6.01 Midterm 1  Fall 2017

Name:  Answers  Section Number:
Kerberos (Athena) name:

<table>
<thead>
<tr>
<th>Section</th>
<th>Design Lab Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:</td>
<td>Thursday 9:30am</td>
</tr>
<tr>
<td>2:</td>
<td>Thursday 2:00pm</td>
</tr>
</tbody>
</table>

Please WAIT until we tell you to begin.

During the exam, you may refer to any written or printed paper material. You may NOT use any electronic devices (including calculators, phones, etc).

If you have questions, please come to us at the front to ask them.

**Enter all answers in the boxes provided.**
Extra work may be taken into account when assigning partial credit, but only work on pages with QR codes will be considered.

**Question 1:** 20 Points
**Question 2:** 23 Points
**Question 3:** 20 Points
**Question 4:** 22 Points
**Question 5:** 23 Points
**Total:** 108 Points
1 The Turning Po(in)t (20 Points)

In Design Lab 4, we connected a potentiometer as a voltage divider. Consider the following circuit, where $R_p = 5k\Omega$.

In this configuration, $V_o$ and $\alpha$ are related as shown in the graph on the right.

For each of the following configurations, sketch the relationship between $V_o$ and $\alpha$ in that circuit. The original curve has been reproduced in gray on each graph, and can be used as a reference.

1.1 Configuration 1
1.2 Configuration 2

\[ V_o = \alpha R_p \]

\[ (1 - \alpha)R_p \]

10V

\[ 2V \]

1.3 Configuration 3

\[ V_o = \alpha R_p \]

\[ (1 - \alpha)R_p \]

10kΩ

10kΩ

\[ 10V \]

\[ 10V \]
1.4 Configuration 4

\[ (1 - \alpha)R_p \]

\[ \alpha R_p \]

\[ V_o \]

1.5 Configuration 5

\[ (1 - \alpha)R_p \]

\[ \alpha R_p \]

\[ V_o \]

\[ 0.5\text{mA} \]
2 Cart Blanche (23 Points)

In this problem, we will consider a small cart on a flat surface:

\[ F \rightarrow \square \rightarrow \bigcirc \]

It is possible to move the cart along the surface by applying a force \( F \) to the back of the cart. The cart has a mass \( m \) (kilograms), and its acceleration \( a \) is governed by Newton’s Second Law of Motion:

\[ F = ma \]

We will start by building a discrete-time model of this system. Complete the sketch of the block diagram of this system below, using only finitely many gains, delays, and/or adders/subtractors. Assume that the cart’s acceleration is set at the start of each timestep based on the current force on the cart, and that the cart maintains that acceleration for the full timestep. Let the signals \( F, A, \) and \( V \) represent the force applied to the cart in Newtons, the cart’s acceleration in meters/sec\(^2\), and the cart’s velocity in meters/sec, respectively, and let \( T \) be the length of one timestep in seconds.
2.1 Friction

We can model friction in the system by adding a feedback loop as shown below. Here, the net force acting on the cart is the sum of an input force \( F_i \) and a force from friction, which is proportional to the cart’s velocity (assume that the constant of proportionality \( k_f \) is nonnegative).

\[
\begin{align*}
F & \rightarrow A \\
F_i & \rightarrow A \\
& \rightarrow \text{Friction} \\
& \rightarrow V \\
& \rightarrow F \\
& \rightarrow V
\end{align*}
\]

What are the units of \( k_f \)?

\[
\frac{N}{m/s} = \frac{kg}{s}
\]

Solving, we find that the system functional of this entire system is:

\[
\frac{V}{F_i} = \frac{TR}{m - mR + k_fTR}
\]

How many poles does this system have? 1

What are the poles of this system? \( 1 - \frac{k_fT}{m} \)

For what values of \( k_f \), if any, is the system stable? \( k_f \leq \frac{2m}{T} \)

For what values of \( k_f \), if any, is the system’s unit sample response oscillatory? \( k_f > \frac{m}{T} \)

2.1.1 Slowing Down

Assume that at time 0, a cart with a mass of 2kg is moving at a velocity of 20m/s. If the length of a timestep is \( T = 0.1s \) and we are running in a world where \( k_f = 8 \), approximately how long will it take for the cart to decelerate to 0.2m/s if no force other than friction is applied?

You do not need to simplify your answer completely; your answer may include numbers, multiplication, \( n \), sin, cos, tan, log, and/or \( \sqrt{\cdot} \).

\[
\frac{\log_{0.6} 0.01}{10}
\]

seconds
2.2 Position

Finally, we would like to expand our model to include the cart’s position \( P \) (meters). Assume that the cart’s velocity is set at the beginning of each timestep, and that it maintains a constant velocity for the entire timestep. Complete the block diagram of this system below, using finitely-many gains, delays, and/or adders/subtractors:

![Block diagram of the system](image)

How many poles does this system \( \left( \frac{P}{R_i} \right) \) have? 2

What are the poles of this system \( \left( \frac{P}{R_i} \right) \)?

\[ 1, 1 - \frac{k_f T}{m} \]

For what values of \( k_f \), if any, is the system stable?

\[ k_f \leq \frac{2m}{T} \]

For what values of \( k_f \), if any, is the system’s unit sample response oscillatory?

\[ k_f > \frac{m}{T} \]
### 2.3 Graphs

Assuming that the cart starts at rest and the external force is 1 Newton at time 0, and 0 Newtons on all other timesteps, which of the graphs on the facing page represents each of the following signals? Enter a single number in each box, or enter None if none of the graphs depict that signal. Note that the scales may be different for each of the graphs. Note that some of the graphs may be used more than once, and some may not be used at all.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Graph Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>The external force applied to the cart ($F_1$)</td>
<td>6</td>
</tr>
<tr>
<td>The net force on the cart ($F$)</td>
<td>11</td>
</tr>
<tr>
<td>The cart’s acceleration ($A$)</td>
<td>11</td>
</tr>
<tr>
<td>The cart’s velocity ($V$)</td>
<td>0</td>
</tr>
<tr>
<td>The cart’s position ($P$)</td>
<td>1</td>
</tr>
<tr>
<td>The friction coefficient ($k_f$)</td>
<td>None</td>
</tr>
</tbody>
</table>
3 Op-Amp (20 Points)

Consider the following circuit:

Solve for the ratio $V_o/V_i$ under each of the conditions below. Make the ideal op-amp assumption and ignore output limitations of the op-amp.

1. When $R_1 \to \infty$, $R_3 \to \infty$, and $R_2$ and $R_4$ are finite and nonzero, what is the ratio $V_o/V_i$?

   \[
   \frac{V_o}{V_i} = 1
   \]

2. When $R_3 \to \infty$, $R_4 = 0$, and $R_1$ and $R_2$ are finite and nonzero, what is the ratio $V_o/V_i$?

   \[
   \frac{V_o}{V_i} = \frac{R_2 + R_1}{R_1}
   \]
3. When $R_1 \to \infty$ and $R_2$, $R_3$, and $R_4$ are finite and nonzero, what is the ratio $V_o/V_i$?

\[
\frac{V_o}{V_i} = \frac{R_4 + R_3}{R_3}
\]

4. When $R_3 \to \infty$ and $R_1$, $R_2$, and $R_4$ are finite and nonzero, what is the ratio $V_o/V_i$?

\[
\frac{V_o}{V_i} = \frac{R_2 + R_4 + R_1}{R_1}
\]

5. When $R_2 = 0$ and $R_1$, $R_3$, and $R_4$ are finite and nonzero, what is the ratio $V_o/V_i$?

\[
\frac{V_o}{V_i} = \frac{R_4 + R_3||R_1}{R_3||R_1}
\]

6. In the general case when all four resistances are finite and nonzero, what is the ratio $V_o/V_i$?

\[
\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1} + \frac{R_4}{R_1} + \frac{R_4}{R_3} + \frac{R_2R_4}{R_1R_3}
\]
4 System Design (22 Points)

Consider an LTI system whose output, in response to a unit sample input, is given by:

\[ y[n] = \begin{cases} 
0 & \text{if } n < 0 \\
9 & \text{if } n \geq 0 \text{ and } n \text{ is even} \\
4 & \text{if } n \geq 0 \text{ and } n \text{ is odd} 
\end{cases} \]

This relationship is shown in the graph below:

4.1 From Rest

Is it possible to design an LTI system whose unit sample response is 9, 4, 9, 4, 9, 4, ... when started from rest? If so, draw a block diagram for such a system in the box below, using only finitely many delays, gains, and/or adders/subtractors and enter the system’s poles in the box below. If not, explain briefly (1-2 sentences) why this is not possible, and enter None in the box for the poles.

What are the poles of this system? ±1
4.2 Initial Conditions

Consider the systems below, where $R(i)$ represents a delay ($\mathcal{R}$) element initialized so that its first output is $i$. Is it possible to set the initial conditions on the delay boxes such that these systems will produce the sequence 9, 4, 9, 4, 9, 4, ... in response to a unit sample input? If so, enter the appropriate values for the initial conditions of the delays. If not, enter None in each box.

![Diagram of the first system](image1)

$$a = 8 \quad b = 4$$

![Diagram of the second system](image2)

$$c = -4 \quad d = 4$$
5 Let There Be Light (23 Points)

In the manila folder at your table, you will find a circuit and a multimeter. This circuit is designed to be a brightness controller for a light. Similar to streetlights, we want our circuit to turn a light on when there is little ambient light and turn it off when there is a lot of ambient light.

The circuit is designed to have two stages:

- a voltage divider that produces a voltage that goes up as ambient light goes down, and
- an amplifier, designed to scale this voltage up by a factor of 11.

and the voltage output from the amplifier is applied across an LED similar to those we used in Design Lab 5.

The circuit you have been given was intended to have these properties. However, it was not properly assembled. This problem will have three parts: creating a schematic representing the circuit you were given, explaining what was wrong with that circuit, and fixing the circuit.

Background

The voltage divider uses a device called a photoresistor, whose resistance depends on the amount of light falling on it. Its resistance in ambient light is around 5kΩ, and its resistance in darkness is around 1.5MΩ. The figure below shows both a picture of a photoresistor and a representation of a photoresistor in a schematic diagram.

For reference, there are four resistor values on the board, identified by their color codes:

- 3kΩ: Orange, Black, Red
- 10kΩ: Brown, Black, Orange
- 100kΩ: Brown, Black, Yellow
- 2.2MΩ: Red, Red, Green

And here is the pinout for the op-amp package on the board (the same op-amp package we have used throughout 6.01):
5.1 Schematic

In the box below, draw a schematic diagram representing **exactly the circuit that was on the board when it was given to you**. Clearly label all resistor values.

If you are having trouble identifying which resistor is which on the board, please don’t hesitate to ask a staff member for clarification.
5.2 What Is Wrong?

Connect your circuit to the power supply (which has been configured to provide 10V). When doing so, do not look directly into the LED. Using circuit theory and/or measurements from the board, determine what is wrong with this circuit with respect to the design goals from above (a voltage divider should produce a voltage that goes up as ambient light goes down, and an amplifier should scale that voltage up by a factor of 11).

Note that you can test the circuit’s behavior in darkness by covering the photoresistor with your hands.

You may assume that nothing is wrong with the "load" (the combination of the 3kΩ resistor and the LED on the right side of the board). You should not modify that part of the circuit.

Briefly explain what is wrong with this circuit given the design goals mentioned earlier, and what changes you would need to make to fix it. Justify your plan using circuit theory.

There are two issues with the circuit as constructed. Currently, decreasing light will result in the voltage from the op-amp going down (causing the light to dim), rather than going up. We can fix this by swapping the positions of the photoresistor and the 2.2MΩ resistor. Also, the 10kΩ and 100kΩ resistors are currently connected in parallel, so the op-amp circuit has a gain of 1. To make a gain of 11, we could reconfigure these so that the op-amp acts as a non-inverting amplifier.

5.3 Fix It!

Make modifications to the physical circuit so that it behaves as described above. When you are finished, carefully put your modified circuit back in the manila envelope.

Original (top) and fixed (bottom) circuits shown on following page. Arrows indicate changes that were made.