Lecture 8: Circuit Abstractions

Lecturer: Adam Hartz (hartz@mit.edu)

As you come in...

- Grab one handout (on the table by the entrance)
- If you plan on using a laptop or smartphone during lecture, please sit near the back.
Modularity and Abstraction

One central theme of 6.01 is the idea of controlling complexity when designing large systems through modularity and abstraction. Modularity and abstraction are essential in the design on complex systems.
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**Python:**
- Primitives: +, *, 7, ’cat’
- Combinations: if, while, list, dict
- Abstractions: def, class
Modularity and Abstraction

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**Python:**
- Primitives: +, *, 7, 'cat'
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**Signals and Systems:**
- Primitives: $k$, $R$
- Combinations: Adders, Cascade, Feedback
- Abstractions: System Functionals, Poles
Modularity and Abstraction in Circuits?

**Circuits** represent systems as connections of elements.

**Currents** flow through elements, and

**Voltages** develop across elements.

We would like to have a modular way of working with circuits. In ideal case, elements would be complex abstractions.

The complication is that **all** of the elements interact with each other.
Buffers can be used to increase modularity:

The voltage divider on the left acts as a module that produces $8\text{V}$, regardless of loading of bulb on the right.

![Diagram of voltage divider and op-amp circuit](image-url)
Buffering with Op-Amps

Buffers can be used to increase modularity:

The voltage divider on the left acts as a module that produces 8V, regardless of loading of bulb on the right.

Op-Amp topologies can also be viewed as modular pieces. For example, non-inverting amplifier:
Buffering Is Not Always An Option

All elements in a circuit model of a neuron interact with each other:

![Diagram of neuron model]

Fortunately, there are other useful ways to deal with interactions.
Check Yourself

Imagine you are given a black box with one of these two circuits inside, but with the terminals exposed. How can you tell which one of the circuits is inside it?
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How can you tell which one of the circuits is inside it?

![Circuit Diagram]

1. 10V
2. 2Ω
3. 5A
4. 2Ω
One-Ports

If a circuit connects to “the world” via just two terminals, then that circuit can be represented by a single generalized element (regardless of how many components are in the circuit).

\[ I \downarrow \quad + \quad \]
\[ \quad \quad V \downarrow \quad - \]

This is analogous to

- replacing delays, gains, adders with system functional
- combining a sequence of operations in a procedure call
- combining diverse data in a list
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\[ I \quad + \quad V \]

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The one-port is characterized by how it constrains the current through it and the voltage across it: its "I-V curve".
In this circuit, changing an element changes voltages and currents systematically. Consider the changes in $V_o$ and $I_o$ when $R_o$ is changed:

$R_o$ (Ω)  $V_o$ (V)  $I_o$ (A)
Patterns

In this circuit, changing an element changes voltages and currents systematically. Consider the changes in $V_o$ and $I_o$ when $R_o$ is changed:

<table>
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- $R_o (\Omega)$ | $V_o (V)$ | $I_o (A)$
- 0    | 0    | 30
- 1    | 20   | 20
- 2    | 30   | 15
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![Circuit Diagram]

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R_o \text{ (Ω)} & V_o \text{ (V)} & I_o \text{ (A)} \\
0 & 0 & 30 \\
1 & 20 & 20 \\
2 & 30 & 15 \\
3 & 36 & 12 \\
4 & 40 & 10 \\
\end{array}
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\begin{array}{|c|c|c|}
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0 & 0 & 30 \\
1 & 20 & 20 \\
2 & 30 & 15 \\
3 & 36 & 12 \\
4 & 40 & 10 \\
\infty & 60 & 0 \\
\hline
\end{array}
$$
Check Yourself!

Why is the relationship between $V_o$ and $I_o$ linear?

1. Ohm’s law for $R_o$ is a linear relation between $I_o$ and $V_o$.
2. The parallel combination of $R_o$ and $6\Omega$ is linear in $R_o$.
3. The equations for the three left-most components are linear.
4. The voltage divider formed by $3\Omega$ and $6\Omega$ resistors is linear.
5. None of the above.
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Current-Voltage Relationships

The straight line is a property of the boxed part of the circuit. In fact, the same relation holds for any circuit outside the box!

\[
\begin{array}{c}
\text{\small 90V} \\
\text{\small 3Ω} \\
\text{\small 6Ω} \\
\hline
\text{\small I}_o
\end{array}
\quad
\begin{array}{c}
\text{\small V}_o \\
\text{\small R}_o
\end{array}
\quad
\begin{array}{c}
\text{\small 3Ω} \\
\text{\small 6Ω} \\
\hline
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Current-Voltage Relationships

The I-V relation summarized all possible behaviors of the circuit, regardless of what is connected to it.

We can think about an entire circuit as a single one-port:
One-Ports

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These representations are **compositional**: replace multiple elements with a single element that can be used in the same way primitives are used.
Current-Voltage Relationships

How does knowing that the I-V curve is a straight line influence our thinking about abstraction in circuits?
Current-Voltage Relationships

I-V relations for primitives (resistors, sources) are linear.

\[ V = R_0 I \]

\[ V = V_0 \]

\[ I = I_0 \]
Check Yourself!

What is the corresponding I-V relation?

1. I  
   V
2. I  
   V
3. I  
   V
4. I  
   V
What is the corresponding I-V relation? 1
Series One-Ports

If the I-V curves for two one-ports are both straight lines, then the I-V curve of the series combination is also a straight line.

\[ V_s = V_1 + V_2 \]

The "horizontal sum" of two straight lines is a straight line.
Parallel One-Ports

If the I-V curves for two one-ports are both straight lines, then the I-V curve of the parallel combination is also a straight line.

The “sum” of two straight lines is a straight line.
Linear I-V Curves

In fact, in a circuit made of only linear components, the I-V curve for any two terminals in the circuit will be linear!
Thévenin Equivalents

If the current-voltage relation is linear, the current-voltage relation can be replicated by a voltage source in series with a resistor.
What is the slope of this I-V curve?

1. $R$
2. $1/R$
3. $V_0/R$
4. $R/V_0$
5. None of the above
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Thévenin Equivalents

If the current-voltage relation is linear, the current-voltage relation can be replicated by a voltage source in series with a resistor.

\[
I = \frac{V - V_0}{R}
\]

If \( I = 0 \), then \( V = V_0 \) (the "V-intercept"

The rate of growth of \( I = \frac{V}{R} - \frac{V_0}{R} \) with \( V \) is the slope \( \frac{1}{R} \).
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Linear Relationship

Goal: Find I-V curve of a subcircuit so that we can replace it with a Thévenin (without affecting other values in the circuit) to simplify analysis. (similar to series/parallel substitutions)

Because we know that the I-V curve of any circuit we can construct with resistors and independent sources is linear, it can be fully characterized by:

- two points, or
- a point and a slope

Intercepts are specified by the source in the equivalent, and the slope is specified by the resistor.
Intercepts

First, find the “V-intercept”: voltage when current is 0 ("open-circuit voltage")

\[ V_0 = I_0 \times R \]

\[ V = V_0 = 1 \text{ V} \]
Intercepts

First, find the “$V$-intercept”: voltage when current is 0 (“open-circuit voltage”)

Set $I = 0$ in the circuit. Then $V = V_0 = 1\text{V}$. 
Next, find the “I-intercept”: current when voltage is 0 (“short-circuit current”)
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Set $V = 0$ in the circuit. Then $I = I_0 = -\frac{1}{2}A$. 
The equivalent resistance is $R = -\frac{V_0}{I_0}$.
Intercepts

The equivalent resistance is \( R = -\frac{V_0}{I_0} \)

Then \( R = \frac{V_0}{-I_0} = \frac{1V}{0.5A} = 2\Omega \).
Equivalent circuits

These circuits are equivalent (where $R = 2\Omega$ and $V_0 = 1V$):

\[
\begin{align*}
I & \quad V \\
I_0 & \quad V_0
\end{align*}
\]

\[
\begin{align*}
I & \quad V \\
\frac{1}{R} & \quad V_0
\end{align*}
\]

\[
\begin{align*}
I & \quad V \\
\frac{1}{R} & \quad V_0
\end{align*}
\]
Check Yourself!

Find the Thévenin equivalent of this circuit:
Check Yourself!

Find the Thévenin equivalent of this circuit:

Open-circuit Voltage:
\[ V_0 = 10V \left( \frac{3\Omega}{3\Omega+1\Omega} \right) = 7.5V \]
Check Yourself!

Find the Thévenin equivalent of this circuit:

Open-circuit Voltage:
\[ V_0 = 10V \left( \frac{3\Omega}{3\Omega+1\Omega} \right) = 7.5V \]

Short-circuit Current:
\[ I_0 = -\frac{10V}{1\Omega} = -10A \]
Check Yourself!

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Short-circuit Current:
\[ I_0 = -\frac{10V}{1\Omega} = -10A \]

Equivalent Resistance:
\[ R = -\frac{V_0}{I_0} = \frac{7.5V}{10A} = 0.75\Omega \]
Check Yourself!

These circuits are “equivalent” in the sense that their I-V curves (and, thus, their terminal behavior) are the same:
Check Yourself!

Consider doubling the voltage of the source in the previous circuit from 10V to 20V. How do the Thévenin voltage and resistance of the equivalent circuit change?

0. $V_T$ increases, $R_T$ increases
1. $V_T$ increases, $R_T$ stays the same
2. $V_T$ increases, $R_T$ decreases
3. $V_T$ decreases, $R_T$ increases
4. $V_T$ decreases, $R_T$ stays the same
5. $V_T$ decreases, $R_T$ decreases
Check Yourself!

Consider doubling the voltage of the source in the previous circuit from $10V$ to $20V$. How do the Thévenin voltage and resistance of the equivalent circuit change?

0. $V_T$ increases, $R_T$ increases
1. $V_T$ increases, $R_T$ stays the same
2. $V_T$ increases, $R_T$ decreases
3. $V_T$ decreases, $R_T$ increases
4. $V_T$ decreases, $R_T$ stays the same
5. $V_T$ decreases, $R_T$ decreases
Alternate Method

The Thévenin resistance *does not depend on the source values!* So that resistance would be the same if the sources all had value 50, or 100, or 0!

**Alternate approach:** Find $V_T$ as before, then *set all source values to zero* and find the equivalent resistance between the two terminals.

This can provide an alternative way of thinking about circuits and their models.
Alternate Method: Example

\[ V = I(7\Omega + 1\Omega) + 2A \cdot 3V \]

\[ V = I \cdot R_T + V_T \]
Alternate Alternate Method

![Diagram](image)
Conceptual Value of Equivalent Circuits

Equivalent circuits have conceptual value. Example: Will closing the switch increase or decrease $I$?

We could just solve two circuits questions (one with switch open and one with switch closed) and compare $I$ values.

But this question can be answered without doing any calculations!
Conceptual Value of Equivalent Circuits

Replace the parts to the left and right of $I$ with equivalents:

![Equivalent Circuit Diagram]

Closing the switch decreases equivalent resistance to the right of $I$. Therefore, closing the switch increases $I$. 
Conceptual Value of Equivalent Circuits

From homework: pick $R_1$ and $R_2$ such that $V_o = 2.5$ Volts

This is a difficult question, made much easier by replacing part of the circuit with its Thévenin equivalent.
Conceptual Value of Equivalent Circuits

Example: solving for $I$ in the following circuit
Conceptual Value of Equivalent Circuits

Strategy: find Thévenin equivalent for red components, viewed through the port labeled $n_+$ and $n_-$
Conceptual Value of Equivalent Circuits

Strategy: find Thévenin equivalent for red components, viewed through the port labeled $n_+$ and $n_-$
Check Yourself!

What is the open-circuit voltage?

0. 10V  
1. 8V  
2. 5V  
3. 3V  
4. 0V  
5. something else
Check Yourself!

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Check Yourself!

What is the Thévenin resistance?

1. \((2\Omega \parallel 2\Omega) + (1\Omega \parallel 4\Omega)\)
2. \((2\Omega + 2\Omega) \parallel (1\Omega + 4\Omega)\)
3. \((2\Omega \parallel 1\Omega) + (2\Omega \parallel 4\Omega)\)
4. \((2\Omega + 1\Omega) \parallel (2\Omega + 4\Omega)\)
5. something else
Check Yourself!

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4. \((2\Omega + 1\Omega) \ || \ (2\Omega + 4\Omega)\)
5. something else
Conceptual Value of Equivalent Circuits

Can be used to simplify complicated circuits iteratively.

\[ V_a = -\frac{3}{2} (V_1 + 2V_2 + 2V_3 + 2V_4) \]

\[ V_a = -V_1 - V_2 - V_3 - V_4 \]
Conceptual Value of Equivalent Circuits

Can be used to simplify complicated circuits iteratively.

\[
V_a = -\frac{3}{2} \left( V_1^3 + 2 \frac{3}{2} \left( V_2^2 + V_3 + V_4^8 \right) \right)
\]
Conceptual Value of Equivalent Circuits

Can be used to simplify complicated circuits iteratively.

\[ V_a = -V_1 \cdot \frac{2}{2} \left( V_2 + 2V_3 + \frac{V_4}{2} \right) = -V_1 - V_2 - \frac{V_3}{4} - \frac{V_4}{8} \]
Conceptual Value of Equivalent Circuits

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\[ V_a = -\frac{1}{2}(V_1 + 2V_2 + V_3 + 4V_4) = -V_1 - V_2 - \frac{1}{2}V_3 - 2V_4 \]
Conceptual Value of Equivalent Circuits

Can be used to simplify complicated circuits iteratively.

\[ V_a = \frac{1}{2} V_1 + \frac{1}{4} V_2 + \frac{1}{4} V_3 + \frac{1}{8} V_4 \]
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\[ V_a = -\frac{3}{2}(V_1 \frac{1}{3} + 2 \frac{3}{4}(V_2 \frac{2}{4} + V_3 \frac{4}{8} + V_4 \frac{8}{8})) \]
Conceptual Value of Equivalent Circuits

Can be used to simplify complicated circuits iteratively.

\[ V_a = -\frac{2V_1}{2} - \frac{V_2}{4} - \frac{V_3}{4} - \frac{V_4}{8} \]
Conceptual Value of Equivalent Circuits

Can be used to simplify complicated circuits iteratively.

\[
V_a = -\frac{V_1}{3} - \frac{2}{3} \left( \frac{V_2}{2} + \frac{V_3}{4} + \frac{V_4}{8} \right)
\]
Conceptual Value of Equivalent Circuits

Can be used to simplify complicated circuits iteratively.

\[ V_a = -3 \left( \frac{V_1}{3} + \frac{2}{3} \left( \frac{V_2}{2} + \frac{V_3}{4} + \frac{V_4}{8} \right) \right) = -\frac{V_1}{2} - \frac{V_2}{2} - \frac{V_3}{4} - \frac{V_4}{8} \]
Conceptual Value of Equivalent Circuits

Potentially useful as a model of things other than circuits. Example: Designing a safe EKG

A Thévenin equivalent of me could be useful.

Norton Equivalents

If the current-voltage relation is linear, the current-voltage relation can also be replicated by a current source in parallel with a resistor.

\[ V = (I - I_0)R \]

If \( V = 0 \), then \( I = I_0 \) (the "I-intercept")

The rate of growth of \( I = I_0 + \frac{V}{R} \) with \( V \) is the slope \( \frac{1}{R} \)
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If the current-voltage relation is linear, the current-voltage relation can also be replicated by a current source in parallel with a resistor.

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\begin{align*}
I &= I_0 + \frac{V}{R} \\
V &= (I - I_0)R
\end{align*}
\]

From the circuit, \( V = (I - I_0)R \)
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Equivalent Circuits

\[ I = \frac{1}{2} A \]

\[ V = 1 V \]
Consequences of linearity: Equivalent Circuits

Any circuit comprised of only sources and resistors viewed from any two terminals has a linear I-V relationship, and can be represented by a Thévenin or Norton equivalent circuit.

Multiple ways to find the equivalent circuit:

- Find two points (open-circuit voltage $V_T$ and short-circuit current $I_N$ are usually good) and extrapolate from there.
- Find one point and a slope (by turning off all sources and finding equivalent resistance between the two terminals). Only works if all sources are independent!
- Hold I or V constant, and solve for the other.
Op-Amp Models

We have looked at several models of op-amps! At least two: Ideal and VCVS.

The ideal op-amp assumption told us something about terminal behavior when the op-amp is connected in negative feedback.

The VCVS model told us something about causality (the output voltage is set, which then affects the voltage on the − terminal).
Going Further

Last week, in order to understand why the op-amp needed to be connected in negative feedback, we built a more complicated model that accounted for temporal dynamics of the op-amp:

\[
\begin{align*}
V_+ &- K(V_+ - V_-) \\
V_- &
\end{align*}
\]

These are all powerful abstractions that allow us to ignore internal details and focus on end-to-end behavior, but we can also understand the op-amp on a deeper level if we need to.
Op-Amps as Abstractions

We have treated op-amps as fundamental building blocks of circuits (which they are).

\[ V_+ - V_- = K(V_+ - V_-) \]

But op-amps are in fact complicated circuits that contain dozens of transistors and resistors (plus a few capacitors).
Op-Amps as Abstractions

Here is the circuit for a $\mu A709$ op-amp:
Op-Amps as Abstractions

The op-amp is itself an abstraction! Details about its internal behavior are ignored, to focus exclusively on terminal behavior:

$$V_o = K(V_+ - V_-)$$

just three terminals ($V_+$, $V_-$, $V_o$) described by two equations rather than 15 transistors, 15 resistors, and a capacitor.
Op-Amps as Abstractions

Op-amps are made from transistors, which constitute a separate level of abstraction.
Look more closely at a simpler op-amp:
Common Transistor Patterns

A *current mirror* sets its output current to equal its input current.

\[ I_1 = I_2 \]

\[ V_0 = 0.6 \text{V} \]

It can be represented by a current-controlled current source:
A pair of transistors can be used to split a current.

\[ \alpha I_i (1 - \alpha) I_i \]

\[ V_+ + V_- \]

\[ I_i \]

The fraction \( \alpha \) is proportional to \( e^{(V_+ - V_-)/v_T} \), where \( v_T \approx 26mV \).

This “differential amplifier” can be represented by two voltage-controlled current sources.
Modeling an Op-Amp

Output current accumulates in capacitor to make output voltage.
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\[ V_o = I_i (1 - \alpha) + \alpha I_i \]

Output current accumulates in capacitor to make output voltage.
Modeling an Op-Amp

\[ V_+ \rightarrow +V \rightarrow \text{Op-Amp} \rightarrow V_- \rightarrow V_o \]

\[ i \rightarrow \alpha I_i \left( 1 - \alpha \right) I_i \rightarrow \text{Op-Amp} \rightarrow \alpha I_i \rightarrow V_o \]

Output current accumulates in capacitor to make output voltage.
Modeling an Op-Amp

\[ V_o = \alpha I_i (1 - \alpha) I_i \]

Output current accumulates in capacitor to make output voltage.
Output *current* accumulates in capacitor to make output voltage.
Circuits can be analyzed at multiple levels of abstraction

<table>
<thead>
<tr>
<th>Primitives</th>
<th>Transistor Level</th>
<th>Op-Amp Level</th>
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</thead>
<tbody>
<tr>
<td>R, C, transistor</td>
<td>R, C, Op-Amp</td>
<td></td>
</tr>
<tr>
<td>Combinations</td>
<td>wires (KVL,KCL)</td>
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</tr>
<tr>
<td>Abstractions</td>
<td>Op-Amp!</td>
<td>Dividers, Summer, Subtractor</td>
</tr>
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</table>
Op-Amp Model

Here is a more accurate circuit model of a \( \mu A709 \) op-amp:
Op-Amp: Physical Structure

This artwork shows the physical structure of a $\mu A709$ op-amp:
Modern semiconductor electronics are microfabricated using optical lithography to define the parts and connections.

A two-input logic gate (CMOS NAND) is shown schematically (left) and as physical layers defined using photolithography (right).

The metal traces make connections between transistors formed in the overlapping non-metal regions.
Circuits can be analyzed at multiple levels of abstraction

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<th>Abstractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semiconductors</td>
<td>silicon, dopants</td>
<td>R, C, transistors</td>
</tr>
<tr>
<td>Transistors</td>
<td>R, C, transistors</td>
<td>Op-Amps</td>
</tr>
<tr>
<td>Op-Amps</td>
<td>R, C, Op-Amp</td>
<td>Adders, Subtractors, etc</td>
</tr>
</tbody>
</table>

Abstraction^3!

This hierarchy is quite powerful. Thinking about transistors and op-amps as units allows the construction of circuits with billions of parts.
Abstractions at one level become primitives at the next.

Allows for construction of circuits with billions of parts.