6.01 Introduction to EECS via Robotics

Lecture 5: Circuits Introduction

Lecturer: Adam Hartz (`hz@mit.edu`)

As you come in...

- Grab one handout (on the table by the entrance)
- Please sit near the front!

Midterm 1

Time: Tuesday, 12 March, 7:30-9:30pm

Room: TBD

Coverage: Everything up to and including week 5

You may refer to any printed materials you bring.
You may not use computers, phones, or calculators.

Review materials have been posted to the web.

Conflict? E-mail `hz@mit.edu` by this Friday, 5pm.

6.01: Big Ideas

Focus on **key concepts**

AI/algorithms

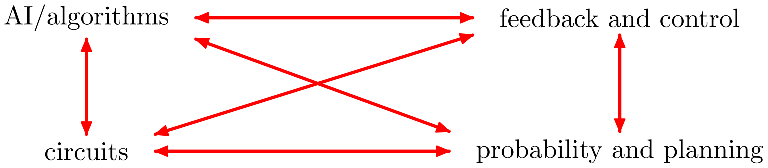
feedback and control

circuits

probability and planning

6.01: Big Ideas

Focus on **key concepts** with **explicit connections**



6.01: Big Ideas

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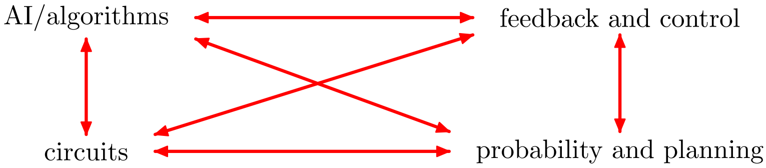
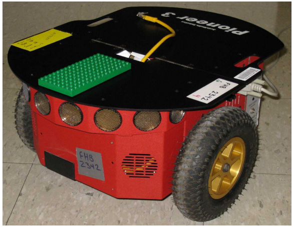


in an **authentic context** with overarching theme:
Modular Design of Complex Systems



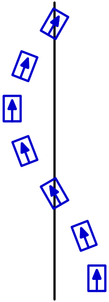
Module 1: Signals and Systems

Focus on:

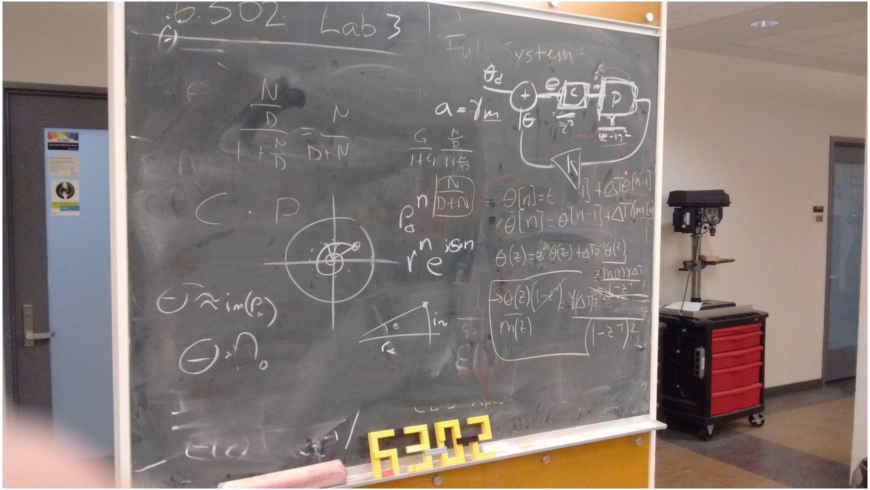
- **modeling** and **simulation** of physical systems
- augmenting physical systems with **computation**

Topics: Discrete-time LTI Feedback Control Systems

Lab Exercises: Robotic Driving, “Jousting”



Module 1: Signals and Systems



Module 1: Signals and Systems

Controlling complexity through modularity and abstraction:

Python:

- Primitives: `+`, `*`, `==`, `!=`, ...
- Combination: `if`, `while`, `f(g(x))`, ...
- Abstraction: `def`, `class`, ...

LTI:

- Primitives: Gains, Delays
- Combination: Adders, Cascade, Feedback, ...
- Abstraction: System Functionals

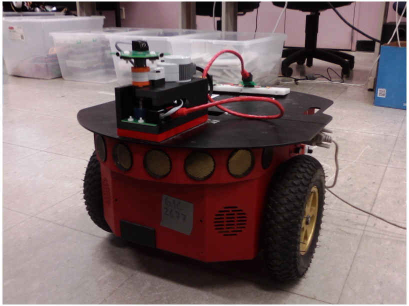
Module 2: Circuits

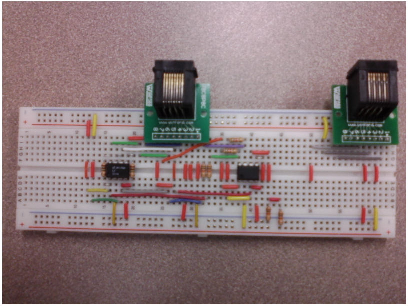
Focus on:

- Designing, constructing, and analyzing physical systems

Topics: Resistive Networks, Op-Amps, Equivalence

Lab Exercises: Design new sensory modality for the robot



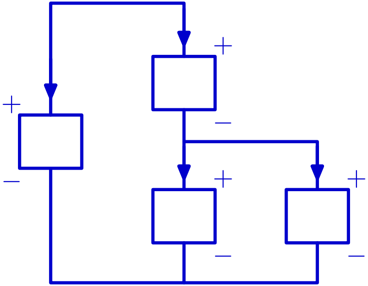


The Circuit Abstraction

Circuits represent systems as *components* connected by *nodes*.

Currents flow through components, and

Voltages develop across components.



Quantities of Interest

- **Voltage**: The difference in electrical potential energy between two points. Measured in Volts (V, J/C)
- **Current**: The rate of flow of positive charge past a point. Measured in Amperes (Amps, A, C/s)

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It is the *difference* in potential between two nodes that drives the currents.

Circuits: Nodes, Wires, and Potential Energy

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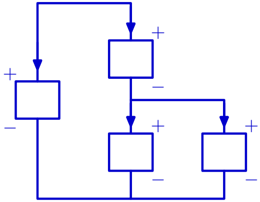
Currents flow through components, and voltages develop across components.



Points that are connected by only wires comprise a **node**. Each node sits at some potential. The difference in potential between nodes (connected by components) drives the flow of current through the circuit.

Components constrain the relationships between the voltage across the component and the current flowing through it.

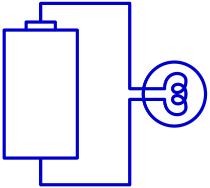
The Circuit Abstraction

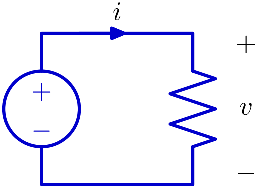
Circuits are useful and important for (at least) two very different reasons:

- as **models** of complex systems
  - biological models
  - thermodynamic models
  - fluid models
- as **physical systems**
  - power (generators, transformers, power lines, etc)
  - electronics (cell phones, computers, etc)
  - sensors (sonars, glucose sensors, etc)

Example: Flashlight

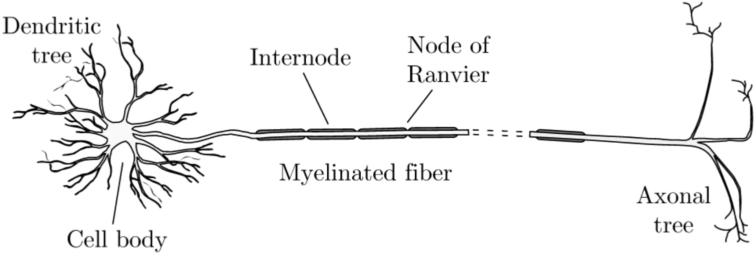
We can represent a flashlight as a voltage source (battery) connected to a resistor (light bulb).



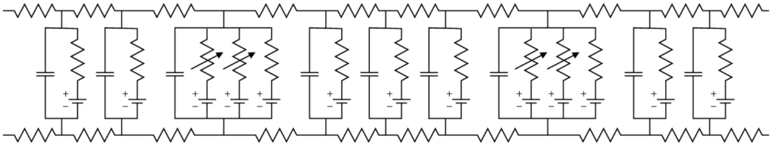


The voltage source generates a voltage v across the resistor and a current i through the resistor.

Example: Myelinated Fiber

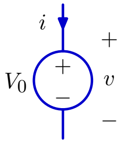


Model of myelinated nerve fiber

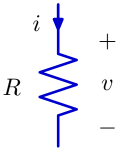


Circuits: Primitives and Combinations

The **primitives** are simple elements: sources and resistors.
The **rules of combination** are the rules that govern the flow of current and the development of voltage.



$v = V_0$



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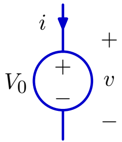

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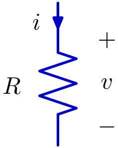

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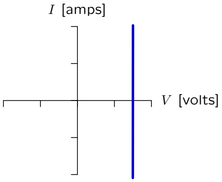
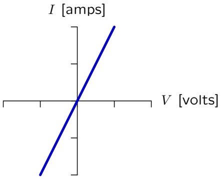




Circuits: Primitives

Resistor

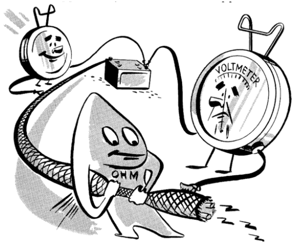
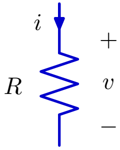


Illustration from *Imperial And Chrysler Reference Guide On Volts, Amps, & Ohms*, 1948.



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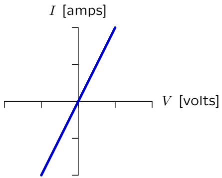
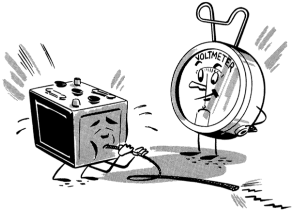


Circuits: Primitives

Voltage Source



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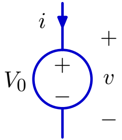

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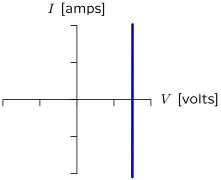
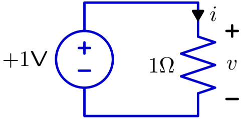


Analyzing Simple Circuits

Example 1:



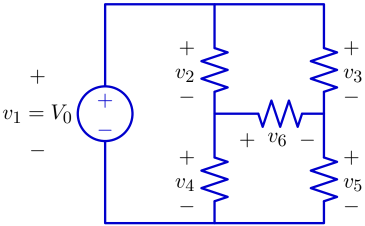
The voltage source determines the voltage across the resistor, so the current through the resistor is $i = v/R = 1V/1\Omega = 1A$.

Analyzing More Complex Circuits

More complicated circuits are more complicated to analyze, but can be analyzed systematically by applying constitutive equations and two conservation laws.

Analyzing Circuits: KVL

KVL: The sum of the voltages around any closed path is zero.



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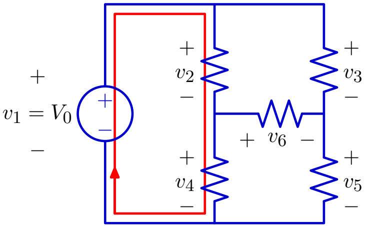
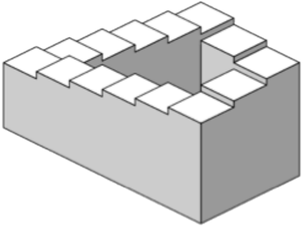


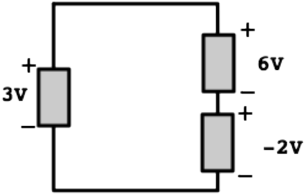
Example: $-v_1 + v_2 + v_4 = 0$

Analyzing Circuits: KVL

Impossible Things



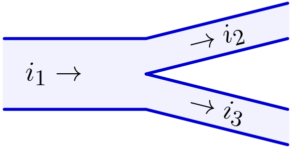
Penrose Staircase



3V −6V − −2V ≠ 0V

Analyzing Circuits: KCL

The flow of electrical current is analogous to the flow of incompressible fluid (e.g., water):



All water that flows into a junction must flow out. All current that flows into a **node** must flow out.

Current i_1 flows in, and two currents i_2 and i_3 flow out:

$i_1 = i_2 + i_3$

Analyzing Circuits: KCL

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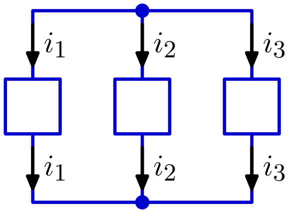

Here, there are two nodes. The net current out of each must be zero.

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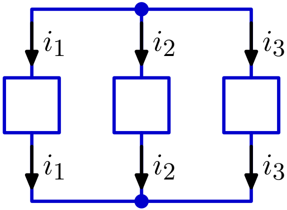

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For the top node: $i_1 + i_2 + i_3 = 0$.

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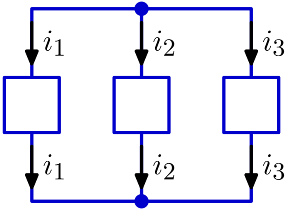

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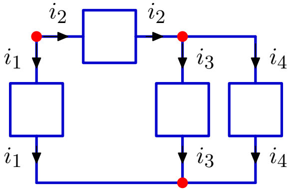
For the top node: $i_1 + i_2 + i_3 = 0$.
Same for the bottom node.

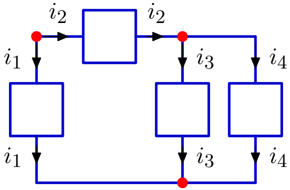
KCL



At each node, KCL must hold.

We can write this relationship for each node:

KCL



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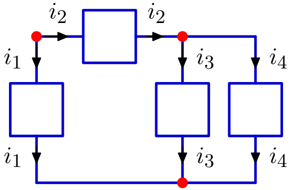

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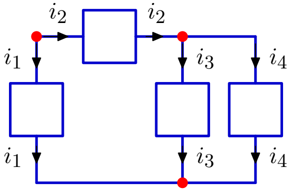

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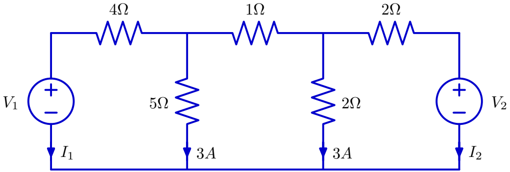

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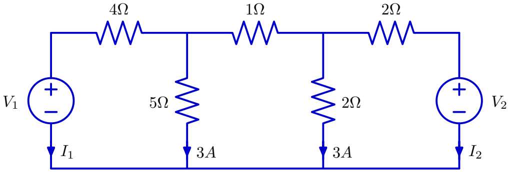

One strategy:

- Give every current and every node potential a name
- Choose a node potential to be our reference 0V
- Write one equation per component (7)
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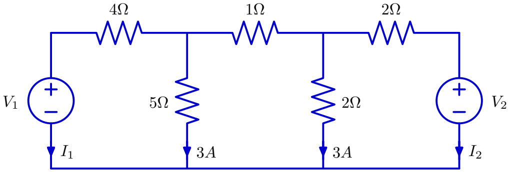

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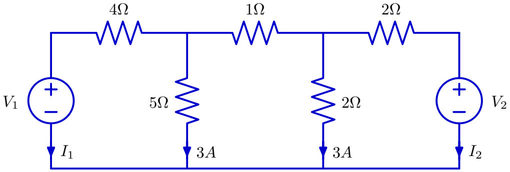

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(version 0.0.1)

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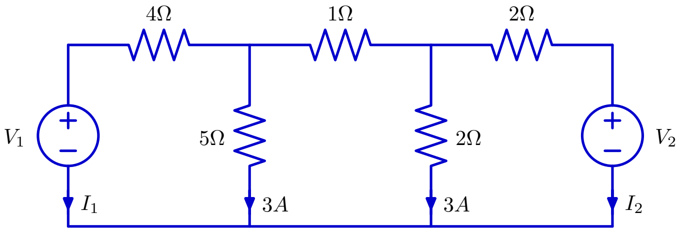
(version 0.0.1)

- pick a node to be reference (0V)
- repeat until circuit solved:
  - Find a component equation or KCL equation with exactly one unknown, and solve for that value directly (it now becomes a known value).

Example: Complicated Circuit



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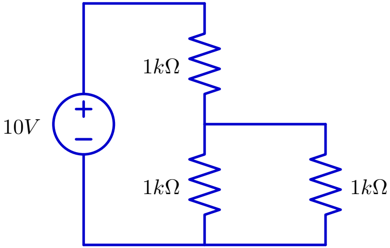
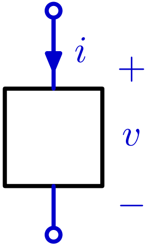


Abstractions: “One-Port”

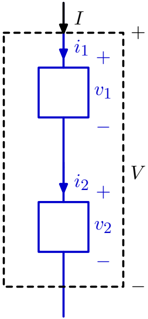
A “one-port” is a circuit that can be represented as a single element:



Current enters one terminal (+) and leaves the other (-). The one-port constrains the relationship between the current i and the voltage v.

Abstractions: Series Combination

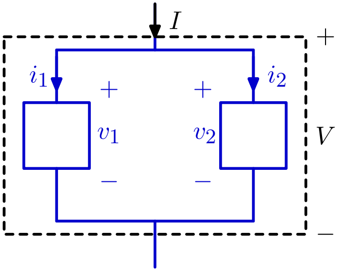
Components can be combined in “series” to form new one-ports:



$V = v_1 + v_2$
$I = i_1 = i_2$

Abstractions: Parallel Combination

Components can be combined in “parallel” to form new one-ports:



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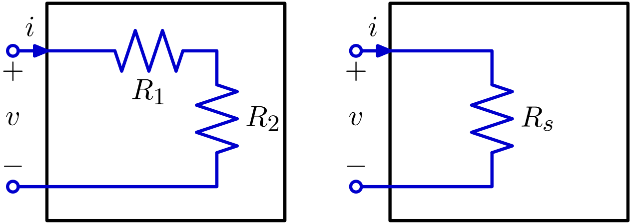
Check Yourself!

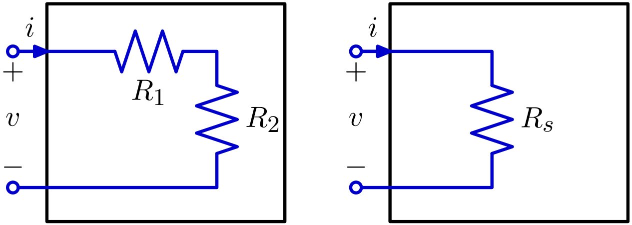


If the two boxed circuits have the same v/i relationship, which of the following, is true?

1. $R_s < R_1$ and $R_s < R_2$
2. $R_1 < R_s < R_2$
3. $R_2 < R_s < R_1$
4. $R_s > R_1$ and $R_s > R_2$
5. None of the above

Series Resistors

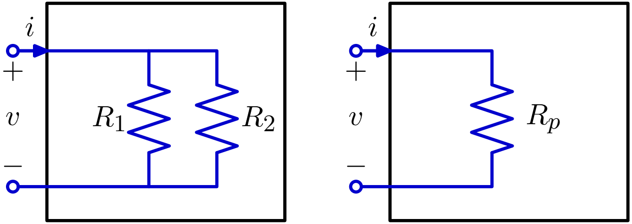
The series combination of two resistors is equivalent to a single resistor whose resistance is the sum of the two original resistances.



$v = iR_2 + iR_1 = iR_s$
$R_s = R_1 + R_2$

The series equivalent resistance is always **larger** than either of the original resistances.

Check Yourself!

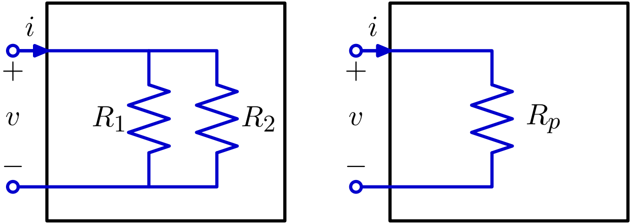


If the two boxed circuits have the same v/i relationship, which of the following, is true?

1. $R_p < R_1$ and $R_p < R_2$
2. $R_1 < R_p < R_2$
3. $R_2 < R_p < R_1$
4. $R_p > R_1$ and $R_p > R_2$
5. None of the above

Parallel Resistors

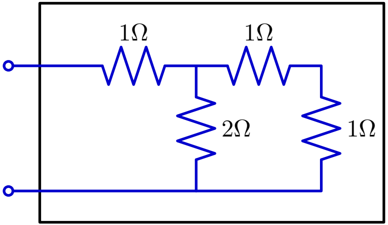
The parallel combination of two resistors is equivalent to a single resistor whose conductance (1/resistance) is the sum of the two original conductances.



$$i = \frac{v}{R1} + \frac{v}{R_2} = \frac{v}{R_p}$$
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$R_p = R_1||R_2 = \frac{R_2 R_1}{R_2+R_1} = \frac{R_1}{1+\frac{R_1}{R_2}} = \frac{R_2}{1+\frac{R_2}{R_1}}$$

The parallel equivalent resistance is always **smaller** than either of the original resistances.

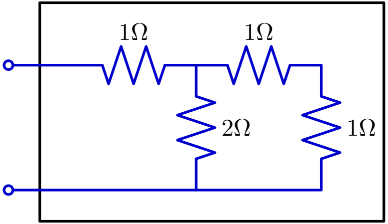
Check Yourself!



What is the equivalent resistance of this one-port?

1. 0.5Ω
2. 1Ω
3. 2Ω
4. 3Ω
5. 5Ω

Check Yourself!



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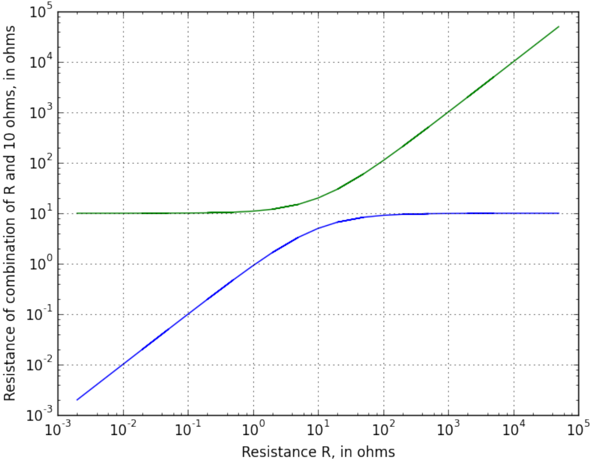
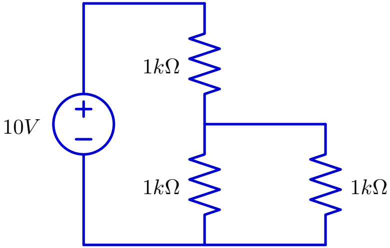


One curve represents the equivalent resistance of R in parallel with 10Ω, and the other represents the equivalent resistance of R in series with 10Ω. Which is which?

Version 0.0.2

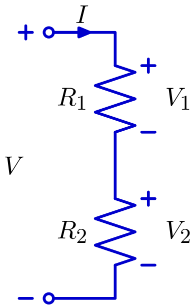
1. Pick a node to be our reference node. All other node potentials will be measured with respect to this node.
2. Look for a constitutive equation with exactly one unknown value. If such an equation exists, solve for the unknown value and GOTO 5.
3. Look for a KCL equation with exactly one unknown current. If such an equation exists, solve for the unknown current and GOTO 5.
4. If no equation with exactly one unknown, look for patterns that can simplify the circuit (series/parallel combinations, etc), and GOTO 2.
5. If the circuit is completely solved, congratulations! If not, GOTO 2.

Example: Complicated Circuit



Voltage Divider

Resistors in series act as voltage dividers:



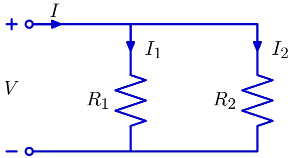
$I = \frac{V}{R_1+R_2}$

$V_1 = R_1 I = \frac{R_1}{R_1+R_2} V$

$V_2 = R_2 I = \frac{R_2}{R_1+R_2} V$

Current Divider

Resistors in parallel act as current dividers:

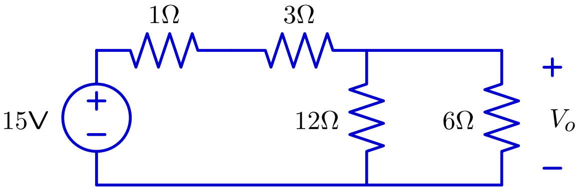


$$V = (R_1||R_2)I$$

$$I_1 = \frac{V}{R_1} = \frac{1}{R_1}\frac{R_1R_2}{R_1+R_2}I = \frac{R_2}{R_1+R_2}I$$

$$I_2 = \frac{V}{R_2} = \frac{1}{R_2}\frac{R_1R_2}{R_1+R_2}I = \frac{R_1}{R_1+R_2}I$$

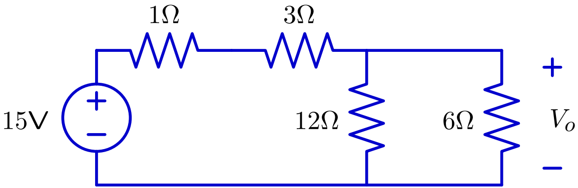
Check Yourself!



Which of the following is true?

1. $V_o \leq 3V$
2. $3V < V_o \leq 6V$
3. $6V < V_o \leq 9V$
4. $9V < V_o \leq 12V$
5. $V_o > 12V$

Check Yourself!



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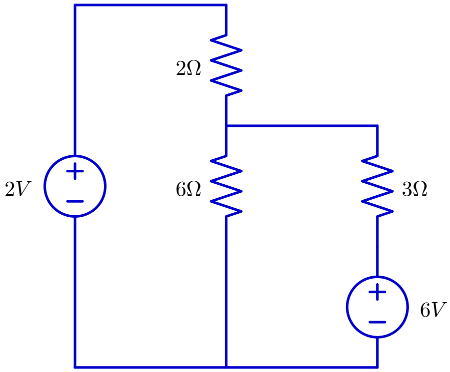
Another Example



Version 1.0

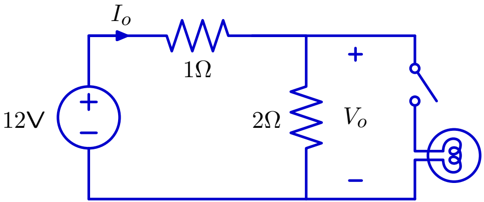
Combining earlier ideas, we can develop a process by which we can solve for *all currents and potentials* in a circuit.

1. Pick a node to be our reference node. All other node potentials will be measured with respect to this node.
2. Look for a constitutive equation with exactly one unknown value. If such an equation exists, solve for the unknown value and GOTO 6.
3. Look for a KCL equation with exactly one unknown current. If such an equation exists, solve for the unknown current and GOTO 6.
4. If no equation with exactly one unknown, look for patterns that can simplify the circuit (series/parallel combinations, etc), and GOTO 2.
5. **Last Resort**: If no simplifications, write a small system of constitutive and KCL equations in terms of node potentials, and solve. GOTO 6.
6. If the circuit is completely solved, congratulations! If not, GOTO 2.

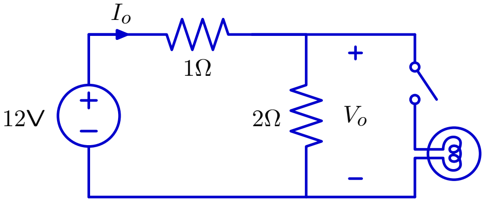
Interaction of Circuit Elements

Circuit design is complicated by interactions among the elements. Adding an element changes voltages and current **throughout** the circuit.

Example: closing a switch is equivalent to adding a new element.



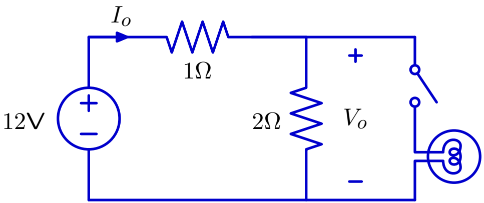
Check Yourself!



How does closing the switch affect V_o and I_o?

1. V_o decreases, I_o decreases
2. V_o decreases, I_o increases
3. V_o increases, I_o decreases
4. V_o increases, I_o increases
5. depends on bulb's resistance

Check Yourself!



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4. V_o increases, I_o increases
5. depends on bulb's resistance

Summary

This time:

- Defined the circuit abstraction ("through" and "across" variables)
- Developed systematic way of solving circuits
- Developed means of thinking about circuits through patterns (series/parallel) and abstractions (one-port)
- Noticed that the ways in which we think about abstraction and modularity in circuits is fundamentally different from the way we thought about these ideas in LTI and programming

Labs This Week:

- ~~*Software*~~ *Hardware Lab*: Dividers, Breadboarding
- *Design Lab*: Joystick-controlled robot!!!