6.01 Introduction to EECS via Robotics

Lecture 5: Circuits Introduction

Lecturer: Adam Hartz (`hz@mit.edu`)

As you come in...

- Grab one handout (on the table by the entrance)
- Please sit near the front!

Notes

Midterm 1

Time: Tuesday, 12 March, 7:30-9:30pm

Room: TBD

Coverage: Everything up to and including week 5

You may refer to any printed materials you bring.
You may not use computers, phones, or calculators.

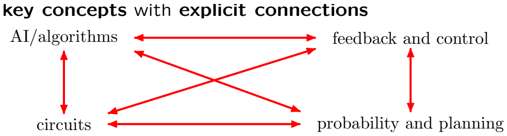
Review materials have been posted to the web.

Conflict? E-mail `hz@mit.edu` by this Friday, 5pm.

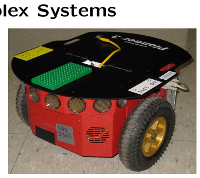
Notes

6.01: Big Ideas

Focus on **key concepts** with **explicit connections**



in an **authentic context** with overarching theme:
Modular Design of Complex Systems



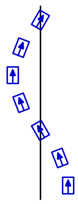
Notes

Module 1: Signals and Systems

Focus on:
- **modeling** and **simulation** of physical systems
- augmenting physical systems with **computation**

Topics: Discrete-time LTI Feedback Control Systems

Lab Exercises: Robotic Driving, "Jousting"



Notes

Module 1: Signals and Systems

Controlling complexity through modularity and abstraction:

Python:
- Primitives: `+`, `*`, `==`, `!=`, ...
- Combination: `if`, `while`, `f(g(x))`, ...
- Abstraction: `def`, `class`, ...

LTI:
- Primitives: Gains, Delays
- Combination: Adders, Cascade, Feedback, ...
- Abstraction: System Functionals

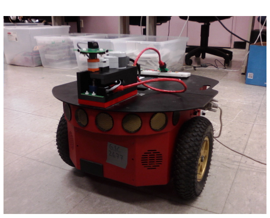
Notes

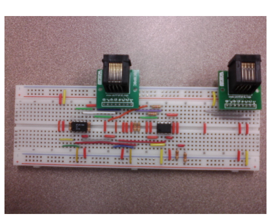
Module 2: Circuits

Focus on:
- Designing, constructing, and analyzing physical systems

Topics: Resistive Networks, Op-Amps, Equivalence

Lab Exercises: Design new sensory modality for the robot





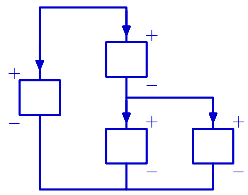
Notes

The Circuit Abstraction

Circuits represent systems as *components* connected by *nodes*.

Currents flow through components, and

Voltages develop across components.



Notes

Quantities of Interest

- **Voltage**: The difference in electrical potential energy between two points. Measured in Volts (V, J/C)
- **Current**: The rate of flow of positive charge past a point. Measured in Amperes (Amps, A, C/s)

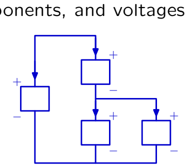
It is the *difference* in potential between two nodes that drives the currents.

Notes

Circuits: Nodes, Wires, and Potential Energy

Circuits represent systems as *components* connected by *nodes*.

Currents flow through components, and voltages develop across components.



Points that are connected by only wires comprise a **node**. Each node sits at some potential. The difference in potential between nodes (connected by components) drives the flow of current through the circuit.

Components constrain the relationships between the voltage across the component and the current flowing through it.

Notes

The Circuit Abstraction

Circuits are useful and important for (at least) two very different reasons:

- as **models** of complex systems
  - biological models
  - thermodynamic models
  - fluid models
- as **physical systems**
  - power (generators, transformers, power lines, etc)
  - electronics (cell phones, computers, etc)
  - sensors (sonars, glucose sensors, etc)

Notes

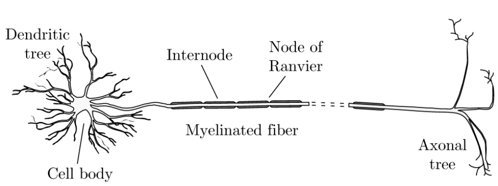
Example: Flashlight

We can represent a flashlight as a voltage source (battery) connected to a resistor (light bulb).

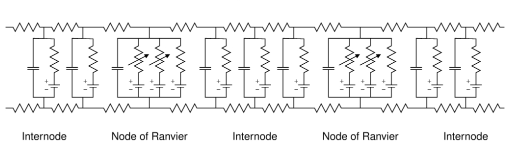
The voltage source generates a voltage v across the resistor and a current i through the resistor.

Notes

Example: Myelinated Fiber



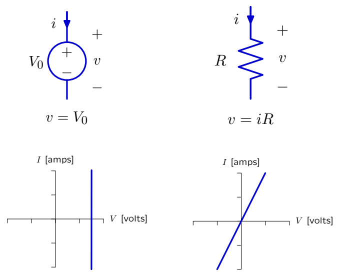
Model of myelinated nerve fiber



Notes

Circuits: Primitives and Combinations

The **primitives** are simple elements: sources and resistors.
The **rules of combination** are the rules that govern the flow of current and the development of voltage.



Notes

Circuits: Primitives

Resistor

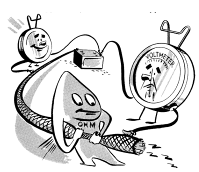


Illustration from *Imperial And Chrysler Reference Guide On Volts, Amps, & Ohms*, 1948.

Notes

Circuits: Primitives

Voltage Source

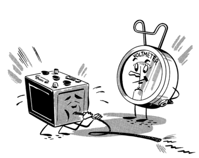
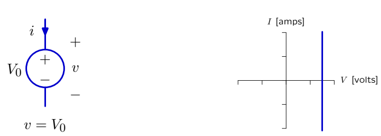


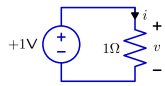
Illustration from *Imperial And Chrysler Reference Guide On Volts, Amps, & Ohms*, 1948.



Notes

Analyzing Simple Circuits

Example 1:



The voltage source determines the voltage across the resistor, so the current through the resistor is $i = v/R = 1V/1\Omega = 1A$.

Notes

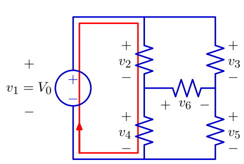
Analyzing More Complex Circuits

More complicated circuits are more complicated to analyze, but can be analyzed systematically by applying constitutive equations and two conservation laws.

Notes

Analyzing Circuits: KVL

KVL: The sum of the voltages around any closed path is zero.

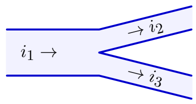


Example: $-v_1 + v_2 + v_4 = 0$

Notes

Analyzing Circuits: KCL

The flow of electrical current is analogous to the flow of incompressible fluid (e.g., water):



All water that flows into a junction must flow out. All current that flows into a **node** must flow out.

Current i_1 flows in, and two currents i_2 and i_3 flow out:

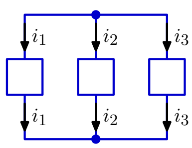
$i_1 = i_2 + i_3$

Notes

Analyzing Circuits: KCL

KCL: The net flow of current into (or out of) a node is zero.

$$\sum i_{in} = \sum i_{out}$$



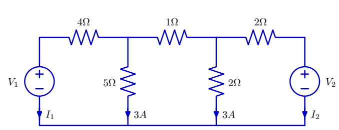
Here, there are two nodes. The net current out of each must be zero.

For the top node: $i_1 + i_2 + i_3 = 0$.
Same for the bottom node.

Notes

Analyzing More Complex Circuits

More complicated circuits are more complicated to analyze, but they can be analyzed systematically by applying constitutive equations and two conservation laws.



One strategy:

- Give every current and every node potential a name
- Choose a node potential to be our reference 0V
- Write one equation per component (7)
- Write one KCL equation per node, except for the reference node (4)

12 equations in 12 unknowns! :(

Notes

Analyzing More Complex Circuits

We will still need to solve all of these equations (to figure out the voltages and currents throughout the circuit), but there may be an easier way.

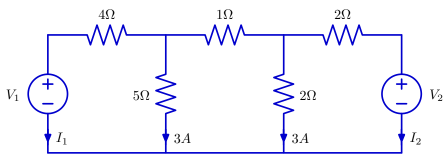
In many circuits, some of the equations can be solved *in isolation*. This suggests an alternative approach.

(version 0.0.1)

- pick a node to be reference (0V)
- repeat until circuit solved:
  - Find a component equation or KCL equation with exactly one unknown, and solve for that value directly (it now becomes a known value).

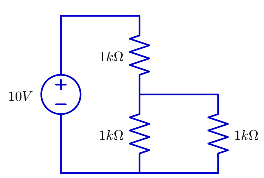
Notes

Example: Complicated Circuit



Notes

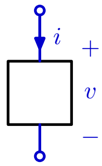
Example: Complicated Circuit



Notes

Abstractions: "One-Port"

A "one-port" is a circuit that can be represented as a single element:

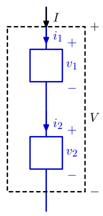


Current enters one terminal (+) and leaves the other (-). The one-port constrains the relationship between the current i and the voltage v.

Notes

Abstractions: Series Combination

Components can be combined in "series" to form new one-ports:

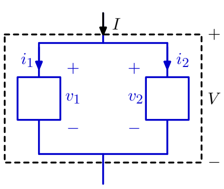


$V = v_1 + v_2$
$I = i_1 = i_2$

Notes

Abstractions: Parallel Combination

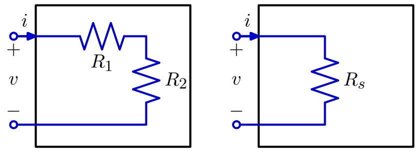
Components can be combined in "parallel" to form new one-ports:



$V = v_1 = v_2$
$I = i_1 + i_2$

Notes

Check Yourself!

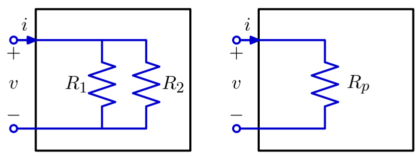


If the two boxed circuits have the same v/i relationship, which of the following, is true?

1. $R_s < R_1$ and $R_s < R_2$
2. $R_1 < R_s < R_2$
3. $R_2 < R_s < R_1$
4. $R_s > R_1$ and $R_s > R_2$
5. None of the above

Notes

Check Yourself!

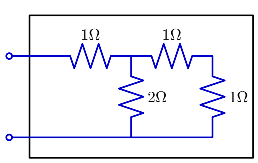


If the two boxed circuits have the same v/i relationship, which of the following, is true?

1. $R_p < R_1$ and $R_p < R_2$
2. $R_1 < R_p < R_2$
3. $R_2 < R_p < R_1$
4. $R_p > R_1$ and $R_p > R_2$
5. None of the above

Notes

Check Yourself!

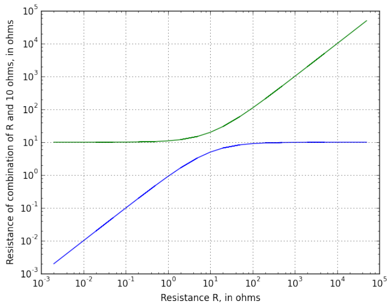


What is the equivalent resistance of this one-port?

1. 0.5Ω
2. 1Ω
3. 2Ω
4. 3Ω
5. 5Ω

Notes

Check Yourself!



One curve represents the equivalent resistance of R in parallel with 10Ω, and the other represents the equivalent resistance of R in series with 10Ω. Which is which?

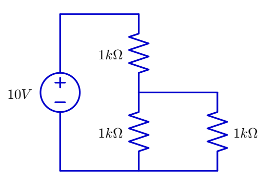
Notes

Version 0.0.2

1. Pick a node to be our reference node. All other node potentials will be measured with respect to this node.
2. Look for a constitutive equation with exactly one unknown value. If such an equation exists, solve for the unknown value and GOTO 5.
3. Look for a KCL equation with exactly one unknown current. If such an equation exists, solve for the unknown current and GOTO 5.
4. If no equation with exactly one unknown, look for patterns that can simplify the circuit (series/parallel combinations, etc), and GOTO 2.
5. If the circuit is completely solved, congratulations! If not, GOTO 2.

Notes

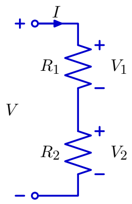
Example: Complicated Circuit



Notes

Voltage Divider

Resistors in series act as voltage dividers:



$I = \frac{V}{R_1+R_2}$

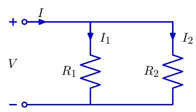
$V_1 = R_1 I = \frac{R_1}{R_1+R_2} V$

$V_2 = R_2 I = \frac{R_2}{R_1+R_2} V$

Notes

Current Divider

Resistors in parallel act as current dividers:



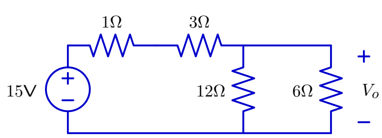
$V = (R_1 || R_2) I$

$I_1 = \frac{V}{R_1} = \frac{1}{R_1} \frac{R_1 R_2}{R_1+R_2} I = \frac{R_2}{R_1+R_2} I$

$I_2 = \frac{V}{R_2} = \frac{1}{R_2} \frac{R_1 R_2}{R_1+R_2} I = \frac{R_1}{R_1+R_2} I$

Notes

Check Yourself!

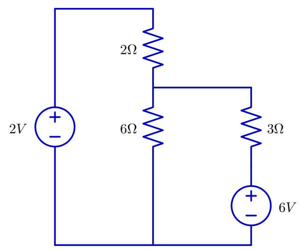


Which of the following is true?

1. $V_o \leq 3V$
2. $3V < V_o \leq 6V$
3. $6V < V_o \leq 9V$
4. $9V < V_o \leq 12V$
5. $V_o > 12V$

Notes

Another Example



Notes

Version 1.0

Combining earlier ideas, we can develop a process by which we can solve for *all currents and potentials* in a circuit.

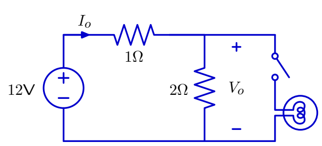
1. Pick a node to be our reference node. All other node potentials will be measured with respect to this node.
2. Look for a constitutive equation with exactly one unknown value. If such an equation exists, solve for the unknown value and GOTO 6.
3. Look for a KCL equation with exactly one unknown current. If such an equation exists, solve for the unknown current and GOTO 6.
4. If no equation with exactly one unknown, look for patterns that can simplify the circuit (series/parallel combinations, etc), and GOTO 2.
5. **Last Resort**: If no simplifications, write a small system of constitutive and KCL equations in terms of node potentials, and solve. GOTO 6.
6. If the circuit is completely solved, congratulations! If not, GOTO 2.

Notes

Interaction of Circuit Elements

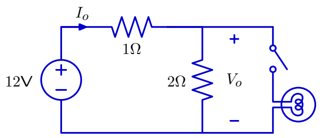
Circuit design is complicated by interactions among the elements. Adding an element changes voltages and current **throughout** the circuit.

Example: closing a switch is equivalent to adding a new element.



Notes

Check Yourself!



How does closing the switch affect V_o and I_o?

1. V_o decreases, I_o decreases
2. V_o decreases, I_o increases
3. V_o increases, I_o decreases
4. V_o increases, I_o increases
5. depends on bulb's resistance

Notes

Summary

This time:

- Defined the circuit abstraction ("through" and "across" variables)
- Developed systematic way of solving circuits
- Developed means of thinking about circuits through patterns (series/parallel) and abstractions (one-port)
- Noticed that the ways in which we think about abstraction and modularity in circuits is fundamentally different from the way we thought about these ideas in LTI and programming

Labs This Week:

- ~~*Software*~~ *Hardware Lab*: Dividers, Breadboarding
- *Design Lab*: Joystick-controlled robot!!!

Notes

Notes