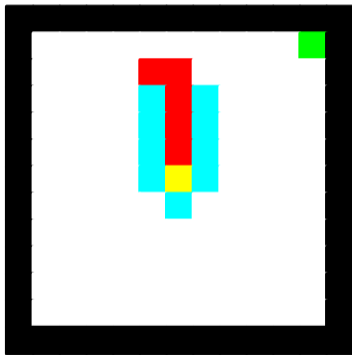
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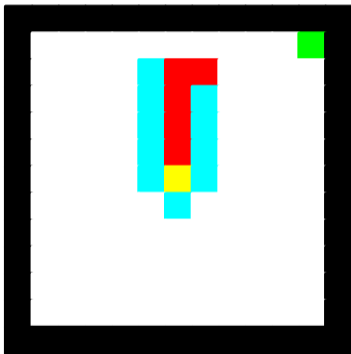
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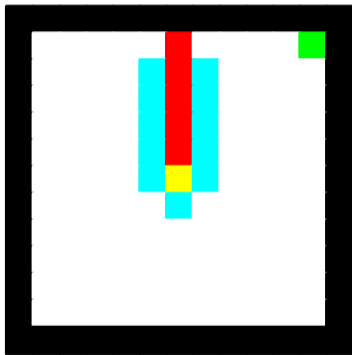
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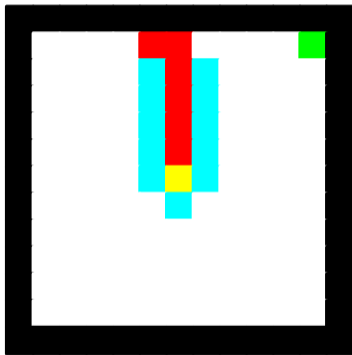
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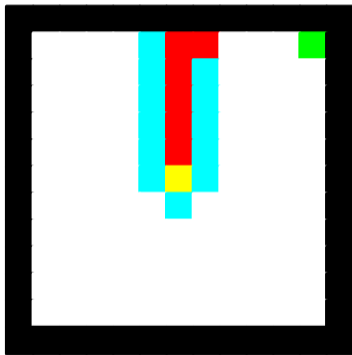
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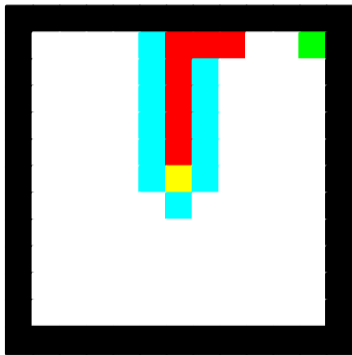
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# Grid: DFS 1

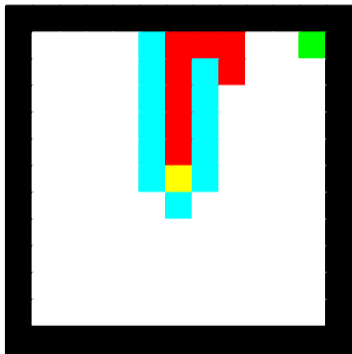
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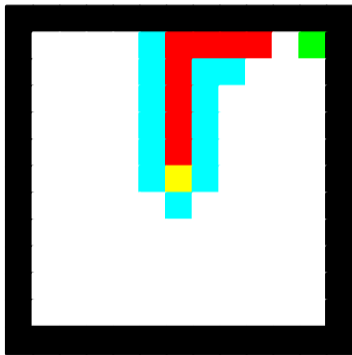
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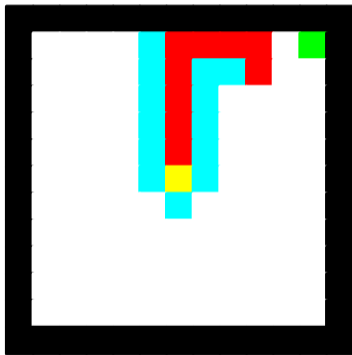
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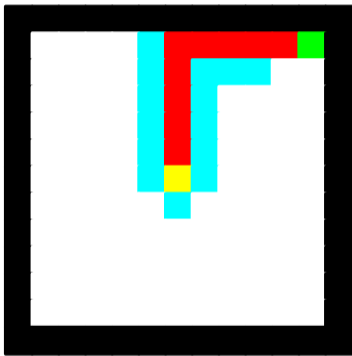
# Grid: DFS 1

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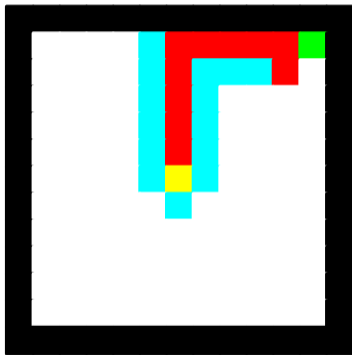
# Grid: DFS 1

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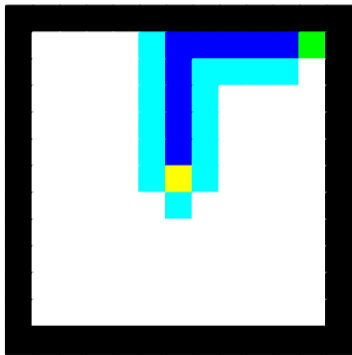
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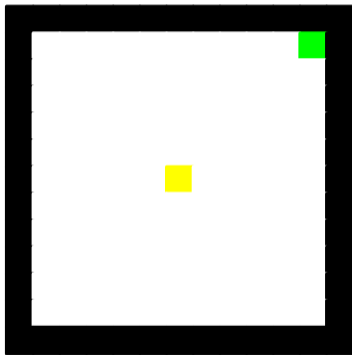
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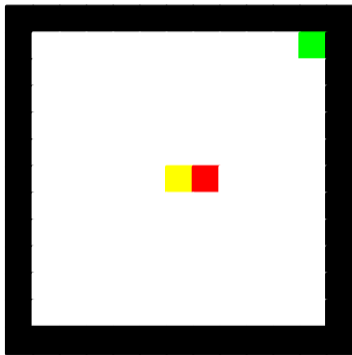
## Grid: DFS 2

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## Grid: DFS 2

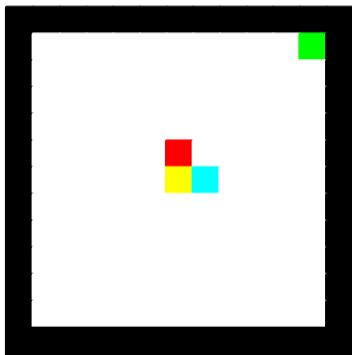
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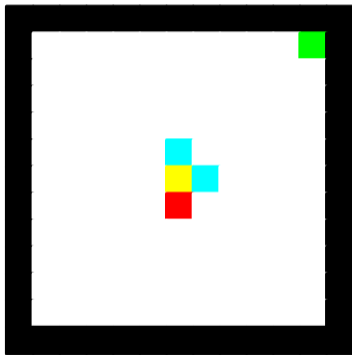
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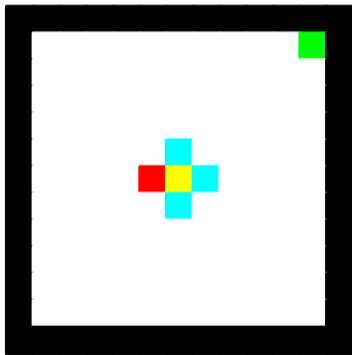
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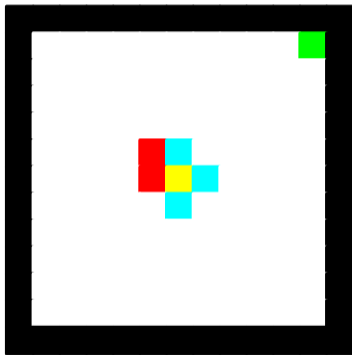
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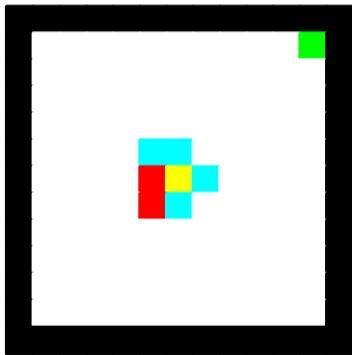
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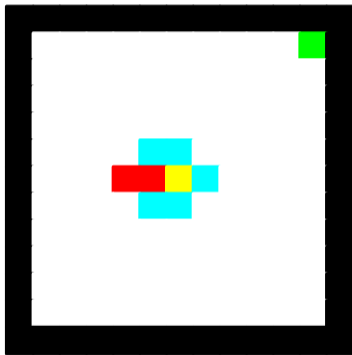
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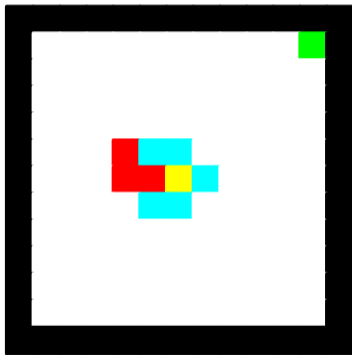
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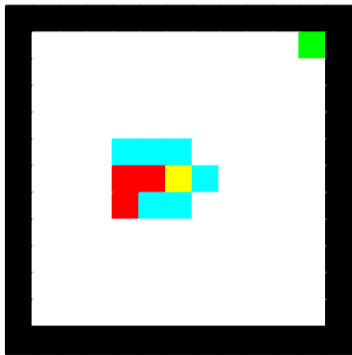
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## Grid: DFS 2

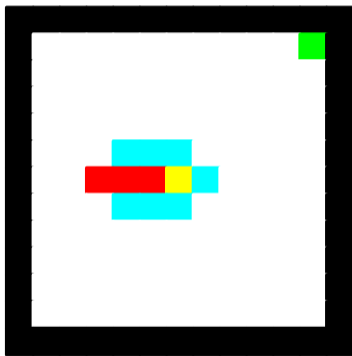
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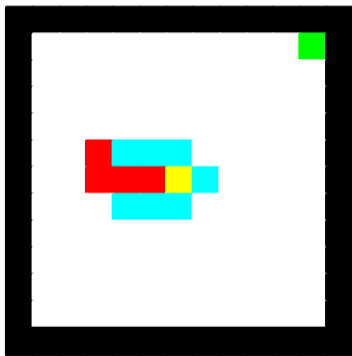
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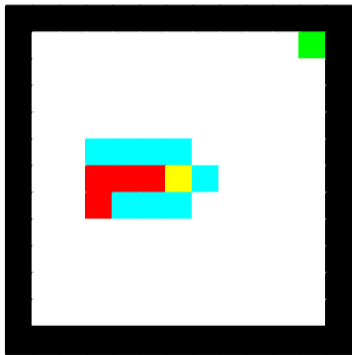
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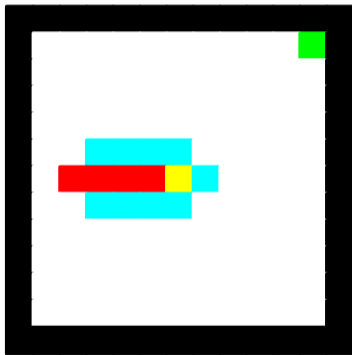
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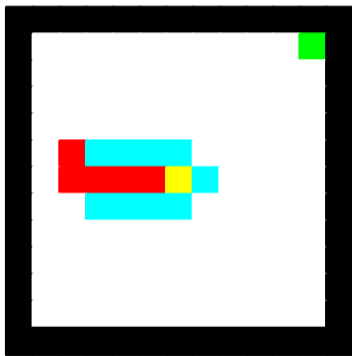
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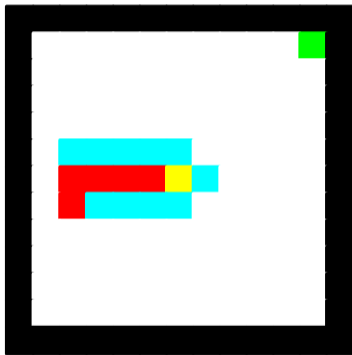
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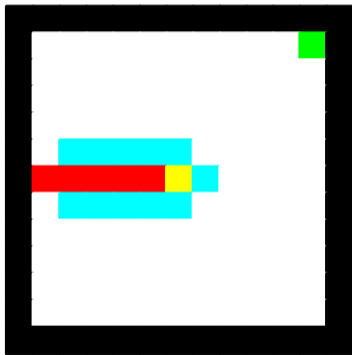
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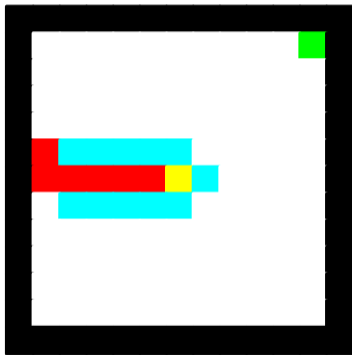
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## Grid: DFS 2

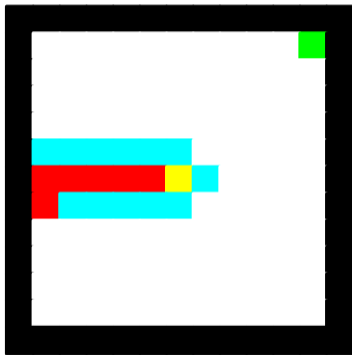
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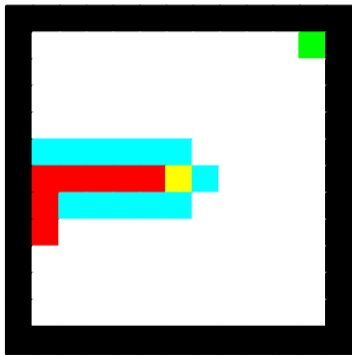
## Grid: DFS 2

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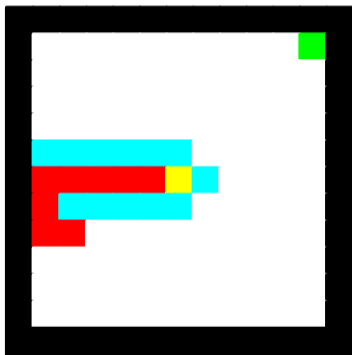
## Grid: DFS 2

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## Grid: DFS 2

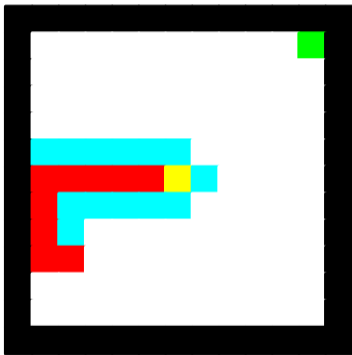
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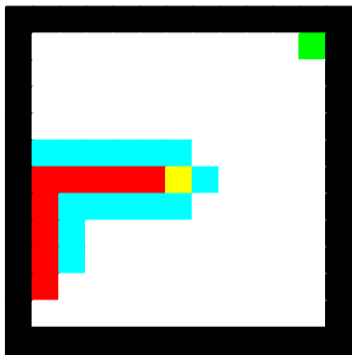
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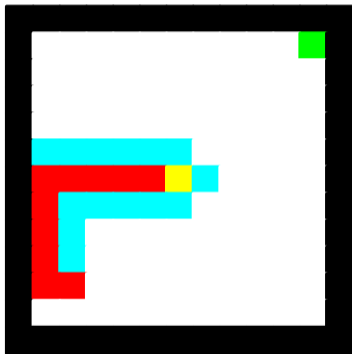
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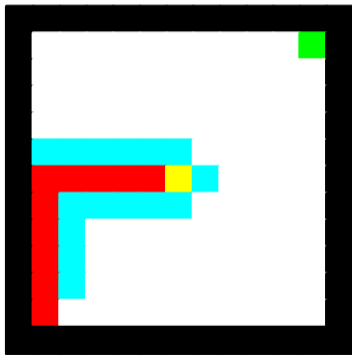
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## Grid: DFS 2

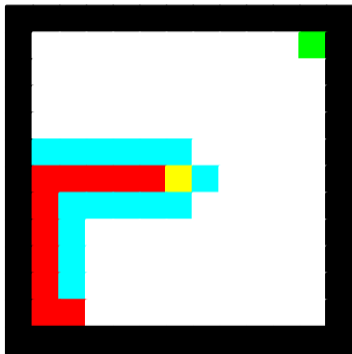
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## Grid: DFS 2

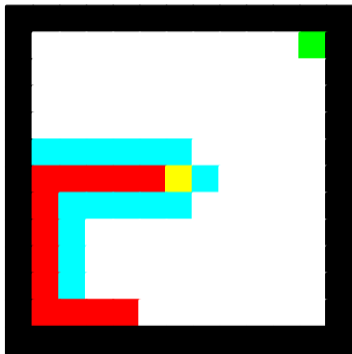
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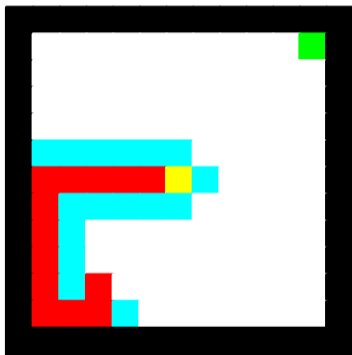
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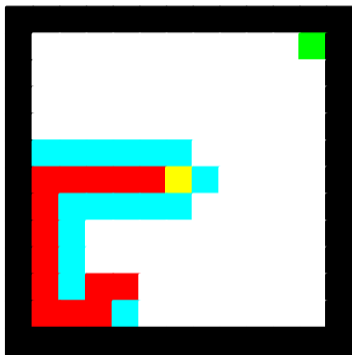
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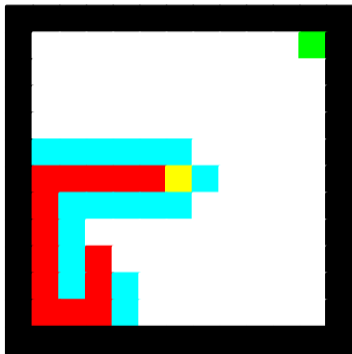
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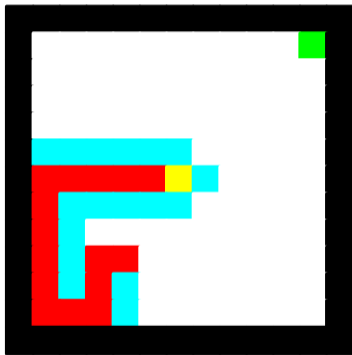
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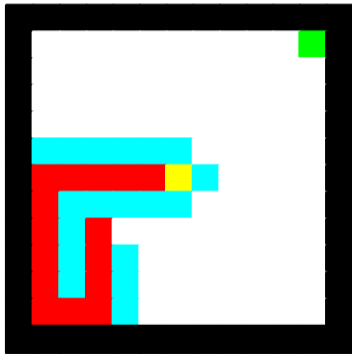
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## Grid: DFS 2

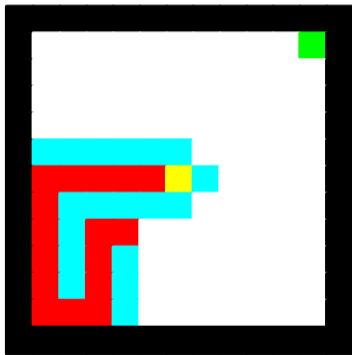
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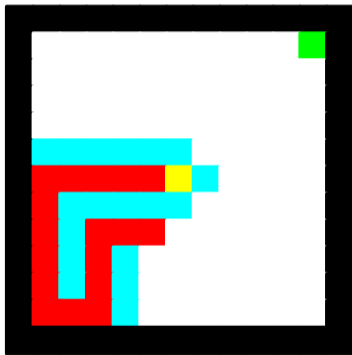
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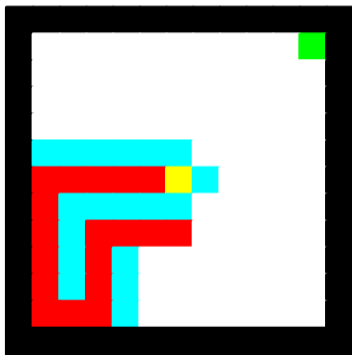
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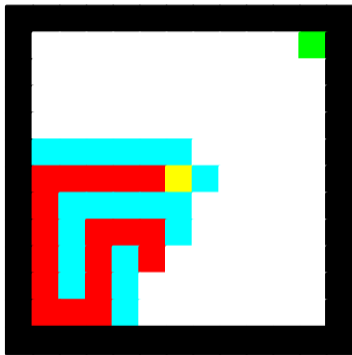
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## Grid: DFS 2

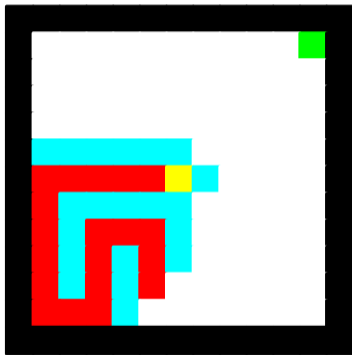
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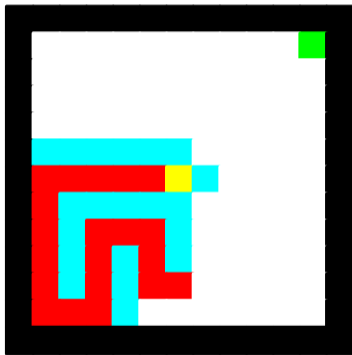
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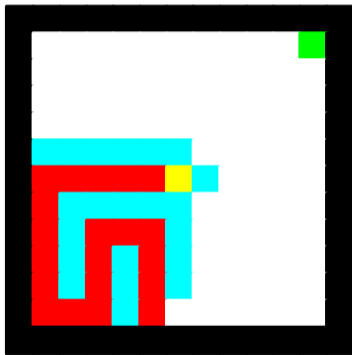
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## Grid: DFS 2

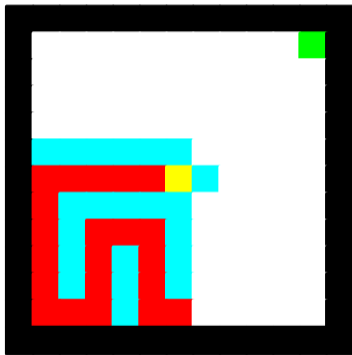
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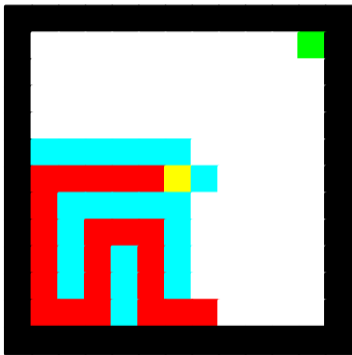
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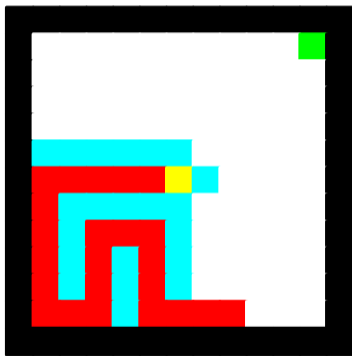
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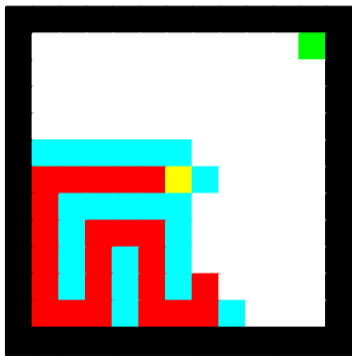
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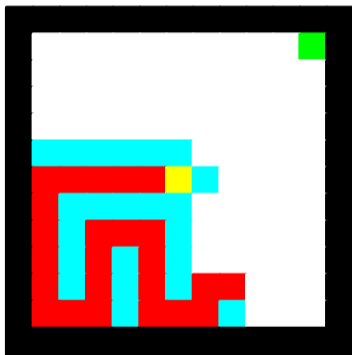
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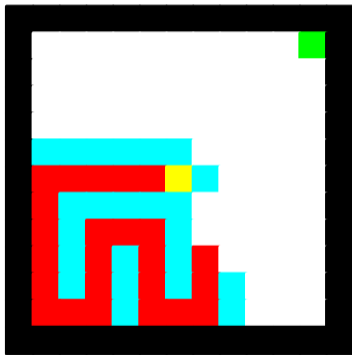
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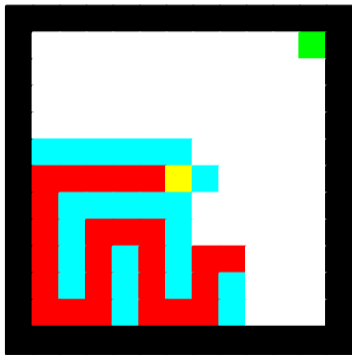
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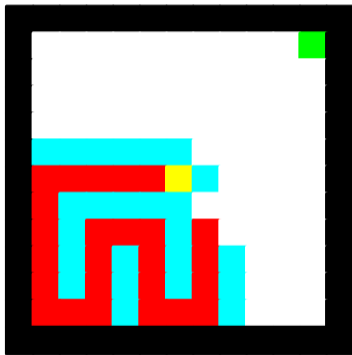
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## Grid: DFS 2

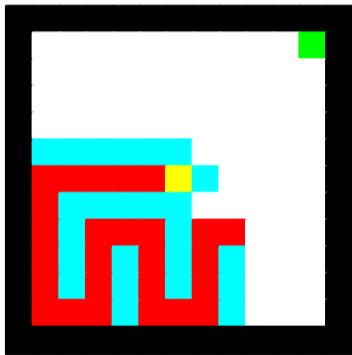
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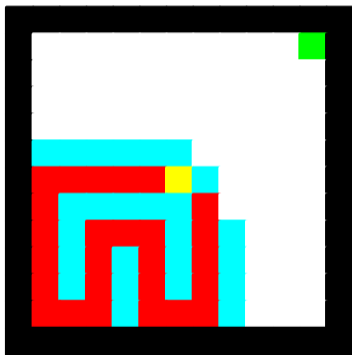
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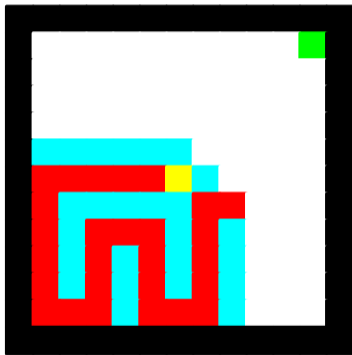
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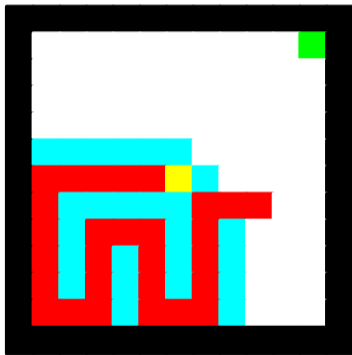
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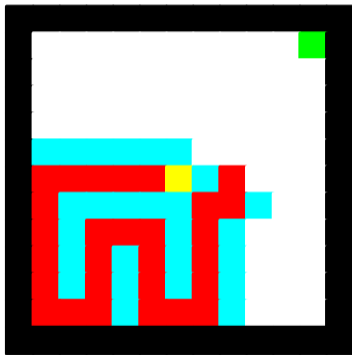
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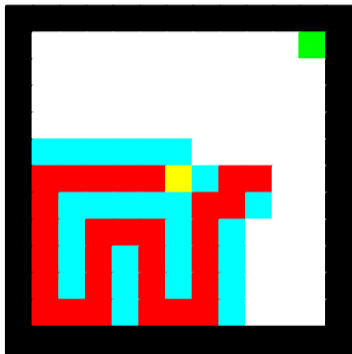
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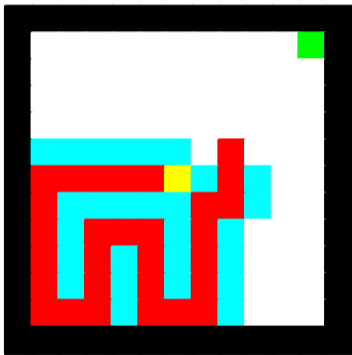
## Grid: DFS 2

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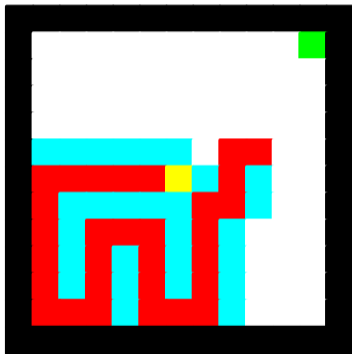
## Grid: DFS 2

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## Grid: DFS 2

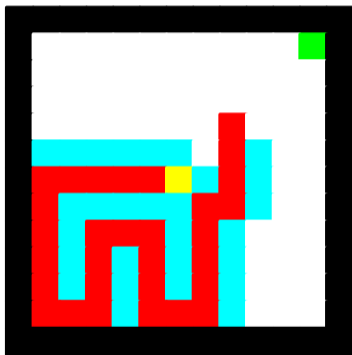
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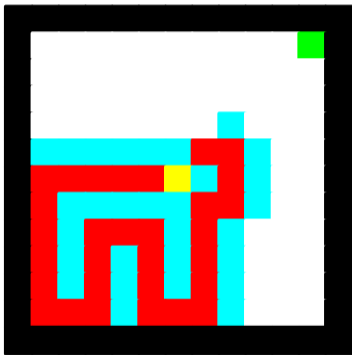
## Grid: DFS 2

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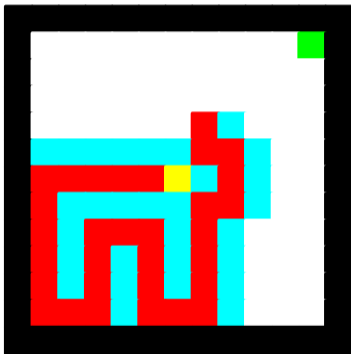
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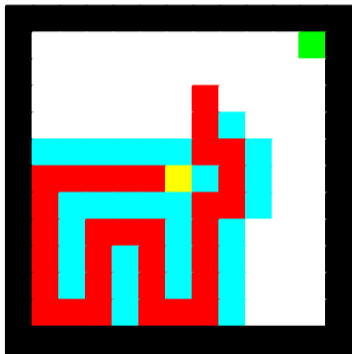
# Grid: DFS 2

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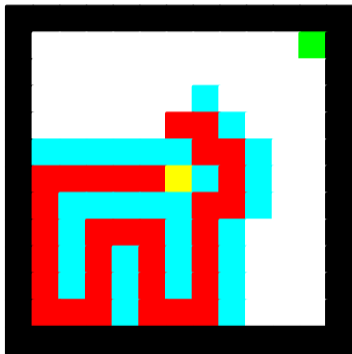
## Grid: DFS 2

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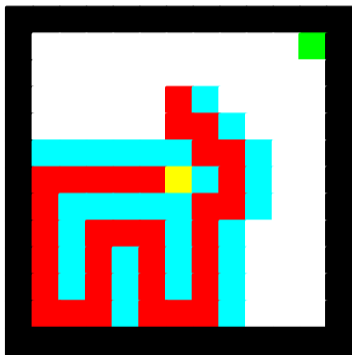
## Grid: DFS 2

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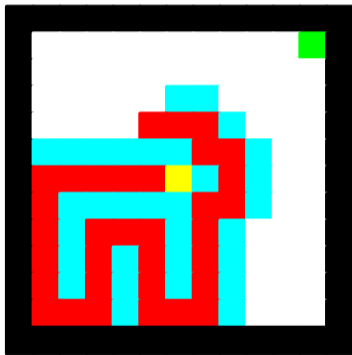
## Grid: DFS 2

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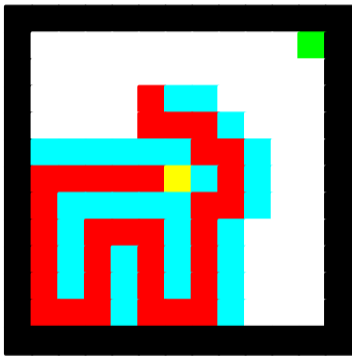
## Grid: DFS 2

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# Grid: DFS 2

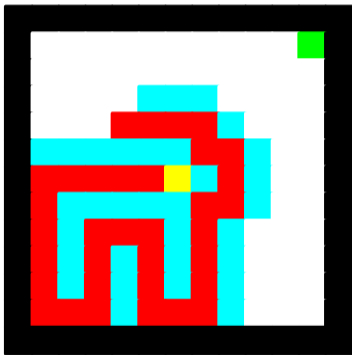
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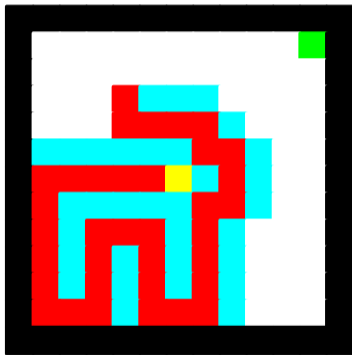
## Grid: DFS 2

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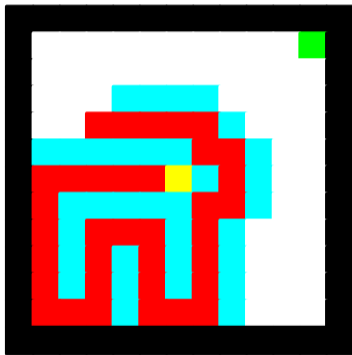
## Grid: DFS 2

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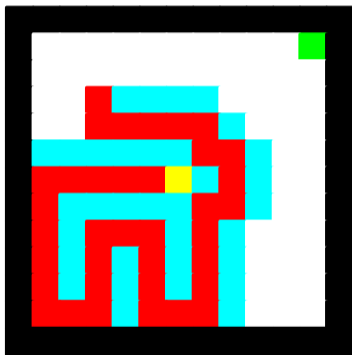
## Grid: DFS 2

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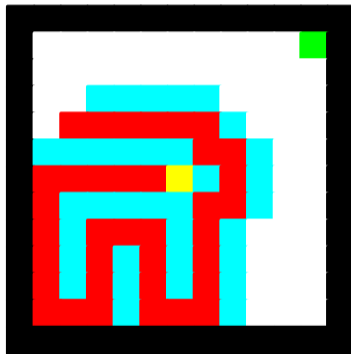
## Grid: DFS 2

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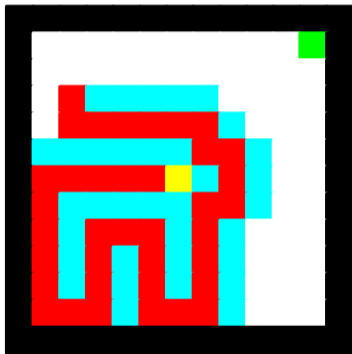
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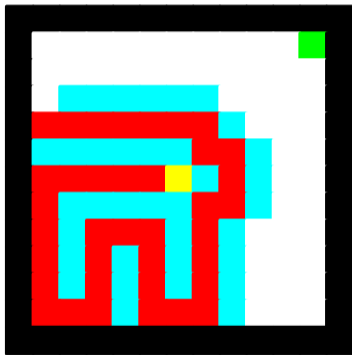
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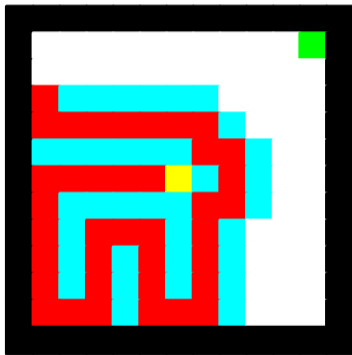
## Grid: DFS 2

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## Grid: DFS 2

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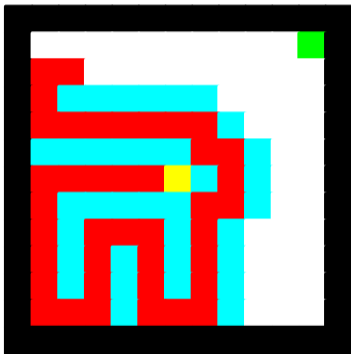






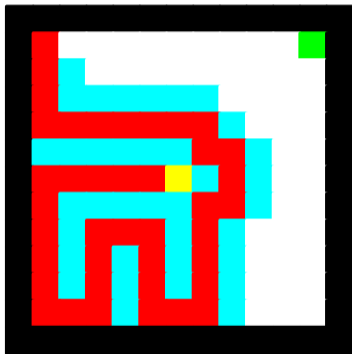
## Grid: DFS 2

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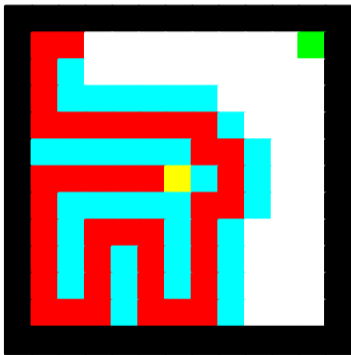
## Grid: DFS 2

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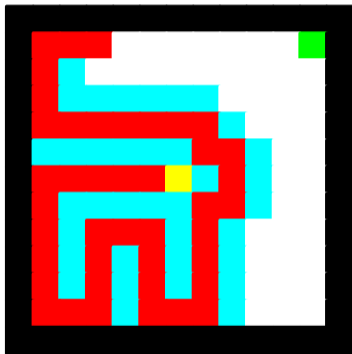
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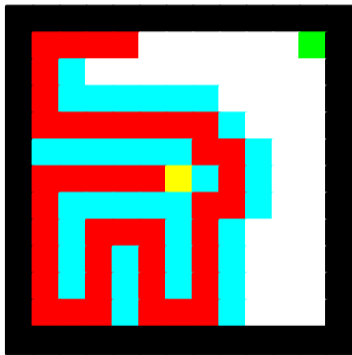
## Grid: DFS 2

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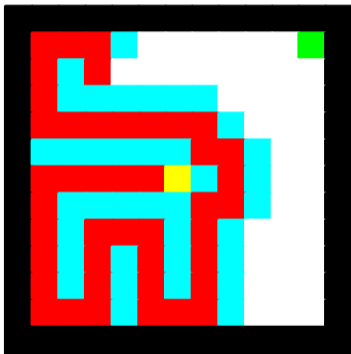
## Grid: DFS 2

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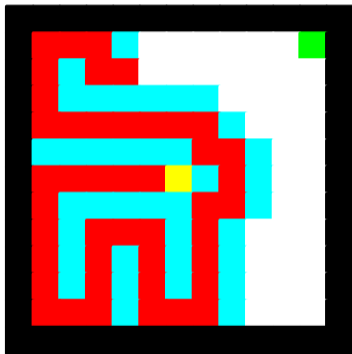
## Grid: DFS 2

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## Grid: DFS 2

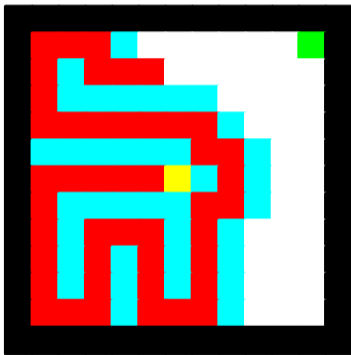
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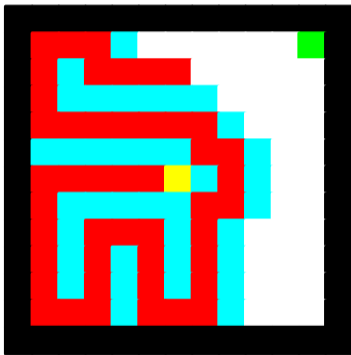
## Grid: DFS 2

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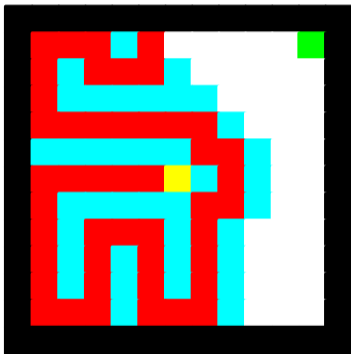
## Grid: DFS 2

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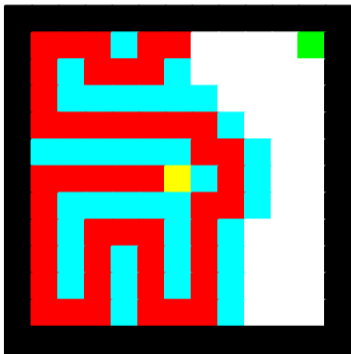
## Grid: DFS 2

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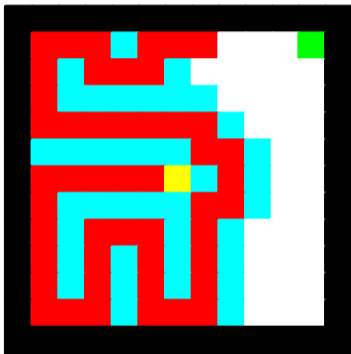
## Grid: DFS 2

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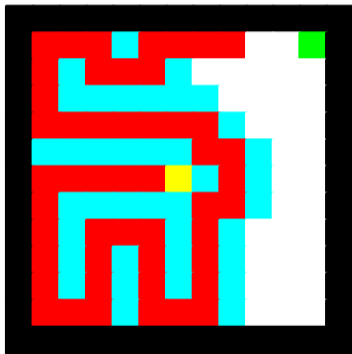
## Grid: DFS 2

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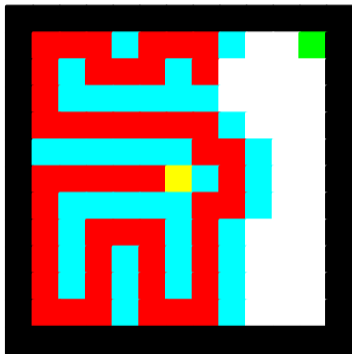
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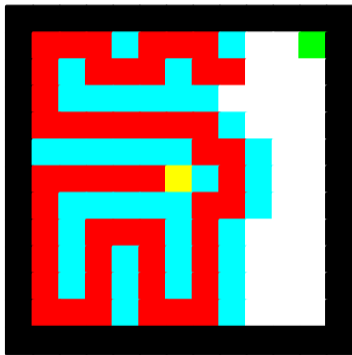
## Grid: DFS 2

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## Grid: DFS 2

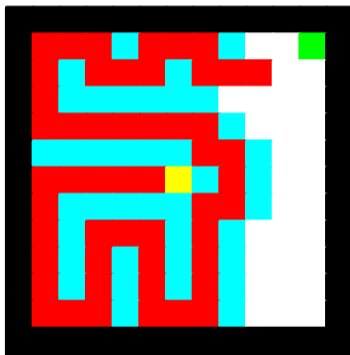
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## Grid: DFS 2

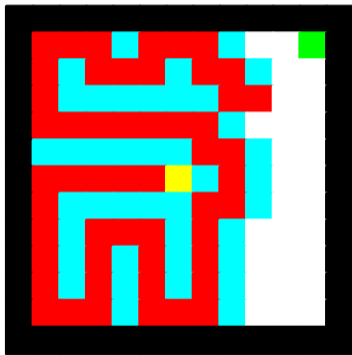
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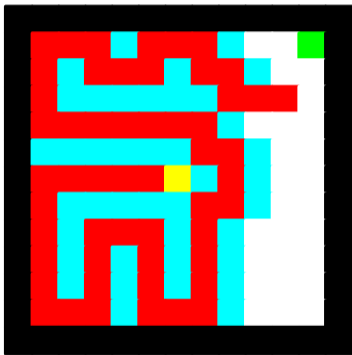
## Grid: DFS 2

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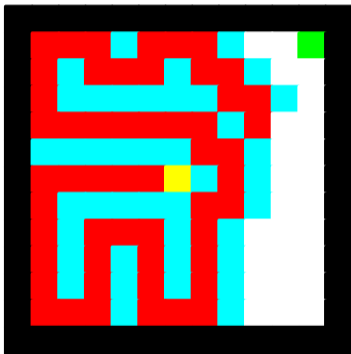
## Grid: DFS 2

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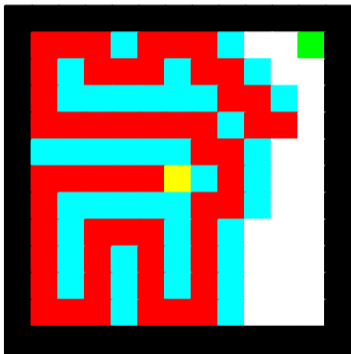
## Grid: DFS 2

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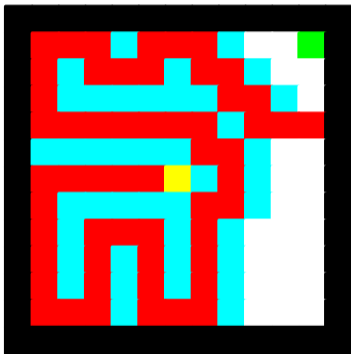
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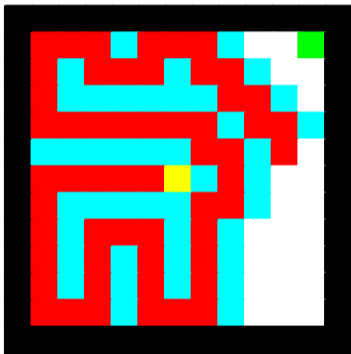
## Grid: DFS 2

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## Grid: DFS 2

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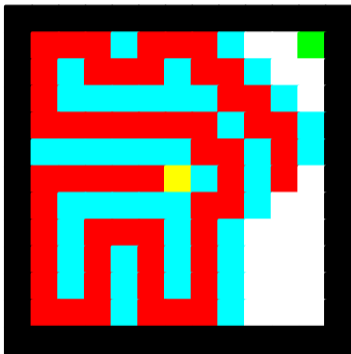






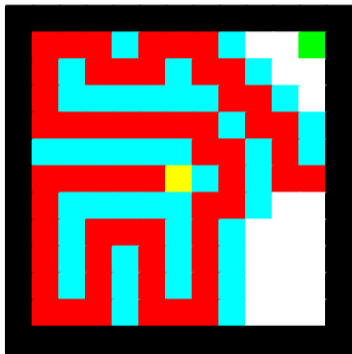
## Grid: DFS 2

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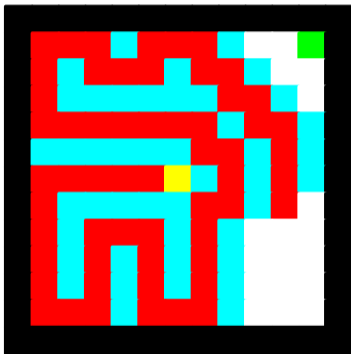
## Grid: DFS 2

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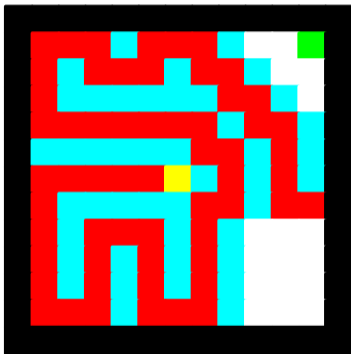
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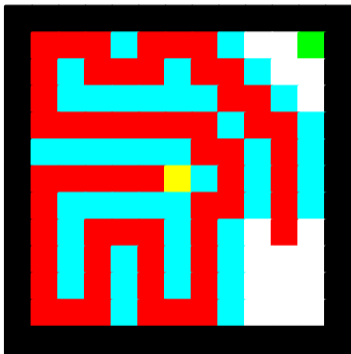
## Grid: DFS 2

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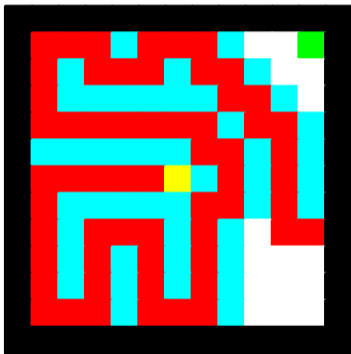
## Grid: DFS 2

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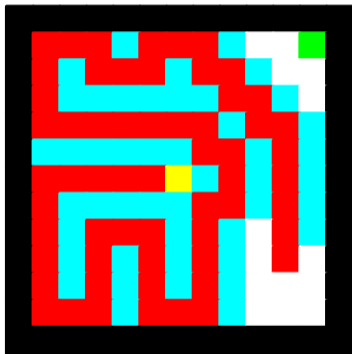
## Grid: DFS 2

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## Grid: DFS 2

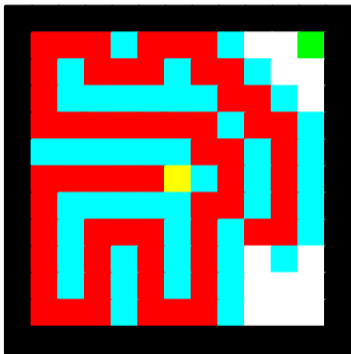
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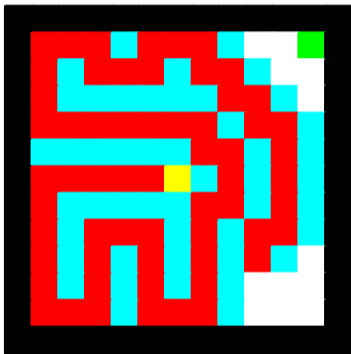
## Grid: DFS 2

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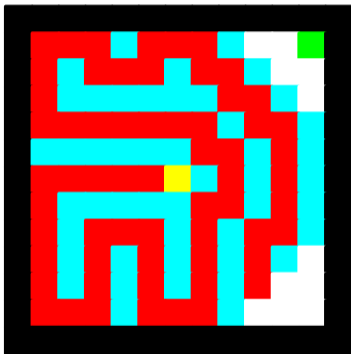
## Grid: DFS 2

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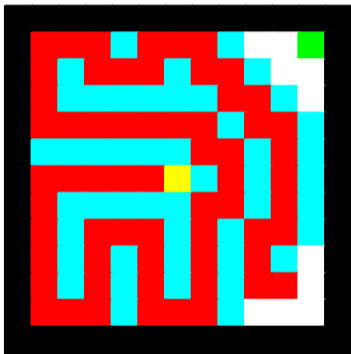
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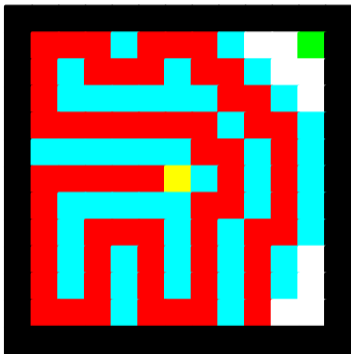
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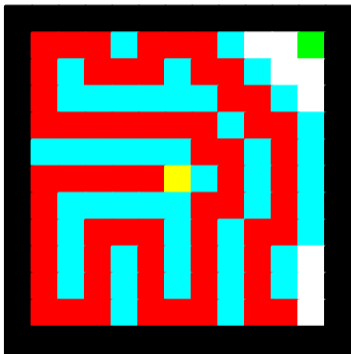
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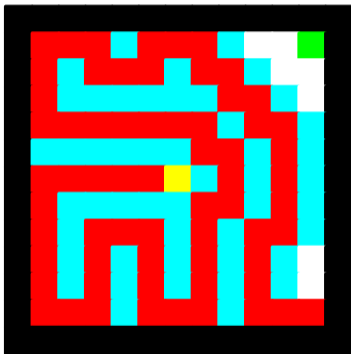
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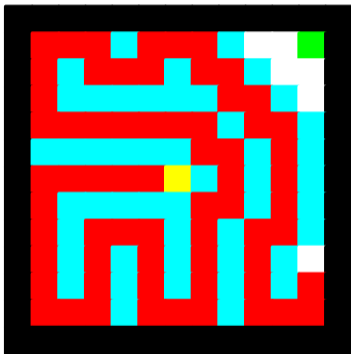
## Grid: DFS 2

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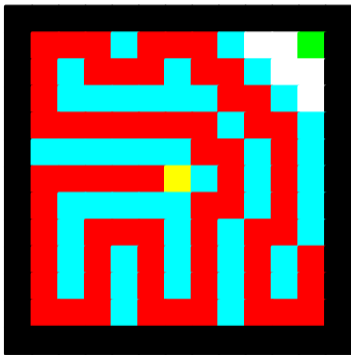
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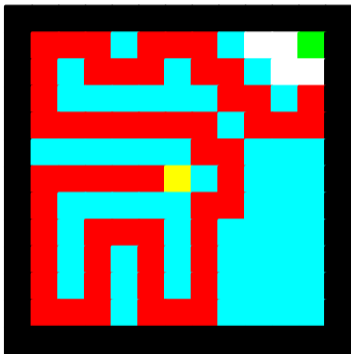
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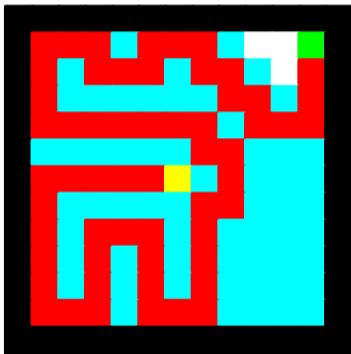
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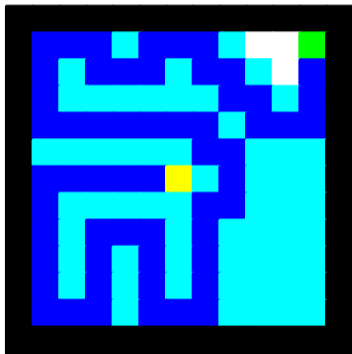
## Grid: DFS 2

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## Grid: DFS 2

---



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flip

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scat

seat

swat

slab

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slam



slap

slav

slaw

slay

blot

clot

plot

scot



shot

soot

sloe

slog

slop

slow

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slum



blip

clip

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span

spar

spas

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duct

dust

duel

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stem

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aria

asia

alga

alma

feat



fiat

flab

flag

flak

flap

flaw

flax

flay



flop

peat

plan

play

scab

scan

scar

beat



best

# Words: BFS vs DFS

---

## **BFS**

quiz → quit → suit → slit → slat → seat → beat → best

## **DFS**

quiz → quit → suit → suet → sues → sups → sops → soys → says → saws →  
sawn → sown → soon → soot → sort → sore → some → same → save →  
wave → wavy → wary → wart → want → wand → wind → wins → wits →  
with → wish → wisp → wasp → rasp → rash → rush → rust → runt → runs →  
ruts → rots → rote → rove → rive → rise → risk → rink → ring → rang →  
sang → sank → sack → suck → such → much → mach → mace → maze →  
raze → rare → tare → tars → tats → vats → vans → vane → vase → vast →  
vest → best

## Search in lib601

---

lib601 procedure called `search`, takes arguments:

- *successors*: function that takes a state and returns a list of successor states
- *start\_state*: the state from which to start the search
- *goal\_test*: a function that takes a state and returns `True` if that state satisfies the goal condition, and `False` otherwise
- *dfs*: boolean; if `True`, run a depth-first search; if `False`, run a breadth-first search

`search` returns a list of states from the root of the tree to the goal, or `None` if no path exists.

# 16 Lines!

---

```
def search(successors, start_state, goal_test, dfs = False):
    if goal_test(start_state):
        return [start_state]
    else:
        agenda = [SearchNode(start_state, None)]
        visited = {start_state}
        while len(agenda) > 0:
            parent = agenda.pop(-1 if dfs else 0)
            for child_state in successors(parent.state):
                child = SearchNode(child_state, parent)
                if goalTest(child_state):
                    return child.path()
                if child_state not in visited:
                    agenda.append(child)
                    visited.add(child_state)
        return None
```

# 16 Lines!

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```

# Casting Problems as Search Problems

---

Biggest issue is choice of **state**.

# Casting Problems as Search Problems

---

Biggest issue is choice of **state**.

From the state, we must be able to:

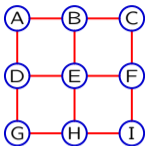
- Determine successors
- Test for goal condition

We'll get a lot of practice with this during the labs and homeworks this week.



# Example: Grid Search

---



```
class Grid:
    def __init__(self, width, height, start, goal):
        self.width = width
        self.height = height
        self.start = start
        self.goal = goal

grid = Grid(3, 3, (0,0), (2,2))

def grid_successors(state):
    r,c = state
    out = []
    for (dr,dc) in [(0,1),(1,0),(0,-1),(-1,0)]:
        if 0<=(r+dr)<grid.height and 0<=(c+dc)<grid.width:
            out.append((r+dr,c+dc))
    return out

result = search(grid_successors, grid.start, lambda x: x==grid.goal, False)
```

# Recap

---

Developed two search algorithms:

- Breadth-first search
- Depth-first search

Discussed the benefits and drawbacks of each

Developed two pruning rules:

- Don't consider paths that revisit states
- Only consider the first path to a given state

# Labs This Week

---

**Software Lab:** Solving Mazes

**Design Lab:** Robots in Mazes

