6.01

Lecture 7: Modularity in Circuits

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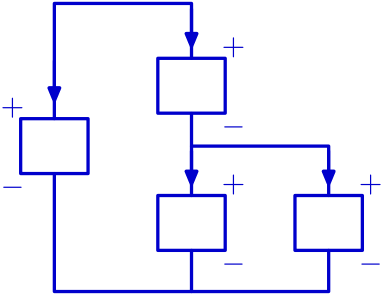
Last Time: The Circuit Abstraction

Circuits represent systems as connections of elements.

Currents flow through elements, and

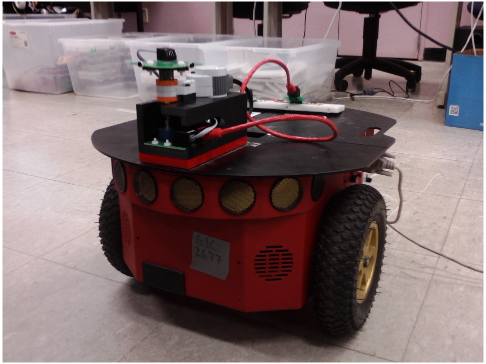
Voltages develop across elements.



Think about system as *constraints* on these variables.

Circuits: Lab Exercise

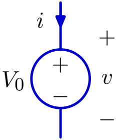
Design a new sensory modality for the robot.

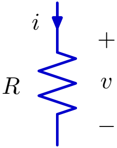


- **DL06:** Pots and loading, Motor Control
- **This Week:** Motor Control, Light Sensor
- **Next Week:** Spring Break
- **Week 8:** Pet Robot!

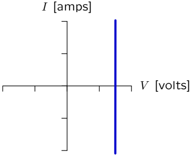
Circuits: Primitives and Combinations

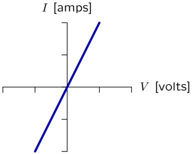
The **primitives** are simple elements: sources and resistors.





$v = V_0$ $\qquad$ $v = iR$





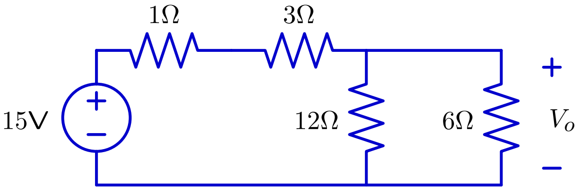
The **rules of combination** are the rules that govern the flow of current and the development of voltage.

Last Time: Analyzing Circuits

Combining component constraints and conservation laws (KCL), we developed a process by which we can solve for *all currents and potentials* in a circuit.

1. Pick a node to be our reference node. All other node potentials will be measured with respect to this node.
2. Look for a constitutive equation with exactly one unknown value. If such an equation exists, solve for the unknown value. GOTO 6.
3. Look for a KCL equation with exactly one unknown current. If such an equation exists, solve for the unknown current. GOTO 6.
4. If no equation with exactly one unknown, look for patterns that can simplify the circuit (series/parallel combinations, etc), and GOTO 2.
5. **Last Resort**: If no simplifications, write a small system of constitutive and KCL equations in terms of node potentials, and solve. GOTO 6.
6. If the circuit is completely solved, congratulations! If not, GOTO 2.

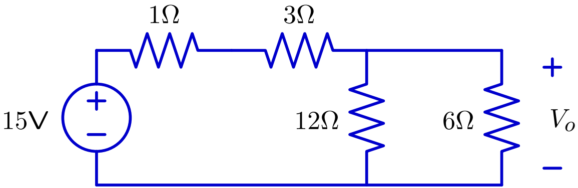
Check Yourself!



Which of the following is true?

1. $V_o \leq 3V$
2. $3V < V_o \leq 6V$
3. $6V < V_o \leq 9V$
4. $9V < V_o \leq 12V$
5. $V_o > 12V$

Check Yourself!



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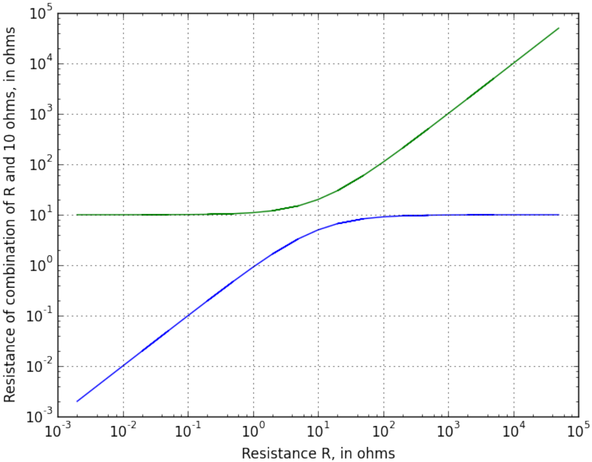
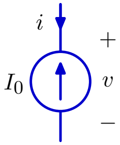


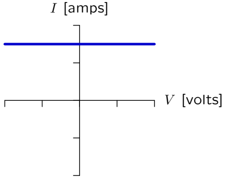
One curve represents the equivalent resistance of R in parallel with 10Ω, and the other represents the equivalent resistance of R in series with 10Ω. Which is which?

Current Source

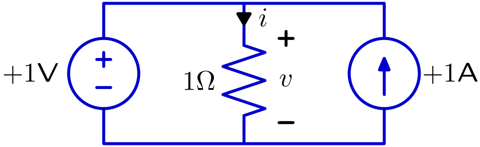
A current source (current **constraint**) ensures that the current flowing through it is exactly some constant value, *regardless of the voltage drop across it*.



$i = -I_0$



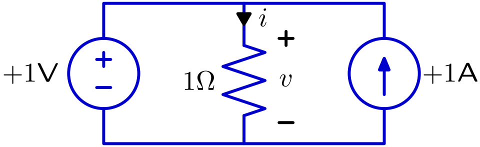
Check Yourself!



What is the current i through the resistor?

1. 1A
2. 2A
3. 0A
4. cannot be determined
5. none of the above

Check Yourself!



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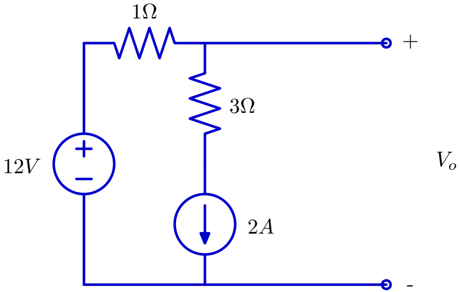
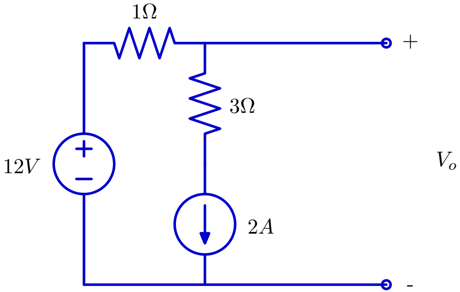
Find the voltage V_o in the circuit below:



0. $14V$
1. $10V$
2. $9V$
3. $6V$
4. something else
5. can't be solved (contradiction)

Check Yourself!

Find the voltage V_o in the circuit below:



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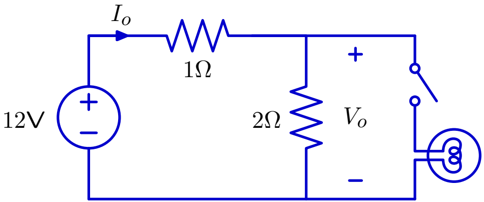
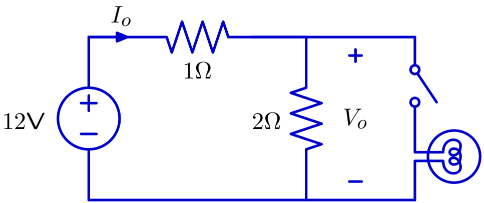
Interaction of Circuit Elements

Circuit design is complicated by interactions among the elements. Adding an element changes voltages and current **throughout** the circuit.

Example: closing a switch is equivalent to adding a new element.



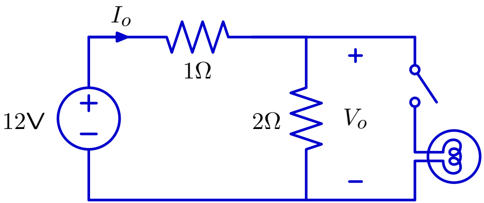
Check Yourself!



How does closing the switch affect V_o and I_o?

1. V_o decreases, I_o decreases
2. V_o decreases, I_o increases
3. V_o increases, I_o decreases
4. V_o increases, I_o increases
5. depends on bulb's resistance

Check Yourself!



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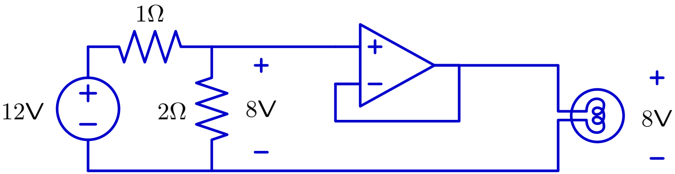
Today

Today: **Modularity in Circuits**
Controlling Complexity, Operational Amplifiers

Buffering with Op-Amps

Interactions between elements can be reduced (or eliminated) by using an operational amplifier as a **buffer**.



Opening and closing the switch has no effect on I_o or V_o.

When the switch is closed, the voltage across the bulb is the same as the voltage at the **input** of the op-amp.

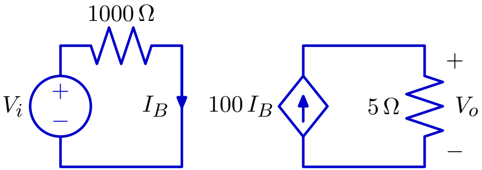
The rest of today: analyzing and designing op-amp circuits

Dependent Sources

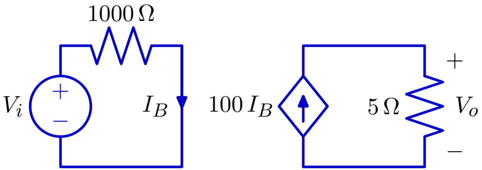
To analyze op-amps, we must introduce a new kind of element: a dependent source.

A dependent source generates a voltage or current whose value depends on another voltage or current.

Example: current-controlled current source



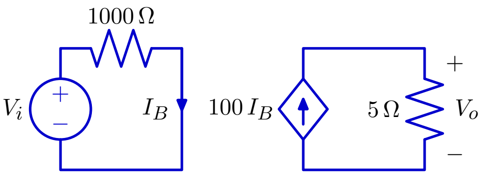
Check Yourself!



Find $\frac{V_o}{V_i}$.

1. 500
2. 1/20
3. 1
4. 1/2
5. none of the above

Check Yourself!

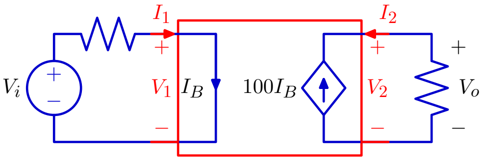


Find $\frac{V_o}{V_i}$.

1. 500
2. 1/20
3. 1
4. 1/2
5. none of the above

Dependent Sources

Dependent sources are **two-ports**: characterized by two equations.

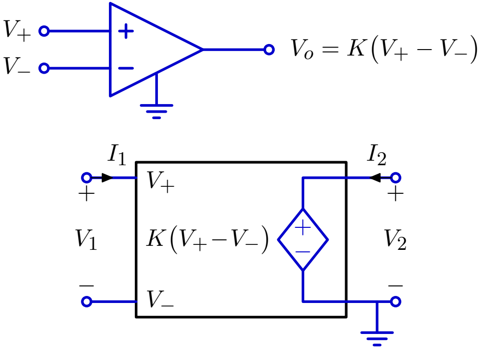


Here $V_1 = 0$ and $I_2 = -100I_1$

By contrast, one-ports (resistors and sources) are characterized by a single equation.

Operational Amplifier

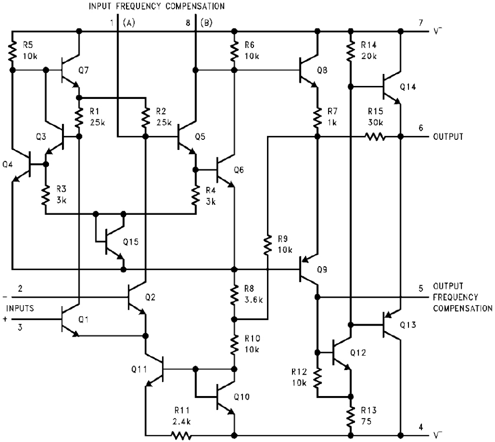
An operational amplifier (op-amp) can be modeled as a voltage-controlled voltage source.



$I_1 = 0$ and $V_2 = KV_1$, where K is large (typically $K > 10^5$).
Not what is actually in an op-amp! This is a **model**.

Operational Amplifier

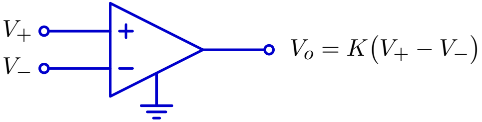
A more accurate circuit model of an op-amp ($\mu A709$):

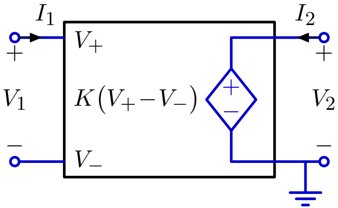


But we won't approach op-amps on this level in 6.01.

Operational Amplifier

An operational amplifier (op-amp) can be modeled as a voltage-controlled voltage source.

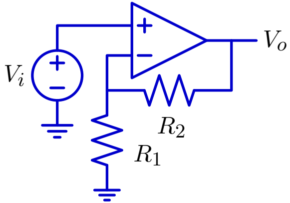




$I_1 = 0$ and $V_2 = KV_1$, where K is large (typically $K > 10^5$).
Not what is actually in an op-amp! This is a **model**.

Op-Amp: Example

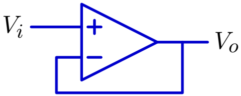
Find $\frac{V_o}{V_i}$ for the following circuit.



The “Ideal” Op-Amp

As $K \to \infty$, the difference between V_+ and V_- goes to zero.

Example:



$$V_o = K(V_+ - V_-) = K(V_i - V_o)$$

$$V_+ - V_- = V_i - V_o = V_i - \frac{K}{1-K}V_i = \frac{1}{1+K}V_i$$

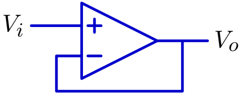
$$\lim_{K\to\infty}(V_+ - V_-) = 0$$

If $V_+ - V_-$ did not go to zero as $K \to \infty$, then $V_o = K(V_+ - V_-)$ could not be finite.

The “Ideal” Op-Amp

The approximation that $V_+ = V_-$ is referred to as the “ideal” op-amp approximation. It greatly simplifies analysis.

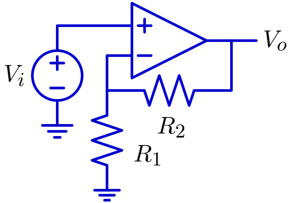
Example:



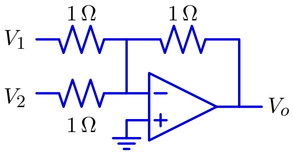
If $V_+ = V_-$, then $V_o = V_i$!

Non-inverting Amplifier

This circuit implements a "non-inverting amplifier."



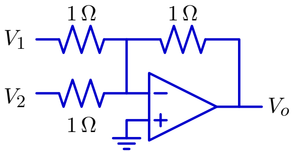
Check Yourself!



Determine the output V_o, making the ideal op-amp assumption.

1. $V_o = V_1 + V_2$
2. $V_o = V_1 - V_2$
3. $V_o = -V_1 - V_2$
4. $V_o = -V_1 + V_2$
5. none of the above

Check Yourself!



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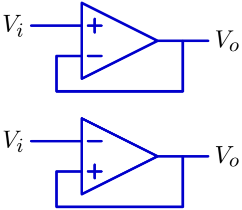
1. $V_o = V_1 + V_2$
2. $V_o = V_1 - V_2$
3. $V_o = -V_1 - V_2$ (inverting summer)
4. $V_o = -V_1 + V_2$
5. none of the above

The "Ideal" Op-amp: Paradox?

The ideal op-amp approximation implies that both of these circuits function identically.

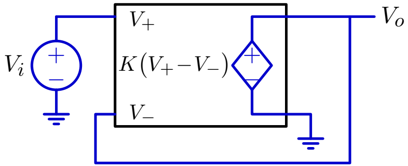


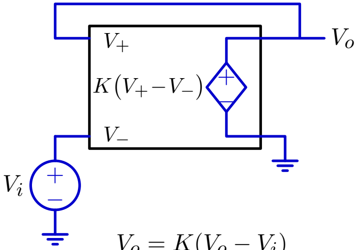
$V_+ = V_- \;\rightarrow\; V_o = V_i$!

However, this seems implausible, given what we know about feedback systems!

Paradox

Analyzing using VCVS model:





$$V_o = K(V_i - V_o)$$

$$(1+K)V_o = KV_i$$

$$\frac{V_o}{V_i} = \frac{K}{1+K} \approx 1$$

$$V_o = K(V_o - V_i)$$

$$(1-K)V_o = -KV_i$$

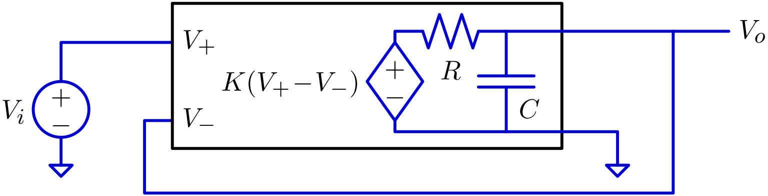
$$\frac{V_o}{V_i} = \frac{-K}{1-K} \approx 1$$

These circuits seem to have identical responses if K is large. Something is wrong!

"Thinking" Like An Op-Amp

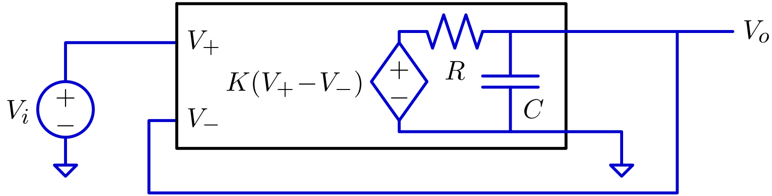
In truth, these systems both have stable (or metastable) points at $V_o = V_i$. However, we need to think about **temporal dynamics**, and what happens when the system gets moved away from the point where $V_o = V_i$.

We can add a resistor and capacitor to our model to account for accumulation of charge in an op-amp.

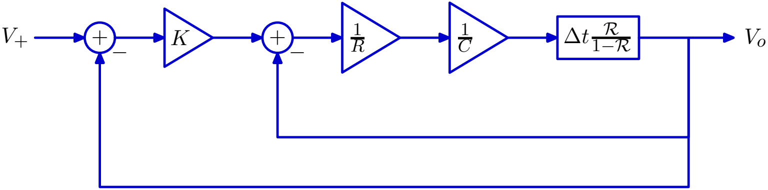


Capacitors accumulate charge.

LTI Model

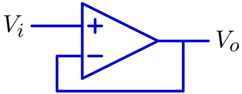


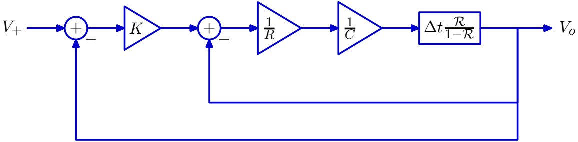
Below is an LTI model of this system (block diagram and `SM`).



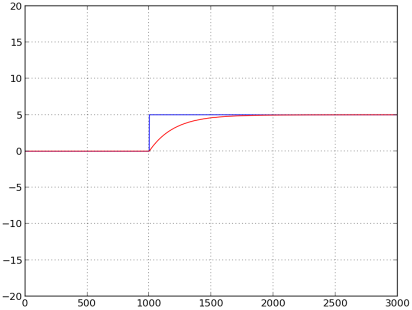

```
integrator = Cascade(Gain(t), FeedbackAdd(R(0),Wire()))
inner = Cascade(Gain(1./R/C),integrator)
topwire = Cascade(Gain(K), FeedbackSubtract(inner,Wire()))
```

Simulation





Simulating a "step" from V_o(red) $= V_i$(blue) $= 0$ to $V_i = 5V$:



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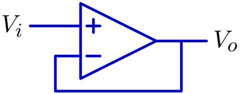
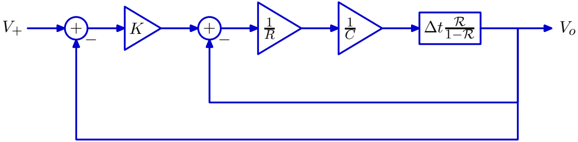
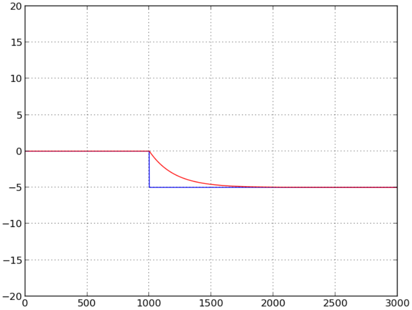




Simulating a "step" from V_o(red) $= V_i$(blue) $= 0$ to $V_i = -5V$:



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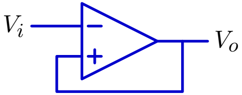

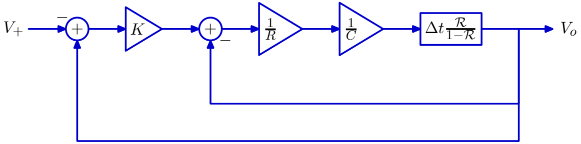

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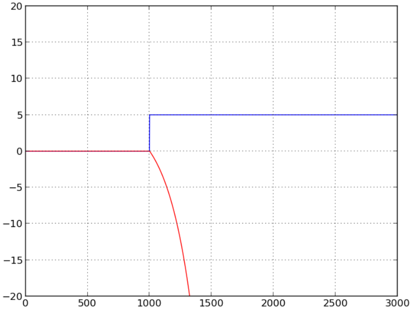

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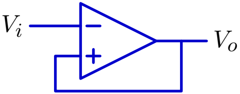
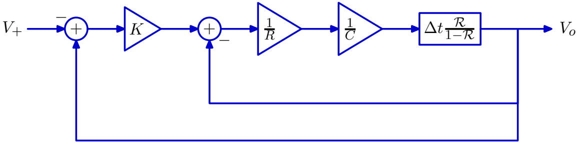
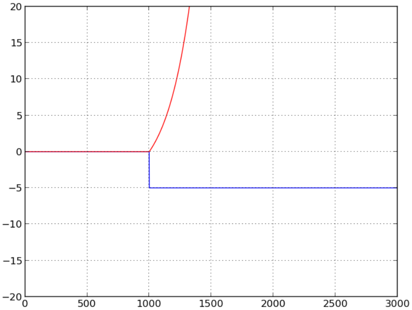


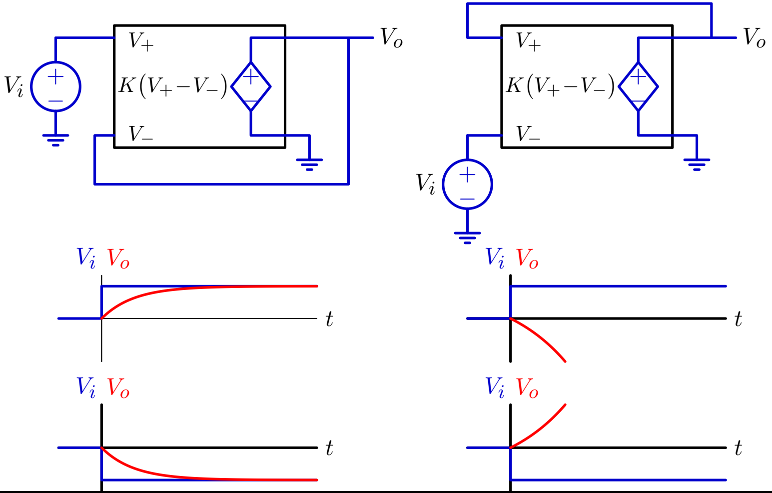


Simulating a "step" from V_o(red) $= V_i$(blue) $= 0$ to $V_i = 5V$:



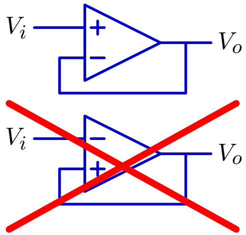
Positive and Negative Feedback

Negative Feedback (left) drives the output **toward** the input.
Positive Feedback (right) drives the output **away from** the input.



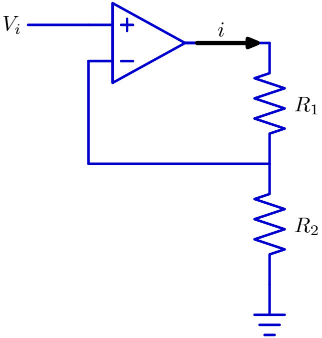
Paradox Resolved!

Although both circuits have solutions with $V_o = V_i$ (for large K), only the first is stable to changes in V_i.



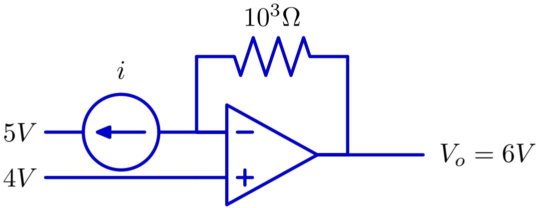
Takeaway: Feedback loop should go to the negative input of the op-amp.

Check Yourself!



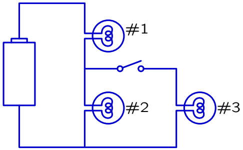
Solve for the current i in the circuit above.

Check Yourself!



Solve for the current i in the circuit above.

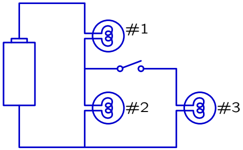
Check Yourself!



Closing the switch will make:

1. Bulb 1 brighter
2. Bulb 2 dimmer
3. Both of the above
4. Bulbs 1, 2, and 3 equally bright
5. none of the above

Check Yourself!



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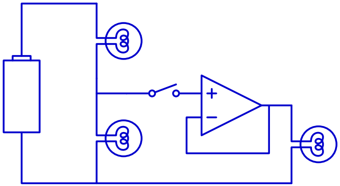
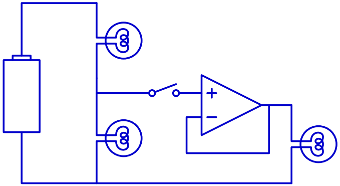
Check Yourself!



When the switch is closed:

1. top bulb is brightest
2. right bulb is brightest
3. right bulb is dimmest
4. all 3 bulbs equally bright
5. none of the above

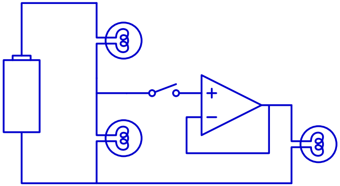
Check Yourself!



When the switch is closed:

1. top bulb is brightest
2. right bulb is brightest
3. right bulb is dimmest
4. all 3 bulbs equally bright
5. none of the above

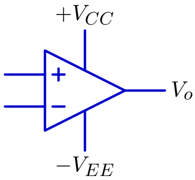
Check Yourself!



The battery provides the power to illuminate the left bulbs.
Where does the power to illuminate the right bulb come from?

Power Rails

Op-amps derive power from connections to a power supply.



Will see this in lab (have to connect pins 2 and 4 of the L272 package to power and ground).

Op-amp's output current comes from the supply.

Typically, the output voltage of an op-amp is constrained by the power supply:

$$-V_{EE} < V_o < V_{CC}$$

Summary

- An op-amp can be modeled as a voltage-dependent voltage source.
- High input resistance means negligibly small current flows into or out of the op-amp's input terminals (though current can flow into or out of the output terminal).
- The "ideal" op-amp approximation is $V_+ = V_-$.
- The ideal op-amp approximation only makes sense when the op-amp is connected with negative feedback.
- The output of an op-amp is typically limited by the supply voltage.

Labs This Week

Exercises: Practice with various op-amp topologies.

Software and Design Lab: Controlling motors, light sensors.

Next Week and Beyond: Designing and Constructing "Eyes" for the Robot.