Please WAIT until we tell you to begin.

During the exam, you may refer to any written or printed paper material. You may NOT use any electronic devices (including calculators, phones, etc).

If you have questions, please come to us at the front to ask them.

Enter all answers in the boxes provided.
Extra work may be taken into account when assigning partial credit.

For staff use:

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1 Real Differences (8 Points)

Consider a system $\mathcal{H}$, which transforms an input signal $X$ into an output signal $Y$:

$$X \rightarrow \mathcal{H} \rightarrow Y$$

Suppose the input signal is a unit sample:

$$x[n] = \delta[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

and for that input, the samples of the output response signal are:

$$y[n] = \begin{cases} (-2)^n & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

1.1 Block diagram

Which of the following block diagrams is an equivalent representation of the system $\mathcal{H}$?

(A) 

(B) 

(C) 

(D) 

Choose exactly one (A, B, C, or D): B
1.2 Composition

Suppose we have another system $\mathcal{F}$, which takes as input $Y$ and gives as output $Z$:

![Diagram]

You are given that $\mathcal{F}$'s system functional is:

$$\frac{Z}{Y} = \frac{1}{1 - \frac{1}{2}R}$$

Compose the systems $\mathcal{H}$ (from the previous parts) and $\mathcal{F}$ to produce a new system which takes $X$ as input, and outputs $Z$:

![Diagram]

Which of the block diagrams on the facing page (page 5) could be equivalent to this new system (assuming the gains were correctly chosen)?

Choose one or more of (1, 2, 3, 4, 5):

2, 4, 5

1.3 Pole Problems

Ben Bitdiddle has a different system with poles at 10, 1, and 0.1. For that system, he reasons that the unit sample response must be a constant value for all $n \geq 0$, since $10^n \cdot 0.1^n \cdot 1^n = 1$ for all $n$.

Is Ben’s reasoning correct? (Yes/No):

No

Briefly explain (1-2 sentences):

The unit sample response of a system will go like a sum of scaled geometric sequences, whose bases are the poles of the system:

$$y[n] \sim \sum \limits_i c_i (p_i)^n$$

So this system’s unit sample response will go more like $10^n + 0.1^n + 1^n$. This system’s response will be dominated by the pole at 10, and will go to infinity as $n$ goes to infinity.
2 ...Dear Liza, Dear Liza (12 Points)

In this question, we will explore “leaky accumulator” systems, whose defining equations are of the following form, where $k$ represents a “leak factor” (with $0 \leq k \leq 1$):

$$y[n] = k \cdot y[n - 1] + x[n]$$

Start by defining a function `make_leaky(k)`, which constructs a model of a system of the form described above, with a leak factor of $k$, started at rest.

You should make use of the `lti` framework from Software Lab 2, using some or all of `R`, `Gain`, `Cascade`, `FeedforwardAdd`, and `FeedbackAdd` (see attached reference) instead of creating an instance of the `System` class directly.

```python
def make_leaky(k):
    return FeedbackAdd(Gain(1),
                       Cascade(R(0), Gain(k)))
```

Define a function `cascade_leaks(leak_factors)`, which constructs a system corresponding to a cascade of leaky accumulators with the specified leak factors (provided as a list). The list of leak factors could be of any length $L \geq 0$. If the list has length 0, the resulting system’s output should be the same as its input.

```python
def cascade_leaks(leak_factors):
    out = Gain(1)
    for k in leak_factors:
        out = Cascade(out, make_leaky(k))
    return out
```
Henry is interested in figuring out the dominant pole of a cascade of leaky accumulators, and does it by calling our procedure to make an instance of the System class, and then calling the dominant_pole method. Eliza says there’s a much more efficient way to figure that out, without even making any instances of System, just by looking at the list of leak rates.

Write a short Python procedure that finds the dominant pole without constructing an instance of System (either directly, via the functions in the previous sections, or through the R, Gain, etc. subclasses):

```python
def leaky_dominant_pole(leak_factors):
    if len(leak_factors) == 0:
        return None
    return max(leak_factors)
```
3 All Systems Go! (12 Points)

Below, you are given a piece of the unit sample response of several different LTI systems, when started from rest. For each, clearly mark with an × the location(s) of that system’s pole(s) in the complex plane.

3.1 System 1

Unit Sample Response

Poles

3.2 System 2

Unit Sample Response

Poles

3.3 System 3

Unit Sample Response

Poles
3.4 System 4

Unit Sample Response

Poles

3.5 System 5

Unit Sample Response

Poles

3.6 System 6

Unit Sample Response

Poles
4 Systemception (10 Points)

The following system contains an LTI subsystem $S$.

If $k = 2$, then

$$H = \frac{Y}{X} = \frac{6R^2}{1 - R + 4R^2}$$

In the box below, draw a block diagram representing the system $S$, using only finitely many delays, gains, and adders:

One possible solution:
5 Solving Circuits (12 Points)

Solve for the specified values in each of the circuits below:

5.1 Circuit 1

\[ V_1 = 8V \quad V_2 = 2V \]

5.2 Circuit 2

\[ I_1 = 1A \quad I_2 = \frac{1}{4}A \]
5.3 Circuit 3

\[ V_1 = 2.5V \quad V_2 = 1V \]